# PMU-based Online Change-Point Detection of Imbalance in Three-Phase Power Systems

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***Abstract*—In this paper, the problem of online change-point detection of voltage imbalance in a three-phase power system using phasor measurement unit (PMU) data is considered within a sequential hypothesis-testing framework. A general model for the positive-sequence data from a PMU measurement at the time domain and off-nominal frequencies is presented. The new formulation, which assumes an additional Gaussian noise, enables fast online detection of imbalance. Closed-form expressions of the cumulative sum (CUSUM) and generalized likelihood ratio (GLR) tests are developed for detection of imbalances. The performance of the change-point detection procedures is evaluated using the average-run-length and the expected detection delay. Numerical simulations show that the proposed method can be used for enhanced situational awareness in future grid management systems and demonstrate the ability to inform strategies for advancing grid capabilities by using change-point detection methods.**

***Index Terms*—Phasor measurement unit (PMU), power system monitoring, unbalanced power system, state estimation, online change-point detection**

## I. INTRODUCTION

The three-phase power system is designed to operate in balanced scenarios [1]. However, in practice, imbalances happen frequently [2]. Imbalances may be a precursor to more serious contingencies leading to possible blackouts [3, 4]. In addition, substantial power imbalance causes excessive losses, overheating, insulation degradation, a reduced lifespan of motors and transformers, and interruptions in production processes [5-8]. Thus, the ability to detect quickly potentially harmful levels of imbalance in various power systems is highly desirable for the benefit of both the utility and customer [4, 6]. To this end, effective algorithms and sophisticated methods are crucial for detecting an abnormal level of imbalance in real time and evaluating the associated effects. It is in this context that modern sensing devices, such as phasor measurement units (PMUs), have the potential to provide rapid detection of contingencies and situational awareness [1].

### A. Summary of results

This paper focuses on online detecting a change in the imbalance condition of a three-phase power system. The main contributions of this paper are threefold. First, we express the power system imbalance change-detection as an online hypothesis testing problem based on time-domain samples. The advantage of using time-domain signals is that the PMU constructs the output frequency-domain signals by using a discrete Fourier transform operator, which may cause time delay. Second, we apply a cumulative sum (CUSUM) test to solve our problem and a generalized likelihood ratio (GLR) procedure that can be used when the system parameters are unknown. The proposed CUSUM and GLR procedures are well-known in statistics and are specifically tailored for the smart grid structure. Finally, using simulation results for a variety of imbalance conditions, we examine the detection performance of the proposed approaches. In particular, we demonstrate that the the performance of the proposed change-point detection procedures are evaluated using the average-run-length (ARL) and the expected detection delay (EDD).

### B. Related works

When system imbalance occurs, the PMU's output exhibits nonstationary frequency deviations [4, 9] and the positive-sequence measurements become non-circular [10, 11]. The main performance measure of imbalances in power systems is the voltage unbalance factor (VUF) [4, 12, 13]. Recently, new parametric imbalance detection methods that outperform the classical parametric methods in terms of probability of error have been proposed. The new methods are based on the GLR test [14, 15] and generalized locally most powerful test (GLMP) [16, 17] approaches. All these methods are performed offline and are based on two- or three-phase data that are not available at the control center. In contrast, the proposed parametric change-detection method is an online method, which employs single-phase data.

In the last decade, modern optimization and statistical methodologies have been shown to be powerful tools in power system problems (see, e.g., [10, 11, 14, 18-21]). In this context, change-point detection of an emerging abrupt change point in a time series seems a promising tool. It is well known that, under some conditions, the optimal change-point detection procedure is the CUSUM procedure (e.g., [22]). However, in the case where there are unknown system parameters, the CUSUM method should be replaced by methods that are also based on a state estimation stage. In the context of power systems, a non-parametric online identification method of the level, location, and effects of voltage imbalance in a distribution network is derived in [6]. Derivation of parametric change-detection methods is expected to improve the detection performance.

