COMPRESSIVE DEMODULATION OF MUTUALLY INTERFERING SIGNALS

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ABSTRACT

The challenge of Multiuser Detection (MUD) is that of demodulating mutually interfering signals given that at any time instant the number of active users is typically small. The promise of compressed sensing is the demodulation of sparse superpositions of signature waveforms from very few measurements. This paper considers signature waveforms that are are drawn from a Gabor frame. It describes a MUD architecture that uses subsampling to convert analog input to a digital signal, and then uses iterative matching pursuit to recover the active users. Compressive demodulation requires $K \log N$ samples to recover K active users whereas standard MUD requires N samples. The paper provides theoretical performance guarantees and consistent numerical simulations.

***Index Terms*—** Multiuser Detection, Compressed Sensing, Gabor Frames

1. INTRODUCTION

Demodulation of mutually interfering signals is central to multiaccess communications. It includes the special case of the Random Access Channel (RAC) that arises in modeling control channels in wireless networks where MUD becomes the recovery of the active users. In general, the goal also includes demodulation of transmitted symbols. The two biggest impediments are the asynchronous character of random access and the lack of Channel State Information (CSI) at the Base Station (BS). The signature waveforms are obtained by modulating a chip waveform by digital sequence of length L. Our goal is to maximize the number of users N that the network can support, and the number of active users K that the BS can reliably demodulate with or without requiring knowledge of the delays or CSI.

A baseline architecture for demodulation of a sparse superposition is a bank of matched filters, each correlating the received signal with a shift of a signature waveform. The first drawback is the number of required filters which is $N(\tau + 1)$ where τ is the maximum delay. A second drawback is that when the signature waveforms are not orthogonal, the noise will be colored and amplified.

We consider an alternative architecture where the analog signal is sampled directly at the chip rate. This approach does not amplify noise but it does require a high-rate Analog-to-Digital (A/D) converter. We frame the challenge of demodulation as a compressive sensing problem where the columns of the measurement matrix are randomly sampled shifts of the digital sequences used to generate the signature waveforms. This is the extension to asynchronous communication of the architecture for synchronous MUD proposed by Xie et. al. [1]. This model of asynchronous Random Access Communication appears in Applebaum et al. [2] where compressive demodulation is accomplished through convex optimization; see also [3] for a treatment of synchronous random access.

Our main theoretical contribution is relating the probability of error for the proposed MUD algorithm to two geometric measures associated with the set of sampled signature waveforms. These measures, the worst case and average coherence, were introduced by Bajwa et.al. [4] in the context of model selection. The average coherence of full Gabor frames is calculated in [4] and here we extend this analysis to the subsampled frames. Compressive demodulation of the subsampled superposition is performed by an iterative matching pursuit algorithm under the assumption that we will only receive one delayed path for each user. We provide explicit performance guarantees in terms of the coherence measures and the distribution of received signal powers. These fundamental limits quantify robustness to the near-far problem in multiple access communication. We demonstrate through simulation that our architecture is able to support more users at a given sampling rate or to support a given number of users at a reduced sampling rate. We also describe how our methods can be extended to support demodulation of symbols in the absence of Channel State Information, that is to enable blind MUD.

2. MODEL

Consider a multiuser system with N users. We assume that users communicate using spread spectrum waveform of the form

$$x_n(t) = \sqrt{P_n}\sum_{l=0}^{L-1} a_{n,l}\, g(t - lT_c),\ t \in [0, T), n = 1, \ldots, N, \quad (1)$$

where $g(t)$ is a unit-energy pulse $\int |g(t)|^2 dt = 1$, T is the symbol duration, T_c is the chip duration, P_n denotes the transmit power of the nth user, and the spreading sequence

$$\tilde{\boldsymbol{a}}_n = [a_{n,0} \quad \cdots \quad a_{n,L-1}]^\top, \quad n = 1, \ldots, N \quad (2)$$

is the L-length (real or complex-valued) codeword assigned to the nth user. The notation $^\top$ denotes transpose of a matrix or vector. As we describe more concretely in a subsequent paragraph, each codeword $\boldsymbol{a}_n$ contains a periodic extension such that the end of the codeword is a repetition of the first symbols. The signal at the receiver is given by

$$y(t) = \sum_{n=1}^{N} g_n \sqrt{P_n}\, \delta_{\{n \in \mathcal{I}\}} b_n x_n(t - \tau_n) + w(t), \quad (3)$$

where $g_n \in \mathbb{C}$ and $\tau_n \in \mathbb{R}_+$ are the channel fading coefficient and the delay associated with the nth user, respectively. We assume binary phase-shift keying (BPSK) transmission, where $b_n \in \{-1, 1\}$ is the transmitted symbol of the nth user, and $w(t)$ is a complex additive white Gaussian noise (AWGN) introduced by the receiver

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circuitry. Denote by $\mathcal{I}$ the set of active users. The Dirac function $\delta_{\{x\}} = 1$ if x is true otherwise it is equal to zero.

