Diversity-Multiplexing-Delay Tradeoff for Multihop MIMO Relay Networks with ARQ

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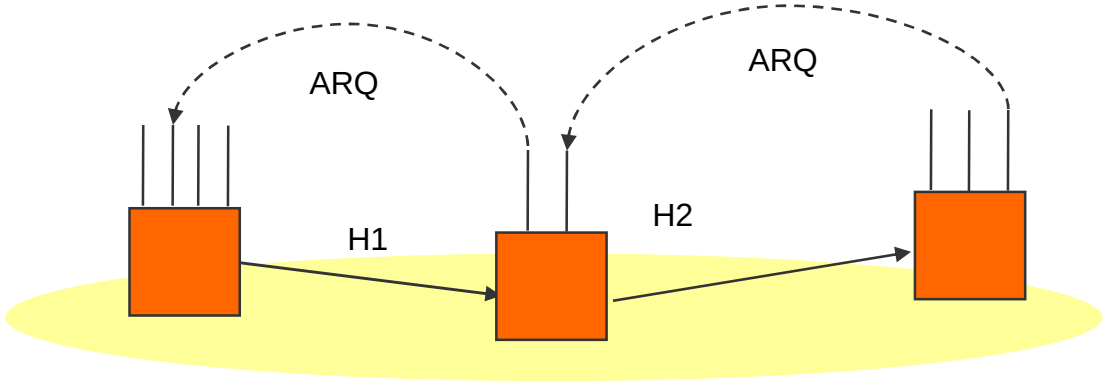
Outline

- Introduction
- Model
  - Multihop MIMO relay network
  - Multihop ARQ protocol
- Diversity-Multiplexing-Delay Tradeoff (DMDT) for Multihop MIMO Relay Networks with ARQ
- Conclusions and Future Work

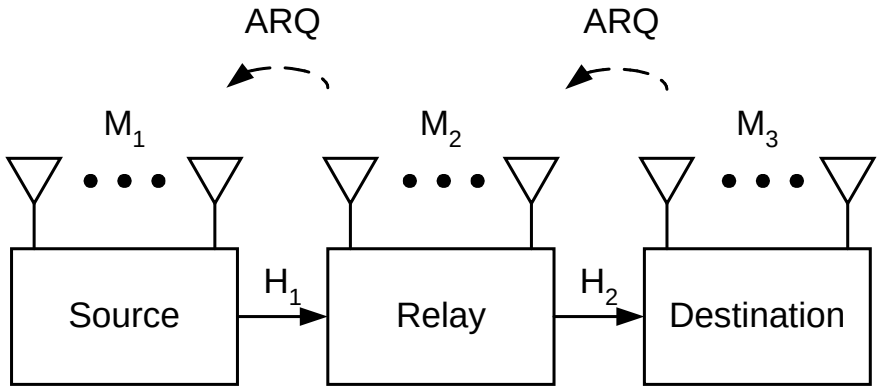
Introduction

- MIMO used to increase data rate or robustness
- Multihop relays used for coverage extension
- ARQ protocol: 1 bit feedback, or time diversity
- Diversity multiplexing (delay) tradeoff
  - Point-to-point MIMO *[Zheng&Tse 2004]*
  - Point-to-point MIMO with ARQ *[El Gamal&Caire&Damen 2006]*
  - Single antenna relay channel with ARQ *[Tabet&Dusad&Knopp2007]*
  - Multihop channel without ARQ *[Gunduz&Goldsmith&Poor 2009]*