## II. MEASUREMENT TIME-DOMAIN MODEL

The voltages in a three-phase power system are assumed to be pure sinusoidal signals of frequency $\omega_0+\Delta$, where $\omega_0$ is the known nominal frequency ($100\pi$ or $120\pi$) and $\Delta$ is the frequency deviation from this nominal value. The magnitudes and phases of the three voltages are denoted by $V_a, V_b, V_c \geq 0$ and $\varphi_a, \varphi_b, \varphi_c \in [0, 2\pi]$, respectively. The PMU samples these real signals $N$ times per cycle of the nominal frequency, $\omega_0$, to produce the following discrete-time, noisy measurement model (e.g., [1], pp. 51-52):

$$\mathbf{x}[n] = \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}\mathbf{v} + \frac{1}{2}e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n}\mathbf{v}^* + \mathbf{w}[n],\ \forall n \in \mathbb{Z}, \quad (1)$$

where $\gamma \triangleq \frac{2\pi}{N}$ and $\mathbf{v} \triangleq \left[V_a e^{j\varphi_a}, V_b e^{j\varphi_b}, V_c e^{j\varphi_c}\right]^T$. The noise sequence, $\{\mathbf{w}[n]\}_{n\in\mathbb{R}}$, is assumed to be a real white Gaussian noise sequence with a known covariance matrix $\sigma^2\mathbf{I}_3$.

We assume that the sequence is given by:

$$\underbrace{\mathbf{x}[0], \mathbf{x}[1], \ldots, \mathbf{x}[\tau-1]}_{\text{balanced system}}, \underbrace{\mathbf{x}[\tau], \ldots, \mathbf{x}[t]}_{\text{unbalanced system}}, \quad (2)$$

where $1 \leq \tau \leq t$ denotes the change-point location. That is, we assume that the vectors $\mathbf{x}[0], \ldots, \mathbf{x}[\tau-1]$ and $\mathbf{x}[\tau], \ldots, \mathbf{x}[t]$ are obtained form the model in (1) with balanced and unbalanced voltage vectors, $\mathbf{v}$ and $\tilde{\mathbf{v}}$, respectively.

The positive voltage sequence, *i.e.,* the "space vector" [23], is calculated from three-phase voltages by using the symmetrical component transformation ([1] pp. 63-67):

$$v_+[n] = \mathbf{g}^T\mathbf{x}[n], \quad n = 0, \ldots, t, \quad (3)$$

where $\mathbf{g} \triangleq \frac{1}{3}\left[1, \alpha, \alpha^2\right]^T$ and $\alpha = e^{j2\pi/3}$. For the special case of a perfectly balanced system, the three-phase voltages satisfy $V_a = V_b = V_c$ and $\varphi_a = \varphi_b + \frac{2\pi}{3} = \varphi_c - \frac{2\pi}{3}$. Therefore, it can be verified that for this case, *i.e.,* before the change, $\mathbf{g}^T\mathbf{v}^* = 0$. By using this result and substituting the model from (1) in (3), one obtains that before the change:

$$v_+[n] = \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}C_+^v + \mu_+[n], \quad (4)$$

$\forall n = 0, \ldots, \tau-1$, and after the change

$$v_+[n] = \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}C_+^{\tilde{v}} + \frac{1}{2}e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n}(C_-^{\tilde{v}})^* + \mu_+[n], \quad (5)$$

$\forall n = \tau, \ldots, t$, where $C_+^v \triangleq \mathbf{g}^T\mathbf{v}$, $C_+^{\tilde{v}} \triangleq \mathbf{g}^T\tilde{\mathbf{v}}$, and $C_-^{\tilde{v}} \triangleq \mathbf{g}^H\tilde{\mathbf{v}}$. In addition, the noise sequence, $\mu_+[n] \triangleq \mathbf{g}^T\mathbf{w}[n]$, $n = 0, 1, \ldots, t$, is time-independent sequence and satisfies

$$\mu_+[n] \sim \mathcal{N}^C\left(0, \frac{\sigma^2}{3}\right). \quad (6)$$

## III. ONLINE CHANGE-POINT DETECTION FORMULATION

### A. The hypothesis-testing problem

The objective of this study is to develop a method for change-point detection based on the PMU output of the positive sequence components. For instance, for the positive sequence shown in Fig. 1, single-phase imbalance occurs at time $n = 50$. The goal is to use the PMU voltage positive-sequence measurements to locate this change point. It can be seen that this task becomes easier as the signal-to-noise ratio (SNR) increases.

