Change-point detection

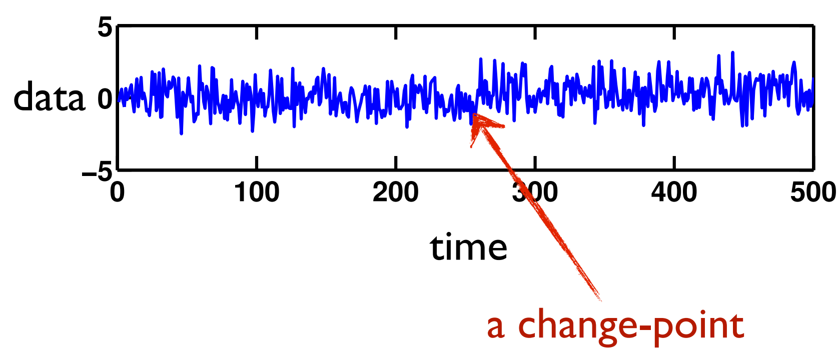
"... to detect abrupt changes in the statistical behavior of an observed signal or time series."

–Quickest detection by Poor and Hadjiliadis, 2008

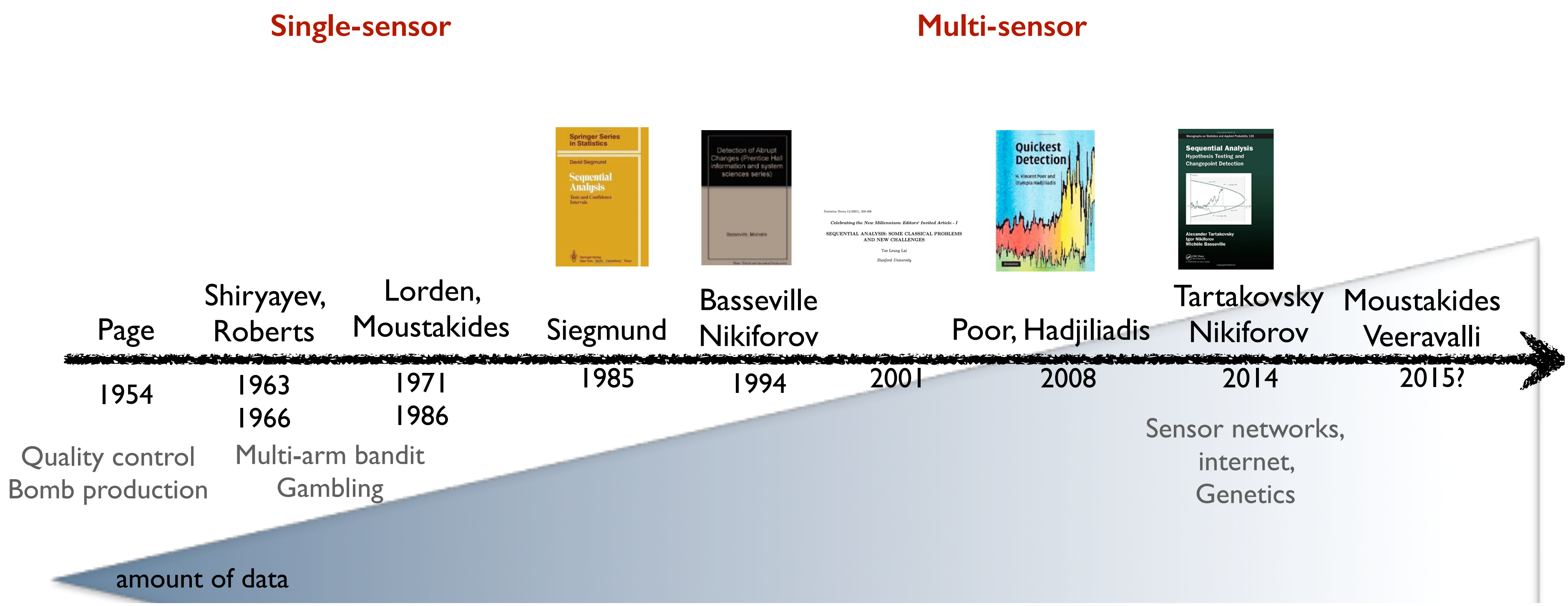
- change-point represents an **interesting event** or **anomaly**