Multihop MIMO Relay Network

- Relays: decode-and-forward

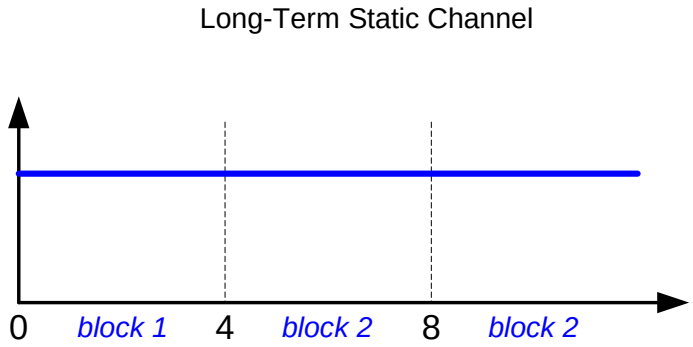


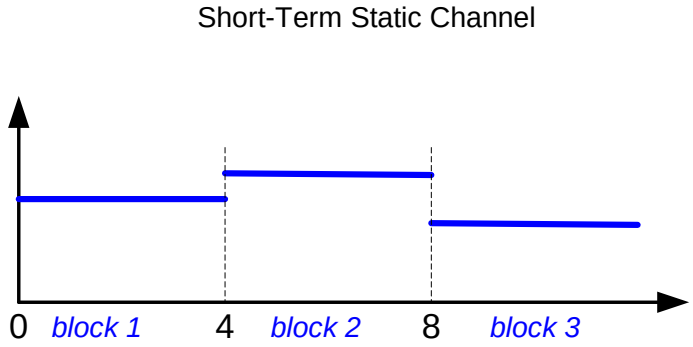
- Channel model

$$Y_{i,l} = \sqrt{\frac{SNR}{M_i}} H_{i,l} X_{i,l} + W_{i,l},$$

$$1 \le l \le L, i = 1,2$$

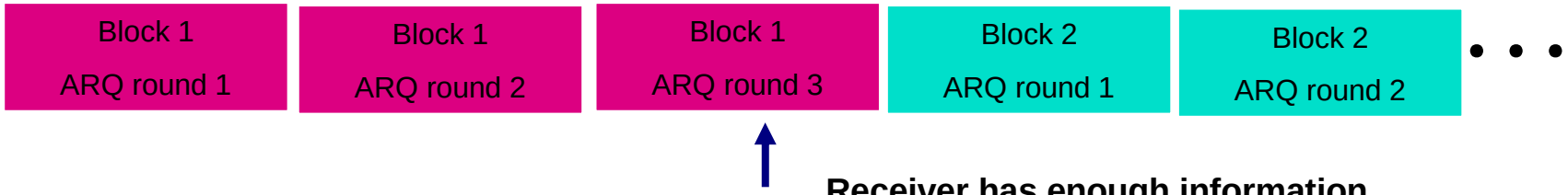
- Long-term/short term static channel





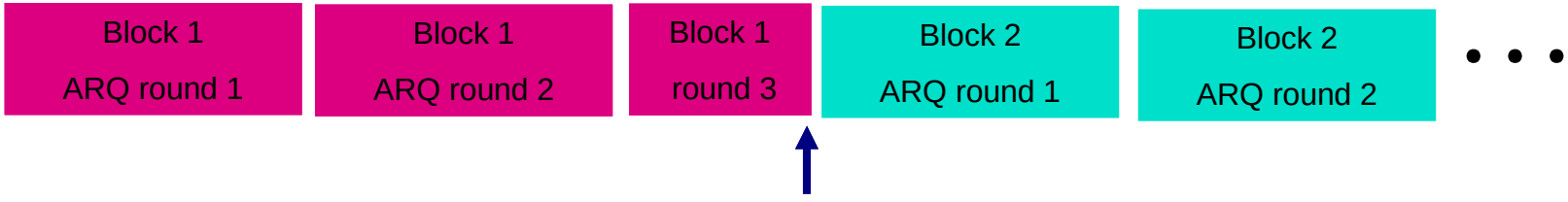
Multihop ARQ Protocols

- Fixed ARQ allocation
  - Maximum ARQ round for ith hop L_i $\quad \sum_{i=1}^{N} L_i \leq L$
- Adaptive ARQ allocation
  - Fixed Block Length (FBL) (easier for synchronization)



Receiver has enough information to decode at this moment

- Variable Block Length (VBL)