Define the individual discrete delays $\tau_n' \triangleq \lfloor \tau_n/T_c \rfloor \in \mathbb{Z}_+$, and the maximum discrete delay $\tau \triangleq \max_n \tau_n' \in \mathbb{Z}_+$. While the values of τ_n' are unknown, τ is assumed to be known by the transmitters and receivers. We consider the vectors $\tilde{\boldsymbol{a}}_n$ as periodic extensions. That is, taking $\boldsymbol{a}_n$ as the last $P = L - \tau - 1$ symbols of $\tilde{\boldsymbol{a}}_n$, we have $\tilde{a}_{n,l} = a_{n,P-\tau-1+l}$ for $l = 1, \ldots, \tau + 1$. As a result, any length P sub-sequence of the vectors $\tilde{\boldsymbol{a}}_n$ will be a cyclic shift of $\boldsymbol{a}_n$. In Section 4 we will show how to construct $\boldsymbol{a}_n$ using Gabor frame.

We assume the codewords are a reasonable length relative to the delays such that $P > M$. The receiver starts sampling from the $\tau + 1$ sample, so that all active users' waveforms have arrived. Then the receiver takes M compressive measurements which come from uniformly random subsampling of the received sequence or its DFT. As a result, the output data vector can be written as

$$\boldsymbol{y} = \boldsymbol{I}_\Omega \boldsymbol{A}\boldsymbol{R}\boldsymbol{b} + \boldsymbol{w} \triangleq \boldsymbol{H}\boldsymbol{R}\boldsymbol{b} + \boldsymbol{w}, \tag{4}$$

where $\boldsymbol{y} \in \mathbb{C}^{M\times 1}$, $\boldsymbol{H} = \boldsymbol{I}_\Omega \boldsymbol{A} \in \mathbb{C}^{M\times N(\tau+1)}$, where M is the number of samples, $\boldsymbol{A} \in \mathbb{C}^{P\times N(\tau+1)}$, and the noise is Gaussian distributed with zero mean and variance $\sigma^2 \boldsymbol{I}_{M\times M}$. The subsampling matrix is $\boldsymbol{I}_\Omega$, where Ω denotes indices of samples. The columns of matrix $\boldsymbol{A}$ have a block structure with each block consisting of a circulantly shifted codeword. Define a Toepliz matrix $\boldsymbol{A}_n \in \mathbb{R}^{P\times(\tau+1)}$ as

$$\boldsymbol{A}_n = \left[\begin{array}{cccc} \mathcal{T}_0\tilde{\boldsymbol{a}}_n & \mathcal{T}_1\tilde{\boldsymbol{a}}_n & \cdots & \mathcal{T}_\tau\tilde{\boldsymbol{a}}_n \end{array}\right], \tag{5}$$

where the notation $\mathcal{T}_k$ denotes circulant shift the matrix by k. Let $\boldsymbol{A} = [\boldsymbol{A}_1 \quad \cdots \quad \boldsymbol{A}_N]$. The vector $\boldsymbol{b} \in \mathbb{C}^{N(\tau+1)}$ contains the transmitted symbols, the vector $\boldsymbol{b}$ is a concatenation of N vectors $\boldsymbol{b}_n'$, each of length $\tau + 1$, with only one non-zero entry at the location of τ_n': $b_{n,m}' = b_n \delta_{\{m=\tau_n'+1\}}$. The diagonal matrix $\boldsymbol{R}$ has entries containing the transmitted power, channel gain, and the symbols: $R_{mm} = g_n\sqrt{P_n}\delta_{\{m=(n-1)(\tau+1)+\tau_n'\}}$, for $n = 1, \ldots, N$ and $m = 1, \ldots, N(\tau + 1)$. We assume the support of active users $\mathcal{I}$ is a uniform random K-subset of $[N] \triangleq \{1, \ldots, N\}$.

3. COMPRESSIVE DEMODULATION

3.1. The Detector

Demodulation is accomplished by a matching pursuit procedure (Algorithm 1) that takes explicit account of the block structure of the measurement matrix $\boldsymbol{A}$ and corresponding block sparsity of the received signal. The key feature is that after selecting one column from a block the demodulator ignores the other columns in that block, since they correspond to alternative shifts of the same signature waveform.