  seismic event, solar flare, human activity

- wish to detect change-point **online**



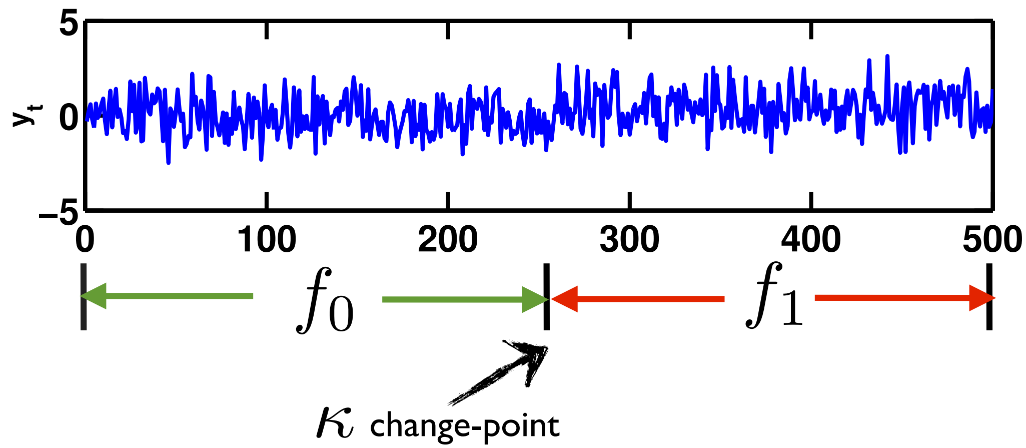
A brief history of change-point detection



Single-sensor change-point detection

Classic "change-point" detection in statistics and quality-control

- a sequence of i.i.d. observations $y_1, y_2, \cdots \in \mathbb{R}$



- sequential hypothesis testing

$$
\begin{aligned}
\mathrm{H}_0: \quad & y_t \sim f_0, \quad t = 1, 2, \ldots \\
\mathrm{H}_1: \quad & y_t \sim f_0, \quad t = 1, \ldots, \kappa, \\
& y_t \sim f_1, \quad t = \kappa + 1, \ldots
\end{aligned}
$$

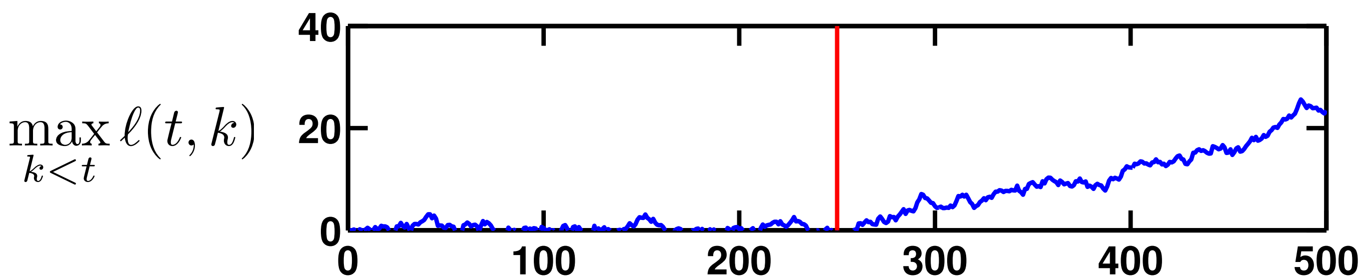
unknown change-point $\kappa > 0$

- **goal**: detect change-point as quickly as possible

Single-sensor likelihood procedure

- for a hypothesized $\kappa = k$:

$$\ell(t,k) = \log \frac{\prod_{i=1}^{k} f_0(y_i) \cdot \prod_{i=k+1}^{t} f_1(y_i)}{\prod_{i=1}^{t} f_0(y_i)} = \sum_{i=k+1}^{t} \log \frac{f_1(y_i)}{f_0(y_i)}$$



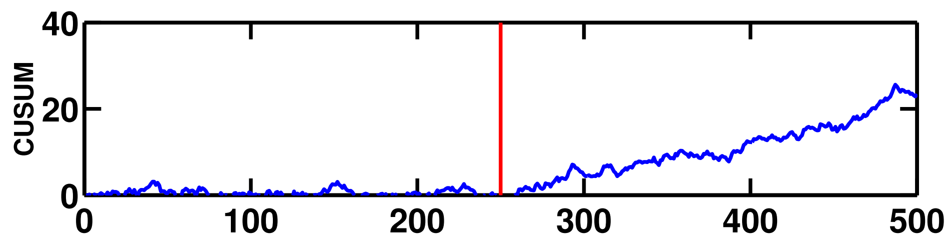
- stop the first time hitting a threshold b

$$T = \inf\{t : \max_{k<t} \ell(t,k) \geq b\}$$

Gaussian case

- $f_0 = \mathcal{N}(0,1)$, $f_1 = \mathcal{N}(\mu, 1)$, $\mu > 0$

CUSUM procedure [P54]: $T = \inf\{t : \max_{k<t} \sum_{i=k+1}^{t} (\mu y_i - \frac{\mu^2}{2}) \geq b\}$



- when μ **unknown**, replace it with maximum likelihood estimator

GLR procedure [SV86]: $T = \inf\{t : \max_{k<t} \frac{(\sum_{i=k+1}^{t} y_i)^2}{t-k} \geq b\}$