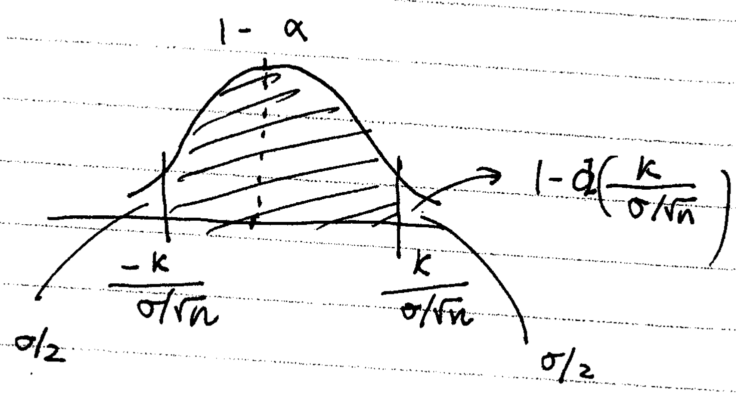
P1

$\boxed{9/25/2013, \text{ Wed}}$

~~Let $X_1, \ldots, X_n$ be iid $n(\mu, \sigma^2)$~~



· by symmetry

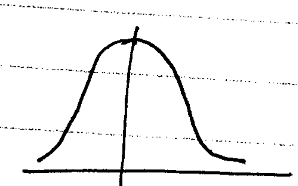
$\cdot\ 1-\Phi\left(\frac{k}{\sigma/\sqrt{n}}\right) = \alpha/2$

$\cdot\ \Phi\left(k/[\sigma/\sqrt{n}]\right) = 1 - \alpha/2$

$\frac{k}{\sigma/\sqrt{n}} = \Phi^{-1}(1-\alpha/2) \triangleq z_{\alpha/2}$

~~e.g. $\alpha = 0.$~~

<u>t distribution</u>



Student t-distribution

pdf $\quad p(x \mid \nu) = \dfrac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma(\nu/2)} \left(1 + \dfrac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$

α : typically 0.05, 0.01, 0.1

interval estimate for t-distribution:

$[\bar{x} - $

(P2)

$\bar{x} \sim N(\mu, \sigma^2/n)$

~~$P($ $<\bar{x}<$~~ $\quad$ α typically 0.05, 0.01, 0.1

$$P(\bar{x} - k < \mu < \bar{x} + k) = 1 - \alpha$$

$$P(-k < \mu - \bar{x} < k) = 1 - \alpha$$

or $P(-k < \bar{x} - \mu < k) = 1 - \alpha$ $\qquad \bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

$$P\left(\frac{-k}{\sigma/\sqrt{n}} < \underbrace{\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}_{Z} < \frac{k}{\sigma/\sqrt{n}}\right) = 1 - \alpha$$

$$\Rightarrow P\left(Z < \frac{k}{\sigma/\sqrt{n}}\right) - P\left(Z < \frac{-k}{\sigma/\sqrt{n}}\right) = 1 - \alpha$$

$$\Rightarrow \Phi\left(\frac{k}{\sigma/\sqrt{n}}\right) - \left(1 - \Phi\left(\frac{k}{\sigma/\sqrt{n}}\right)\right) = 1 - \alpha$$

$\Phi(-x) = 1 - \Phi(x)$

$$\Phi\left(k/(\sigma/\sqrt{n})\right) = 1 - \alpha/2 \qquad = \cancel{1-\alpha/2}\ \alpha/2$$

$$\Rightarrow \frac{k}{\sigma/\sqrt{n}} = z_{\alpha/2}$$

$$\Rightarrow k = \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$$

interval estimate : for μ $\left(\bar{x} - \frac{\sigma}{\sqrt{n}} z_{\alpha/2},\ \bar{x} + \frac{\sigma}{\sqrt{n}} z_{\alpha/2}\right)$

then the chance that ~~$\bar{x}$~~ μ belong to this interval is $1-\alpha$.

E.g. ~~$\alpha = 0.1$~~ $\qquad$ ~~$\mu =$~~ $\quad \sigma^2 = 0.1$, $n = 5$

~~$\alpha/2 = 0.05$~~ $\qquad$ ~~98.2~~

~~$\Phi(k/(\sigma/\sqrt{n})) = 1 - 0.05$~~ $= 0.95$

~~$k/(\sigma/\sqrt{n}) = 1.605.$~~

<u>Example</u>:

digital thermometer, take 6 measurements

$98.2,\ 98.6,\ 97.4,\ 98.2,\ 97.9,\ 98.9,\quad \sim N(\mu, \sigma^2)$

$n = 6$

$\sigma^2 = \cancel{0.1}\ 1$

$\alpha = 0.1$

$\alpha/2 = 0.05$

$\Phi\left(\frac{k}{\sigma/\sqrt{n}}\right) = 1 - 0.05 = 0.95$

$\frac{k}{\sigma/\sqrt{n}} = 1.605$ (use normal table)

$k = \frac{\sqrt{\cancel{0.1}\ 1}}{\sqrt{6}} 1.605 = \cancel{0.2072}\ \ 0.66\cancel{52}$

$\bar{x} = \frac{1}{6}\sum_{i=1}^{6} x_i = 98.2$

interval estimator for μ:

$$\mu \in [98.2 - \cancel{0.2072}\ \cancel{0.6552}\ 0.66,\ 98.2 + \cancel{0.6552}\ 0.66]$$

$$= [97.54,\ 98.86]$$

(with 95% chance,
true temperature is in this
interval)