3.2. Performance Guarantee

To state the performance guarantee for Algorithm 1, we need to introduce two fundamental measures of coherence among the normalized columns of the $M \times N(\tau + 1)$ matrix $\boldsymbol{H} = \boldsymbol{I}_\Omega\boldsymbol{A}$: *worse-case coherence* $\mu(\boldsymbol{H}) \triangleq \max_{n\neq m} |\boldsymbol{h}_n^H \boldsymbol{h}_m|$, and *average coherence* $\nu(\boldsymbol{H}) \triangleq \frac{1}{N(\tau+1)-1} \max_n \left|\sum_{m\neq n} \boldsymbol{h}_n^H \boldsymbol{h}_m\right|$. We also define that the matrix $\boldsymbol{H}$ having unit l_2-norm columns is said to obey the *coherence property* if the following two conditions hold:

$$\mu(\boldsymbol{H}) \leq 0.1/\sqrt{2\log N(\tau+1)}, \quad \nu(\boldsymbol{H}) \leq \mu/\sqrt{M}. \tag{6}$$

Algorithm 1 Matching Pursuit Detector for Asynchronous MUD

1: Input: matrices $\boldsymbol{H}$ and $\boldsymbol{R}$, signal vector $\boldsymbol{y}$, number of active users K
2: Output: active user set $\mathcal{I}$, transmitted symbols b_n, $n \in \mathcal{I}$
3: Initialize: $\mathcal{I}_0 :=$ empty set, $\hat{\boldsymbol{b}}_0 := \boldsymbol{0}$, $\boldsymbol{v}_0 := \boldsymbol{y}$, $\mathcal{H}_0 = \{1, \ldots, N(\tau+1)\}$
4: **for** $j = 0 \to K - 1$ **do**
5: Compute: $\boldsymbol{f} := \boldsymbol{H}^H \boldsymbol{v}_j$
6: Find $i = \arg\max_{n\in\mathcal{H}_j} |f_n|$
7: Detect active users: $\mathcal{I}_{j+1} = \mathcal{I}_j \cup \{\lceil i/(\tau+1)\rceil\}$
8: Update: $\mathcal{H}_{j+1} = \mathcal{H}_j \backslash \{\lfloor i/(\tau+1)\rfloor(\tau+1) + 1, \cdots \lceil i/(\tau+1)\rceil(\tau+1), \}$
9: Detect symbols: $[\hat{\boldsymbol{b}}_{j+1}]_i = \mathbf{sgn}(r_i f_i)$, and $[\hat{\boldsymbol{b}}_{j+1}]_n = [\hat{\boldsymbol{b}}_j]_n$ for $n \neq i$.
10: Update residual: $\boldsymbol{v}_{j+1} = \boldsymbol{v}_j - \boldsymbol{H}\boldsymbol{R}\boldsymbol{b}_{j+1}$
11: **end for**
12: $\hat{\mathcal{I}} = \mathcal{I}_K$, $\hat{\boldsymbol{b}} = \hat{\boldsymbol{b}}_K$

We sort the amplitude of the diagonal entries of $\boldsymbol{R}$, $|r_n|$, from the largest to the smallest for the active users and denote as $|r|_{(1)}, \ldots, |r|_{(K)}$. Then define the following fundamental quantities, the nth signal-to-noise ratio (SNR) and the nth-largest to average ratio (LAR_n):

$$\text{SNR}_n = \frac{|r|_{(n)}^2}{\mathbb{E}\{\|\boldsymbol{w}\|^2\}/K}, \quad \text{LAR}_n = \frac{|r|_{(n)}^2}{\|\boldsymbol{r}_{\mathcal{I}_n}\|^2/K}, \tag{7}$$

for $n = 1, \ldots, K$, where $\|\boldsymbol{r}_{\mathcal{I}_n}\|^2 = \sum_{i=n}^k |r|_{(i)}^2$.

Theorem 1. *Suppose that the matrix $\boldsymbol{H}$ satisfies the coherence property and let the noise $\boldsymbol{w}$ be distributed as $\mathcal{N}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{M\times M})$. We write $\mu(\boldsymbol{H})$ as $\mu = c_1 M^{-1/\gamma}$ for some $c_1 > 0$ (c_1 may depend on $N(\tau + 1)$ and $\gamma \in \{0\} \cup [2, \infty)$). Then Algorithm 1 satisfies $\mathbb{P}\{\hat{\boldsymbol{b}} \neq \boldsymbol{b}\} \leq 6N^{-1}(\tau + 1)^{-1}$ as long as $N(\tau + 1) \geq 128$ and the number of measurements M satisfies*

$$M > \max\left\{2K\log N(\tau+1), \frac{c_2 K \log(N(N-n)(\tau+1)^2)}{\text{SNR}_n}, \left(\frac{c_3 K\log(N(\tau+1))}{\text{LAR}_n}\right)^{\gamma/2}\right\}_{n=1}^{K},$$

where $c_2 = 8(1-t)^{-2}$ and $c_3 = 800c_1^2 t^{-2}$.