Receiver has enough information to decode at this moment

Diversity Multiplexing Delay Tradeoff

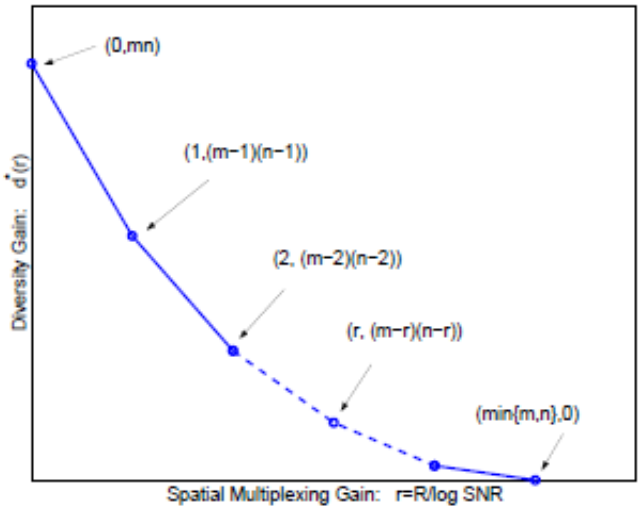
- Multiplexing rate r_e
  - Average throughput of ARQ η
  - Half duplex, using renewal theory:

$$\eta = \frac{R(SNR)}{2}, \quad r_e = \lim_{SNR\to\infty} \frac{\log \eta}{\log SNR} = \frac{r}{2}$$

- Diversity d
  - Exponent of information outage event

$$d = -\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR}$$

- Diversity Multiplexing Tradeoff [Zheng&Tse04]
  - Diversity and multiplexing rate are related by piece-wise linear function



DMDT: **Long Term** Static Channel

- Fixed ARQ Allocation

$$d_F(r_e, L_1, L) = \min\left\{ f_{M_1,M_2}\left(\frac{2r_e}{L_1}\right), f_{M_2,M_3}\left(\frac{2r_e}{L-L_1}\right)\right\}$$

Performance limited by the weakest link

- Adaptive FBL

$$d_{FBL}(r_e, L_1, L) = \min_{l=2,\cdots L-1}\left\{ f_{M_1,M_2}\left(\frac{2r_e}{L-1}\right), f_{M_1,M_2}\left(\frac{r_e}{\quad}\right) \quad \left(\frac{2r}{\quad}\right)\right\}$$

Optimal ARQ equalizes the link performance

- Adaptive VBL: close form solution in some special cases

$$d_{VBL}(r_e, L) = \inf_{(\alpha_1,\alpha_2)\in O} \sum_{i=1}^{2}\sum_{j=1}^{M_i^*}\left(2j-1+\left|M_i - M_{i+1}\right|\right)\alpha_{i,j}$$

$$O \triangleq \left\{ (\alpha_1,\alpha_2) \in \mathbb{R}^{M_1^*\times M_2^*} : \alpha_{i,1} \geq \cdots \geq \alpha_{i,M_i^*} \geq 0, \frac{2r_e}{L} > \frac{S_i(\quad}{S_1(} \right.$$

$$S_i(\alpha_i) \triangleq \sum_{j=1}^{M_i^*}\left(1-\alpha_{i,j}\right)^+$$

Adaptive ARQ do this optimization automatically

DMDT: **Short Term** Static Channel

- Adaptive ARQ: FBL

Gain by a factor due to time diversity of channel variation

$$d_{FBL}(r_e, L_1, L) = \min_{l=2,\cdots L-1} \left\{ \begin{array}{l} (L-1) f_{M_1,M_2}\left(\frac{2r_e}{L-1}\right), \\ (l-1) f_{M_1,M_2}\left(\frac{r_e}{l-1}\right) + (L-l) f_{M_2,M_3}\left(\frac{2r_e}{L-l}\right), \\ (L-1) f_{M_2,M_3}\left(\frac{2r_e}{L-1}\right) \end{array} \right\}$$