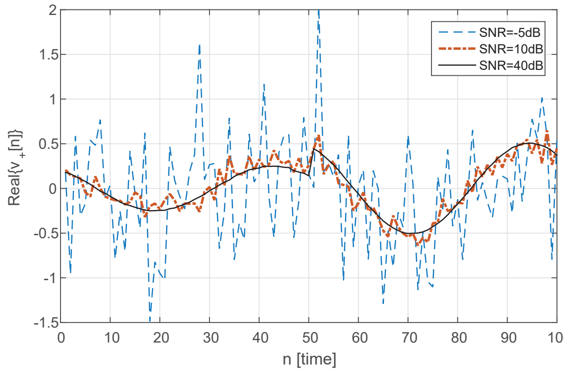

Fig. 1. Positive-sequence measurement of balanced system and unbalanced system before and after the change, respectively, where the change point is at $n = 50$ and with the parameters $C_+^v = 0.3709 + 0.3353j$, $C_+^{\tilde{v}} = 0.8654 + 0.7825j$, and $C_-^{\tilde{v}} = 0.1400 - 0.6518i$, $N = 48$, $\omega_0 = 60$Hz, and $\Delta = 1$Hz.

The change-point detection problem can be mathematically formulated as the following composite sequential hypothesis testing problem:

$$\begin{cases} \mathcal{H}_0: \text{The system is balanced till (at least) } n = t \\ \mathcal{H}_1: \text{ There is a change point at time } \tau \end{cases}, \quad (7)$$

where the measurement model is according to (4) and (5). The change-point detection is a well-studied problem in statistical signal processing and the CUSUM procedure is a popular algorithm in the literature that has some optimality properties for known system parameters [22].

### B. CUSUM approach

The desired stopping rules are determined by likelihood ratios, as described in the following. By using (4) and (5), as well as the noise statistics from (6) and the hypothesis testing from (7), the log-likelihood ratio of hypothesis $\mathcal{H}_1$ versus $\mathcal{H}_0$ for a given change-point time, $1 \leq \tau \leq t$, is given by:

$$\begin{aligned}\mathcal{L}_t(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}) &\triangleq \log f(\nu_+; \mathcal{H}_1) - \log f(\nu_+; \mathcal{H}_0) \\ &= Q_0\left(C_+^v\right) - Q_1\left(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}\right), \quad (8)\end{aligned}$$

where $f(\nu_+; \mathcal{H}_i)$ is the pdf of $\nu_+$ under hypothesis $\mathcal{H}_i$,

$$Q_0\left(C_+^v\right) \triangleq \frac{3}{2\sigma^2}\sum_{n=0}^{t}\left|v_+[n] - \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}C_+^v\right|^2, \quad (9)$$

and

$$\begin{aligned}&Q_1\left(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}\right) \triangleq \frac{3}{2\sigma^2} \times \\ &\left(\sum_{n=0}^{\tau-1}\left|v_+[n] - \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}C_+^v\right|^2 + \right. \\ &\left.\sum_{n=\tau}^{t}\left|v_+[n] - \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n}C_+^{\tilde{v}} - \frac{1}{2}e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n}(C_-^{\tilde{v}})^*\right|^2\right). \quad (10)\end{aligned}$$

By substituting (9) and (10) in (8), one obtains the decision function:

$$\mathcal{L}_t(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}) = S[t] - S[\tau - 1], \tag{11}$$

where the cumulative sum from 0 to $k$ is defined as:

$$S[k] \triangleq \sum_{n=0}^{k} l[n], \quad k \geq 0 \tag{12}$$

and

$$\begin{aligned} l[n] \;&\triangleq\; \frac{1}{4}\left(\left|C_+^v\right|^2 - \left|C_+^{\tilde{v}}\right|^2 - \left|C_-^{\tilde{v}}\right|^2\right) \\ &- \;\text{Real}\left\{\left(C_+^v - C_+^{\tilde{v}}\right) e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n} v_+^*[n]\right\} \\ &+ \;\text{Real}\left\{C_-^{\tilde{v}} e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n} v_+[n]\right\} \\ &- \;\frac{1}{2}\text{Real}\left\{C_+^{\tilde{v}} C_-^{\tilde{v}} e^{2j\gamma\frac{\omega_0+\Delta}{\omega_0}n}\right\}, \; \forall n \geq 0. \end{aligned} \tag{13}$$