The proof for this theorem is omitted here due to space limitations. Note that we can achieve blind MUD by performing joint estimation of the CSI using least-squares estimate on the support after solving the bit ambiguity. These details will be presented in the journal paper.

4. SIGNATURE SEQUENCES FROM A GABOR FRAME

In the following we will construct the signature sequences $\tilde{\boldsymbol{a}}_n$ from Gabor frames. Define $\boldsymbol{g} \in \mathbb{C}^P$ be a seed vector with each entry $|g_i|^2 = 1/M$ and let $\boldsymbol{T}(\boldsymbol{g}) \in \mathbb{C}^{P\times P}$ be the circulant matrix generated from $\boldsymbol{g}$ as $\boldsymbol{T}(\boldsymbol{g}) = [\mathcal{T}_0\boldsymbol{g} \quad \cdots \quad \mathcal{T}_\tau\boldsymbol{g}]$. Its eigen-decomposition can be written as $\boldsymbol{T}(\boldsymbol{g}) = \boldsymbol{F}\text{diag}(\boldsymbol{F}^H\boldsymbol{g})\boldsymbol{F}^H \triangleq \boldsymbol{F}\text{diag}(\hat{\boldsymbol{g}})\boldsymbol{F}^H$, where $\boldsymbol{F} = \frac{1}{\sqrt{P}}[\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_{P-1}]$ is the DFT matrix with columns

$$\boldsymbol{\omega}_m = [e^{j2\pi\frac{m}{P}\cdot 0}, e^{j2\pi\frac{m}{P}\cdot 1}, \ldots, e^{j2\pi\frac{m}{P}\cdot(P-1)}]^\top,$$

and we define corresponding diagonal matrices $\boldsymbol{W}_m = \mathrm{diag}[\boldsymbol{\omega}_m]$, for $m = 0, 1, \ldots, P-1$. Then the Gabor frame generated from $\boldsymbol{g}$ is an $P \times P^2$ block matrix of the form

$$\boldsymbol{\Phi} = [\boldsymbol{W}_0\boldsymbol{T}(\boldsymbol{g}),\ \boldsymbol{W}_1\boldsymbol{T}(\boldsymbol{g}),\ \ldots,\ \boldsymbol{W}_{P-1}\boldsymbol{T}(\boldsymbol{g})]. \tag{8}$$

where each column has norm $\sqrt{P/M}$. If we apply DFT to the Gabor frame $\boldsymbol{\Phi}$, and get $\hat{\boldsymbol{\Phi}} = \boldsymbol{F}^H\boldsymbol{\Phi}$, the order of time-shift and frequency modulation is reversed, and therefore $\hat{\boldsymbol{\Phi}}$ is composed of circulant matrices with proper ordering of columns. In fact, if we index each column m from P^2 to $P \times P$ by $m = Pq + \ell$. The matrix $\boldsymbol{\Phi}_\ell$ is obtained by keeping all columns with $r = \ell \pmod P$. So $\boldsymbol{\Phi}_\ell$ can be written as $\boldsymbol{\Phi}_\ell = \sqrt{P}\cdot\mathrm{diag}(\boldsymbol{S}^\ell\boldsymbol{g})\boldsymbol{F}$, where $\boldsymbol{S}$ is the right-shift matrix by one, and $\hat{\boldsymbol{\Phi}}_\ell = \boldsymbol{F}\boldsymbol{\Phi}_\ell = \sqrt{P}\boldsymbol{T}(\boldsymbol{W}_\ell\hat{\boldsymbol{g}})$ is a circulant matrix. We use $[\boldsymbol{\Phi}_1, \cdots, \boldsymbol{\Phi}_{P-1}]$ as the matrix $\boldsymbol{A}$.

At the receiver, a partial DFT is applied to the received symbol, so $\boldsymbol{I}_\Omega = \boldsymbol{F}_\Omega$ is a partial DFT matrix, and the resulted matrix $\boldsymbol{H} = \boldsymbol{\Phi}_\Omega$ is a subsampled Gabor frame defined in (8), with unit-norm columns. The Gabor frame is known to satisfy the coherence property [4] and Section 4.1 shows that this is also true of subsampled Gabor frames.