- A… cle…

Performance limited by the weakest link

$$d_{VBL}(r_e, L) = \inf_{(\alpha_{i,j})} \sum_{i=1}^{2} \sum_{j=1}^{M_i^*} \sum_{l=1}^{L} \left(2j - 1 + \left|M_i - M_{i+1}\right|\right) \alpha_{i,j}^{l}$$

Optimal ARQ equalizes the link performance

$$\mathrm{O} \triangleq \left\{ \sum_{l=1} S_2 \left(\alpha_2^{\lfloor t \rfloor} \right) < r \right\}$$

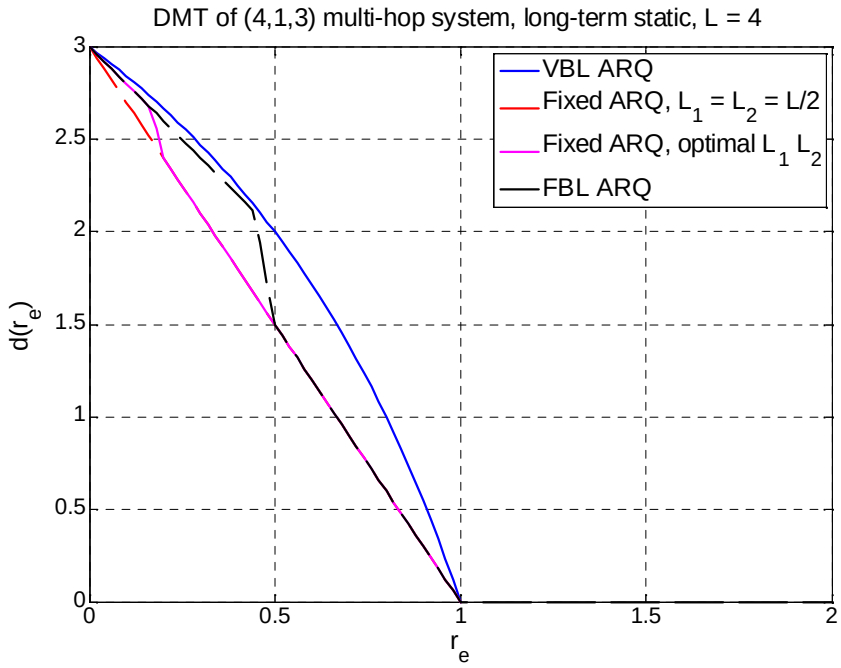
$$t_1 \triangleq \inf \left\{ t \in \mathbb{R} : \sum_{l=1}^{\lfloor t \rfloor} S_1\left(\alpha_1^{l}\right) + \left(t - \lfloor t \rfloor\right) S_1\left(\alpha_1^{\lfloor t \rfloor + 1}\right) = r \right\}$$