Denote by $\hat{\tau}_C$ the time at which the CUSUM algorithm declares an imbalance condition. Then, under the assumption that the parameters $C_+^v, C_+^{\tilde{v}}$ and $C_-^{\tilde{v}}$ are known, $\hat{\tau}_C$ is given by

$$\hat{\tau}_C \;=\; \arg\max_{1\leq\tau\leq t} \mathcal{L}_t(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}), \tag{14}$$

such that $\mathcal{L}_t(\tau, C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}) > \eta$, where $\eta > 0$ is a threshold set by the user. By substituting (11) in (14), we obtain that the change-time estimator from (14) can be rewritten as [24]

$$\hat{\tau}_C \;=\; \arg\min_{1\leq\tau\leq t} S[\tau - 1], \tag{15}$$

such that $G[t] > \eta$, where

$$\begin{aligned} G[0] \;&=\; 0 \\ G[t] \;&=\; \left\{G[t-1] + l[t]\right\}^+, \quad t \geq 1,, \end{aligned} \tag{16}$$

$\{z\}^+ = \max\{z, 0\}$, and $l[t]$ is defined in (13).

The idea is that before any imbalance scenario occurs, the mean of the log-likelihood ratio from (8) is negative. As a result, $G[t]$ would remain close to or at 0 prior to the imbalance situation. On the other hand, when the system becomes unbalanced, the mean of the log-likelihood ratio from (8) is positive. As a result, $G[t]$ increases this change. Hence, the CUSUM algorithm declares the occurrence of an imbalance situation at the first time that $G[t]$ reaches the pre-determined threshold, $\eta$. The algorithm is summarized in Algorithm 1.

## IV. GLR TEST

In practice, the phasors $C_+^v, C_+^{\tilde{v}}$ and $C_-^{\tilde{v}}$ are unknown. Therefore, the CUSUM algorithm cannot be implemented since the likelihood function depends on the unknown parameters. In this context, we can apply the GLR approach. In this approach, we compute the LR statistics combined with maximization w.r.t. the unknown parameters. In order to implement the sequential GLR test, first we derive the maximum likelihood (ML) state estimators under each hypothesis in Section IV-A and then we develop the corresponding GLR procedure based on these estimators in Section IV-B.

**Algorithm 1:** CUSUM for imbalance change detection

```
Input: Positive sequence observations $v_+[n]$,
$n = 0, \ldots, t$, phasors $C_+^v, C_+^{\tilde{v}}, C_-^{\tilde{v}}$, and the nominal
frequency $\omega_0$.
Output: Change-point decision
Initialization:
  • Set the detection threshold $\eta > 0$
  • $S[-1] = G[-1] = 0$
  • Fix $n = 0$
for $n = 0, \ldots, t$ do
    while the algorithm is not stopped do
        1) Set $l[n]$ according to (13) and by using $v_+[n]$
        2) Set $S[n] = S[n-1] + l[n]$
        3) Evaluate the function
               $G[n] = \{G[n-1] + l[n]\}^+$
        4) if $G[n] > \eta > 0$ then
            • The change time estimator from (15) is given
              by
                   $\hat{\tau}_C = \arg\min_{1\leq\tau\leq n} S[\tau - 1]$
            • Stop or reset the algorithm
        end if
        $n = n + 1$
    end
end
```

### A. State estimation

Let us define

$$z_{t_1,t_2} \;\triangleq\; 2\sum_{n=t_1}^{t_2} e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n} v_+[n] \tag{17}$$

$$y_{t_1,t_2} \;\triangleq\; 2\sum_{n=t_1}^{t_2} e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n} v_+^*[n] \tag{18}$$

$$\Psi_\tau \;\triangleq\; \sum_{n=\tau}^{t} e^{2j\gamma\frac{\omega_0+\Delta}{\omega_0}n}, \; \forall 1 \leq \tau \leq t, \tag{19}$$

for any $t_1, t_2 \in [0, t]$. According to the hypothesis problem in (7), the ML estimator of $C_+^v$ maximizes the likelihood function under hypothesis $\mathcal{H}_0$, *i.e.,* minimizes $Q_0\left(C_+^v\right)$ from (9). By equating the derivative of (9) w.r.t. $C_+^v$ to zero, one obtains:

$$\hat{C}_+^{v|0} = \frac{z_{0,t}}{t+1}. \tag{20}$$

The notation $\hat{C}_+^{v|i}$, $i = 0, 1$ denotes the fact that this is the estimator of $C_+^v$ under hypothesis $\mathcal{H}_i$. Similarly, the ML estimators of $C_+^v$, $C_+^{\tilde{v}}$, and $C_-^{\tilde{v}}$ maximize the likelihood function under hypothesis $\mathcal{H}_1$, and are obtained by equating the derivative of (10) w.r.t. $C_+^v$ to zero:

$$\hat{C}_+^{v|1} = \frac{z_{0,\tau-1}}{\tau}, \tag{21}$$

$$\hat{C}_+^{\tilde{v}|1} = \frac{z_{\tau,t}}{t+1-\tau} - \frac{\left(C_-^{\tilde{v}}\right)^* \Psi_\tau^*}{t+1-\tau}, \tag{22}$$

and

$$\hat{C}_{-}^{\tilde{v}|1} = \frac{y_{\tau,t}}{t+1-\tau} - \frac{\left(C_{+}^{\tilde{v}}\right)^{*}\Psi_{\tau}^{*}}{t+1-\tau}, \tag{23}$$

for any $1 \leq \tau \leq t$. Equations (22) and (23) imply that

$$\hat{C}_{+}^{\tilde{v}|1} = \frac{(t+1-\tau)z_{\tau,t} - y_{\tau,t}^{*}\Psi_{\tau}^{*}}{(t+1-\tau)^{2} - |\Psi_{\tau}|^{2}} \tag{24}$$

and

$$\hat{C}_{-}^{\tilde{v}|1} = \frac{(t+1-\tau)y_{\tau,t} - z_{\tau,t}^{*}\Psi_{\tau}^{*}}{(t+1-\tau)^{2} - |\Psi_{\tau}|^{2}}, \tag{25}$$

for any $1 \leq \tau \leq t$. It can be seen that, as expected, for $\tau = t+1$, *i.e.,* there is no change, $\hat{C}_{+}^{\tilde{v}|1} = \hat{C}_{-}^{\tilde{v}|1} = 0$ and $\hat{C}_{+}^{v|1} = \hat{C}_{+}^{v|0}$. If the noise variance $\sigma^2$ is also unknown, its ML estimator can be computed, similar to [17].

### *B. GLR method*

In this section, the CUSUM method from Algorithm 1 is replaced with the GLR method, in which the unknown parameters in the CUSUM terms are replaced by their ML estimators. For this case, the relevant statistic from (8) is replaced by the following GLR statistic:

$$\begin{aligned}
&\tilde{\mathcal{L}}_t(\tau, \hat{C}_{+}^{v|0}, \hat{C}_{+}^{v|1}, \hat{C}_{+}^{\tilde{v}|1}, \hat{C}_{-}^{\tilde{v}|1}) \\
&\quad = Q_0(\hat{C}_{+}^{v|0}) - Q(\tau, \hat{C}_{+}^{v|1}, \hat{C}_{+}^{\tilde{v}|1}, \hat{C}_{-}^{\tilde{v}|1}) = \frac{3}{2\sigma^2} \times \qquad (26) \\
&\Bigg(\frac{(t+1)}{4}\left|\hat{C}_{+}^{v|0}\right|^2 - \frac{1}{2}\text{Real}\left\{\hat{C}_{+}^{v|0} z_{0,t}^{*}\right\} \\
&\quad -\frac{\tau}{4}\left|\hat{C}_{+}^{v|1}\right|^2 - \frac{t+1-\tau}{4}\left|\hat{C}_{+}^{\tilde{v}|1}\right|^2 - \frac{t+1-\tau}{4}\left|\hat{C}_{-}^{\tilde{v}|1}\right|^2 \\
&\quad +\frac{1}{2}\text{Real}\left\{\hat{C}_{+}^{v|1} z_{0,\tau-1}^{*}\right\} + \frac{1}{2}\text{Real}\left\{\hat{C}_{+}^{\tilde{v}|1} z_{\tau,t}^{*}\right\} \\
&\quad +\frac{1}{2}\text{Real}\left\{\hat{C}_{-}^{\tilde{v}|1} y_{\tau,t}^{*}\right\} - \frac{1}{2}\text{Real}\left\{\hat{C}_{+}^{\tilde{v}|1}\hat{C}_{-}^{\tilde{v}|1}\Psi_{\tau}\right\}\Bigg). \qquad (27)
\end{aligned}$$