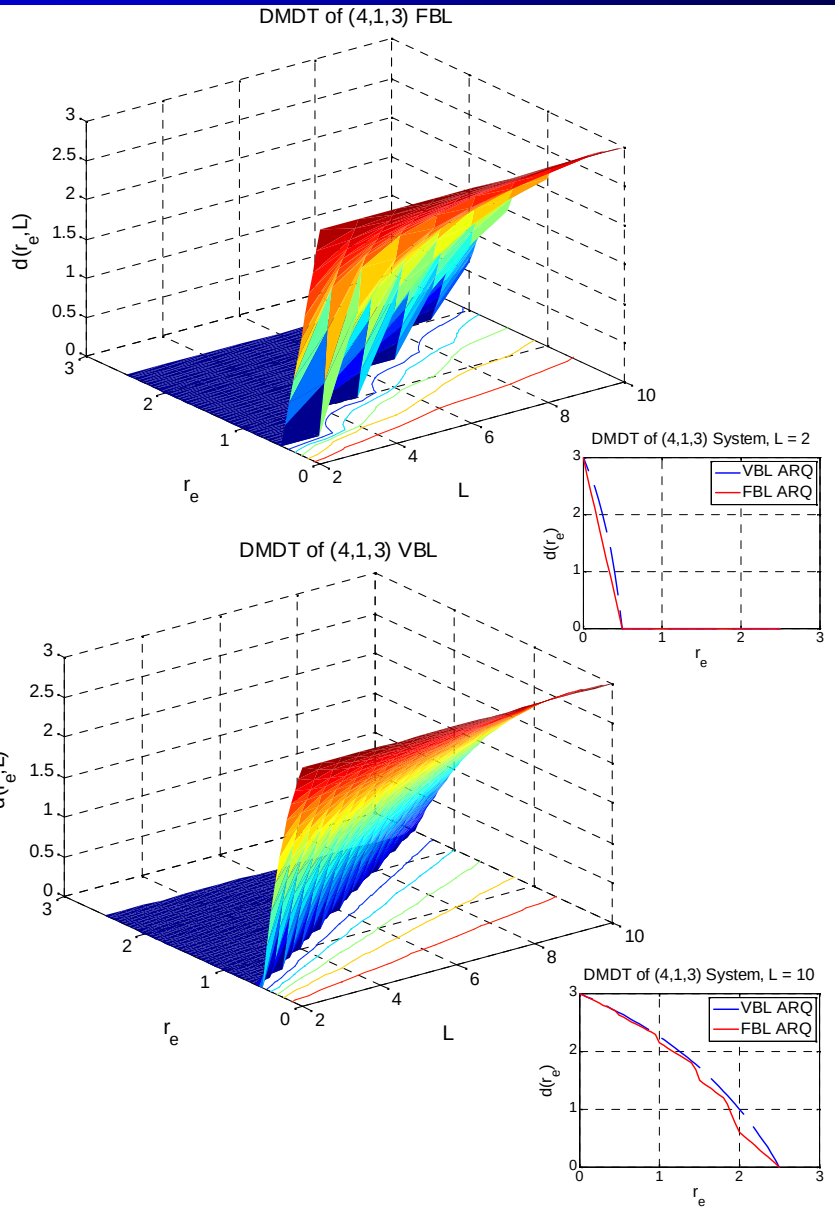
$$S_i(\alpha_i) \triangleq \sum_{j=1}^{M_i^*} \left(1 - \alpha_{i,j}\right)^{+}$$

Example: (4, 1, 3) network

- Long term static channel

$$d_{VBL}(r,L)=\begin{cases}\min\{M_1,M_3\}\dfrac{1-2r/L}{1-r/L} & 0\le r\le L/2\\ 0 & \textit{Otherwise}\end{cases}$$

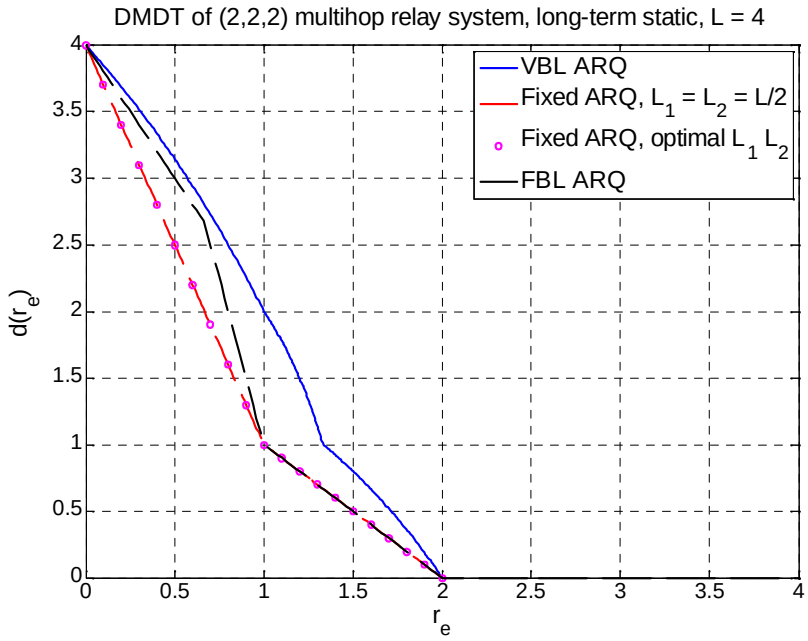




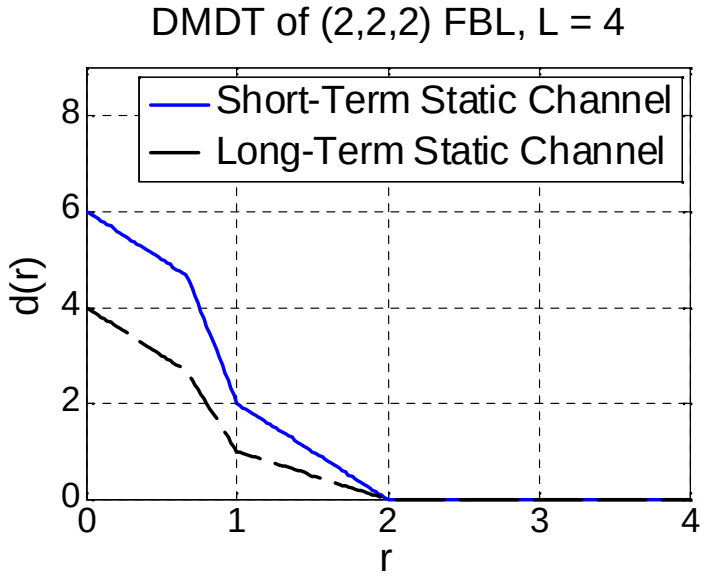
Example: (2, 2, 2) network

- Long term static channel

$$d_{VBL}(r,L)=\begin{cases}\dfrac{2(r-5r/L)}{2-r/L} & 0\le r\le L/2\\ \dfrac{3-4r/L}{1-r/L} & L/2\le r\le 2/3L\\ \dfrac{4(1-r/L)}{2-r/L} & 2/3L\le r\le L\end{cases}$$