By substituting the state estimators from (20), (21), and (25) in (26), one obtains

$$\begin{aligned}
&\tilde{\mathcal{L}}_t(\tau, \hat{C}_{+}^{v|0}, \hat{C}_{+}^{v|1}, \hat{C}_{+}^{\tilde{v}|1}, \hat{C}_{-}^{\tilde{v}|1}) = \frac{3}{2\sigma^2} \times \\
&\Bigg(-\frac{t+1}{4}\left|\hat{C}_{+}^{v|0}\right|^2 + \frac{\tau}{4}\left|\hat{C}_{+}^{v|1}\right|^2 + \frac{t+1-\tau}{4}\left|\hat{C}_{+}^{\tilde{v}|1}\right|^2 \\
&\quad + \frac{t+1-\tau}{4}\left|\hat{C}_{-}^{\tilde{v}|1}\right|^2 + \frac{1}{2}\text{Real}\left\{\hat{C}_{+}^{\tilde{v}|1}\hat{C}_{-}^{\tilde{v}|1}\Psi_{\tau}\right\}\Bigg). \qquad (28)
\end{aligned}$$

Therefore, the GLR rule for the change is

$$\hat{\tau}_{\text{GLR}} = \arg\max_{1\leq\tau\leq t} \tilde{\mathcal{L}}_t(\tau, \hat{C}_{+}^{v|0}, \hat{C}_{+}^{v|1}, \hat{C}_{+}^{\tilde{v}|1}, \hat{C}_{-}^{\tilde{v}|1}), \tag{29}$$

where the r.h.s. should be higher than the threshold $\eta$. Similar to the recursive CUSUM formulation in (15), the recursive version of the GLR procedure can be written as

$$\hat{\tau}_{\text{GLR}}^{\text{recursive}} = \min\left\{t : \max_{\tau<t} \tilde{\mathcal{L}}_t(\tau, \hat{C}_{+}^{v|0}, \hat{C}_{+}^{v|1}, \hat{C}_{+}^{\tilde{v}|1}, \hat{C}_{-}^{\tilde{v}|1}) > \eta\right\}. \tag{30}$$

It can be see that the change-time estimator, $\hat{\tau}_{\text{GLR}}$, is a function of the amplitude of the estimator of the negative phasor, $\hat{C}_{-}^{\tilde{v}|1}$, and of the difference between the positive sequence before and after the change, $\hat{C}_{+}^{v|1} - \hat{C}_{+}^{\tilde{v}|1}$. That is, the GLR detector in (29) can be interpreted as a detector of the presence of a negative-sequence phasor and the presence of a change in the positive-sequence phasor, which fits a typical situation in which the system becomes unbalanced. However, recursive implementation of a GLR test in a CUSUM way is intractable in the presence of unknown parameters. Therefore, an altered window-limited version of the GLR test from (29) has to be used in practical real-time settings.

## V. SIMULATIONS

In this section, the performance of the proposed real-time unbalanced system detection using the CUSUM algorithm in Algorithm 1 and the GLR rule from (29) is evaluated via $10,000$ Monte-Carlo simulations. We consider a single PMU with the parameters, $N = 48$, $\omega_0 = 60$Hz, frequency deviation of $\Delta = 1$ Hz, and different values of noise variance $\sigma^2$. The SNR is defined as SNR $\triangleq \frac{3|C_{+}|^2}{\sigma^2}$. To analyze the detection performance of the proposed methods, we consider a single-phase amplitude imbalance. That is, the three voltage phases, satisfy $\varphi_a = 0.234\pi$, $\varphi_b = \phi_a - \frac{2\pi}{3}$ and $\phi_c = \phi_a + \frac{2\pi}{3}$, with voltage amplitudes fixed at 1 p.u. for a balanced system. To introduce imbalance, the $c$ phasor is multiplied by $\beta = 1.5$, which causes magnitude imbalance.

Two standard performance metrics used to characterize the performance of a change detection algorithms are: the expected duration in between two false alarms, called the ARL, and the EDD, which is the expected time to stop in the extreme case when the change occurs immediately at $\tau = 0$. It is desired that the ARL quantity will be as small as possible to minimize the reaction time of the change-detection method. In general, a tradeoff occurs between high accuracy and minimum delay.