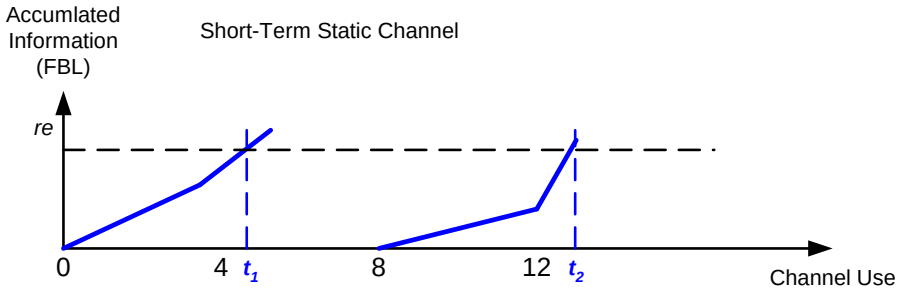


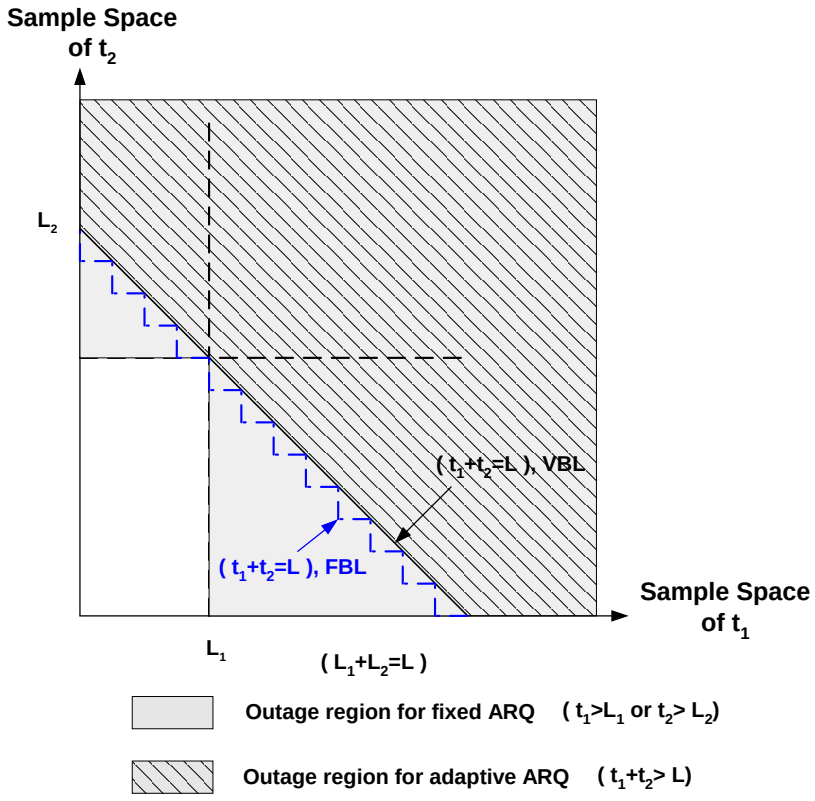
- Short term static channel
  - Compare DMDT of FBL



DMDT Optimality

- **Theorem:** VBL ARQ achieves the optimal DMDT in multihop MIMO relay networks
- Proved by cut-set bound
- An intuitive explanation by boundary crossing probability



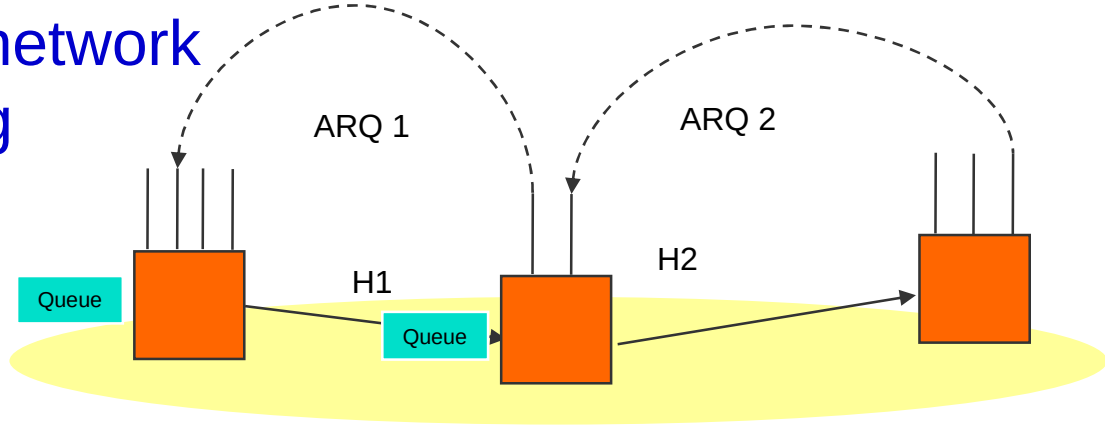


Conclusions

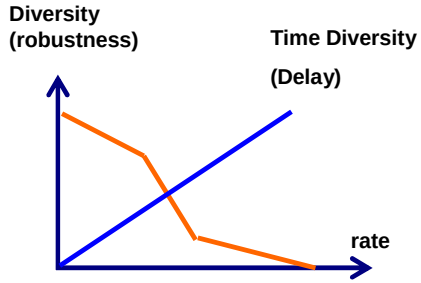
- Derived DMDT for multihop MIMO relay networks with ARQ
- Insights gained
  - System performance limited by the weakest link
  - Optimal ARQ equalizes the link performance by allocating ARQ rounds
- Adaptive variable block length (VBL) ARQ achieves the optimal DMDT

Ongoing Work

- More realistic model: finite SNR →queueing delay cannot be ignored
  - Form a queueing network
  - Combine queueing network theory and DMDT analysis



- Where to operate on this curve?
  - Joint source-to-channel coding



Optimal Operational Point

Thank You!