In Figs. 2 and 3 the ARL$(k = 25)$ and the EDD are presented, respectively, versus SNR, where SNR $\overset{\triangle}{=} \frac{|C_{+}|}{\sigma^2}$. Fig. 2 shows that, in terms of ARL for a change that happens at $\tau = 25$, the GLR achieves the CUSUM performance for high SNR, while for low SNR it has a higher ARL. Fig. 3 shows that, in terms of EDD, The GLR's behavior is similar to the CUMSUM performance, where for both methods, the EDD decreases as the SNR increases. These figures demonstrate that the performance of the CUSUM procedure, which assumes perfect knowledge of the unknown parameters, can be used as a lower bound to the ARL of the GLR decision rules that are based on the estimation of the parameters.

## VI. CONCLUSION

In this paper, we introduce two techniques, based on the CUSUM and GLR tests, for online change-point detection of imbalance condition of three-phase power systems. The proposed methods are based on a statistic model for the positive-sequence data from a PMU measurement at off-nominal frequencies. The performance of the CUSUM procedure, which assumes perfect knowledge of the unknown parameters, can be used as a benchmark on the performance of the GLR decision

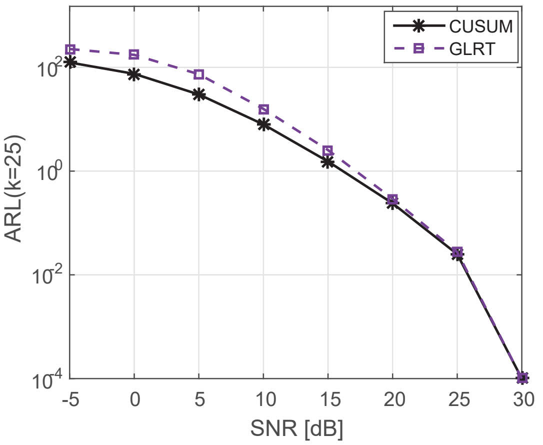

Fig. 2. The ARL versus SNR for single-phase imbalance at time $\tau = 25$ for the CUSUM and GLR tests.

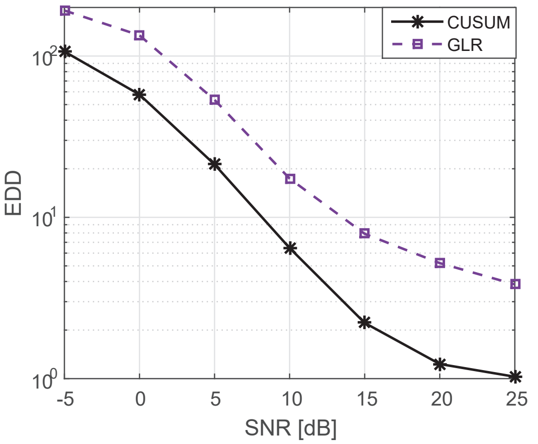

Fig. 3. The expected detection delay versus SNR for single-phase imbalance for the CUSUM and GLR tests.

rule, which estimates the unknown parameters. Numerical simulations show that the proposed GLR-based method can be used for enhanced situational awareness in future grid management systems and it has good performance in terms of ARL and EDD. In particular, the GLR-test achieves the ARL performance of the CUSUM procedure at high SNR. It should be noted that in current power systems, the PMUs transmits only positive sequence, due to lack of communication bandwidth. In the future, it is expected that wide area monitoring systems will be configured for three-phase measurements. The proposed CUSUM and GLR methods can be readily extended to the three-phase measurement case.

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## REFERENCES

[1] A. Phadke and J. Thorp, *Synchronized Phasor Measurements and Their Applications*. New York: Springer Science, 2008.

[2] "Electric power systems and equipment voltage ratings (60 Hertz)," *ANSI Standard Publication*, no. ANSI C84.1, 1995.

[3] S. Zhong and A. Abur, "Effects of nontransposed lines and unbalanced loads on state estimation," in *IEEE Power Engineering Society Winter Meeting*, vol. 2, Jan. 2002, pp. 975–979.

[4] A. Von Jouanne and B. Banerjee, "Assessment of voltage unbalance," *IEEE Trans. Power Delivery*, vol. 16, no. 4, pp. 782–790, Oct. 2001.

[5] N. D. Tleis, *Power Systems Modelling and Fault Analysis: Theory and Practice*. Oxford : Newnes, 2008.

[6] N. Woolley and J. Milanovic, "Statistical estimation of the source and level of voltage unbalance in distribution networks," *IEEE Trans. Power Delivery*, vol. 27, no. 3, pp. 1450–1460, July 2012.

[7] M. T. Bina and A. Kashefi, "Three-phase unbalance of distribution systems: Complementary analysis and experimental case study," *International Journal of Electrical Power & Energy Systems*, vol. 33, no. 4, pp. 817–826, 2011.

[8] T. H. Chen, C. H. Yang, and N. C. Yang, "Examination of the definitions of voltage unbalance," *International Journal of Electrical Power & Energy Systems*, vol. 49, pp. 380–385, 2013.

[9] A. G. Phadke, J. S. Thorp, and M. G. Adamiak, "A new measurement technique for tracking voltage phasors, local system frequency, and rate of change of frequency," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, no. 5, pp. 1025–1038, May 1983.

[10] Y. Xia, S. Douglas, and D. Mandic, "Adaptive frequency estimation in smart grid applications: Exploiting noncircularity and widely linear adaptive estimators," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 44–54, Sep. 2012.

[11] Y. Xia and D. Mandic, "Widely linear adaptive frequency estimation of unbalanced three-phase power systems," *IEEE Trans. Instrumentation and Measurement*, vol. 61, no. 1, pp. 74–83, Jan. 2012.

[12] P. Pillary and M. Manyage, "Definitions of voltage unbalance," *IEEE Power Eng. Rev.*, vol. 21, no. 5, pp. 49–51, May 2001.

[13] M. H. J. Bollen, "Definitions of voltage unbalance," *IEEE Power Eng. Rev.*, vol. 22, no. 11, pp. 49–50, Nov. 2002.

[14] T. Routtenberg, Y. Xie, R. M. Willett, and L. Tong, "PMU-based detection of imbalance in three-phase power systems," *IEEE Trans. Power System*, vol. 30, no. 4, pp. 1966–1976, July 2015.

[15] M. Sun, S. Demirtas, and Z. Sahinoglu, "Joint voltage and phase unbalance detector for three phase power systems," *IEEE Signal Processing Letters*, vol. 20, no. 1, pp. 11–14, Jan. 2013.

[16] R. Concepcion, T. Routtenberg, and L. Tong, "Local detection of voltage imbalance in three-phase power systems based on PMU output," in *IEEE Conference on Innovative Smart Grid Technologies (ISGT2015)*, Feb. 2015, pp. 1–5.

[17] T. Routtenberg, R. Concepcion, and L. Tong, "PMU-based detection of voltage imbalances with tolerance constraints," *IEEE Trans. Power Delivery*, vol. 32, no. 1, pp. 484–494, Feb. 2017.

[18] T. Routtenberg and L. Tong, "Joint frequency and phasor estimation under the KCL constraint," *IEEE Signal Processing Letters*, vol. 20, no. 6, pp. 575–578, June 2013.

[19] Y. C. Chen, T. Banerjee, A. D. Domínguez-García, and V. V. Veeravalli, "Quickest line outage detection and identification," *IEEE Trans. Power Systems*, vol. 31, no. 1, pp. 749–758, Jan. 2016.

[20] O. Kosut, L. Jia, R. J. Thomas, and L. Tong, "Malicious data attacks on the smart grid," *IEEE Trans. Smart Grid*, vol. 2, no. 4, pp. 645–658, Dec 2011.

[21] T. Routtenberg and L. Tong, "Networked detection of voltage imbalances for three-phase power system," in *The 24th IEEE International Symposium on Industrial Electronics (ISIE 2015)*, June 2015, pp. 1345–1350.

[22] D. O. Siegmund, *Sequential Analysis: Tests and Confidence Intervals*, ser. Springer Series in Statistics. Springer, Aug. 1985.

[23] F. Neves, H. Souza, E. Bueno, M. Rizo, F. Bradaschia, and M. Cavalcanti, "A space-vector discrete fourier transform for detecting harmonic sequence components of three-phase signals," in *IEEE Industrial Electronics (IECON).*, Nov. 2009, pp. 3631–3636.

[24] M. Basseville and I. Nikiforov, *Detection of abrupt changes: theory and application*. Englewood Cliffs, N.J: Prentice Hall, 1993.