The maximum discrete delay τ this Gabor frame construction can support is $P-1$, where $\boldsymbol{W}_\ell\hat{\boldsymbol{g}}$, $\ell = 1, \cdots, P$ can be assigned as signature sequences ($\tilde{\boldsymbol{a}}_n$'s) to a user, so the maximum number of total user should satisfy $N \le P$. In general, if $\tau < P-1$, we can split $\boldsymbol{\Phi}_\ell$ into blocks to support multiple users, and send $\mathcal{T}_{d(\tau+1)}\boldsymbol{W}_\ell\hat{\boldsymbol{g}}$ as signature sequences for $d = 0, \cdots, \lceil P/\tau\rceil$ and $\ell = 1, \cdots, P$, so the maximum number of total user satisfies $N \le P\lfloor P/\tau\rfloor$ in general.

4.1. Coherence Properties for subsampled Gabor frames

We first consider $\mu(\boldsymbol{\Phi}_\Omega)$. The coherence between two columns of the subsampled Gabor frame $\boldsymbol{\Phi}_\Omega$ is given as $m \ne m'$,

$$\langle\phi_m^\Omega, \phi_{m'}^\Omega\rangle = \sum_{i\in\Omega}\phi_m^H(i)\phi_{m'}(i), \tag{9}$$

with the expectation $\mathbb{E}\langle\phi_m^\Omega, \phi_{m'}^\Omega\rangle = \langle\phi_m, \phi_{m'}\rangle$, whose absolute value is upper bounded by μ, the worst case coherence of the Gabor frame. Applying the triangle inequality and the Hoeffding's inequality we have for $\gamma > 0$,

$$\Pr\{|\langle\phi_m(\Omega), \phi_{m'}(\Omega)\rangle| - \mu \ge \gamma\} \le 4\exp\left(-\frac{\gamma^2 M}{4}\right),$$

Now we consider all pairs of different inner products and apply the union bound,

$$\Pr\{\mu_\Omega - \mu \ge \gamma\} < 2P^4\exp\left(-\frac{\gamma^2 M}{4}\right). \tag{10}$$

Let $P^4\exp\left(-\frac{\gamma^2 M}{4}\right) = \exp(-4\epsilon)$, then $\gamma = 4\sqrt{\frac{\log P+\epsilon}{M}}$. With probability $1 - 2\exp(-4\epsilon)$, we have

$$\mu_\Omega \le \mu + 4\sqrt{\frac{\log P + \epsilon}{M}}. \tag{11}$$

We next consider $\nu(\boldsymbol{\Phi}_\Omega)$. Let $m = Pq + r$, $m' = Pq' + r'$, the average coherence $\nu_\Omega = \frac{1}{P^2-1}\max_n\left|\sum_{n\ne m}\langle\phi_m^\Omega, \phi_n^\Omega\rangle\right|$, where

$$\sum_{n\ne m}\langle\phi_m^\Omega, \phi_n^\Omega\rangle = \sum_{i\in\Omega}\sum_{n\ne m}\phi_m^H(i)\phi_n(i). \tag{12}$$

Since for each column it can be written as,

$$\phi_m = [g_{(1-r)_P}e^{j2\pi\frac{q}{P}\cdot 0}, \ldots,\ g_{(P-r)_P}e^{j2\pi\frac{q}{P}\cdot(P-1)}]^T,$$

If $r \ne r'$, we have

$$\sum_{q'=0}^{P-1}\sum_{r\ne r'}\phi_m^H(i)\phi_{m'}(i) = Pg^*_{(1-r)_P}\sum_{r\ne r'}g_{(1-r')_P}\cdot 1_{\{i=1\}}$$

If $r = r'$, $q \ne q'$, we have

$$\sum_{q'\ne q}\phi_m^H(i)\phi_{m'}(i) = \frac{1}{M}\left[(P-1)\cdot 1_{\{i=1\}} - 1_{\{i\ne 1\}}\right].$$

where we use the fact $|g_i|^2 = 1/M$. To sum up, we have

$$\sum_{m'\ne m}\phi_m^H(i)\phi_{m'}(i) = \begin{cases} Pg^*_{(1-r)_P}\sum_{r\ne r'}g_{(1-r')_P} + (P-1)/M & i = 1\\ -1/M & i \ne 1\end{cases}$$

Then $\left|\sum_{m'\ne m}\langle\phi_m^\Omega, \phi_{m'}^\Omega\rangle\right| \le \frac{P}{\sqrt{M}}\|g\|_1 + 1 = \frac{P^2}{M} + 1$, and the average coherence $\nu_\Omega = \frac{P^2+M}{P^2-1}\cdot\frac{1}{M}$.

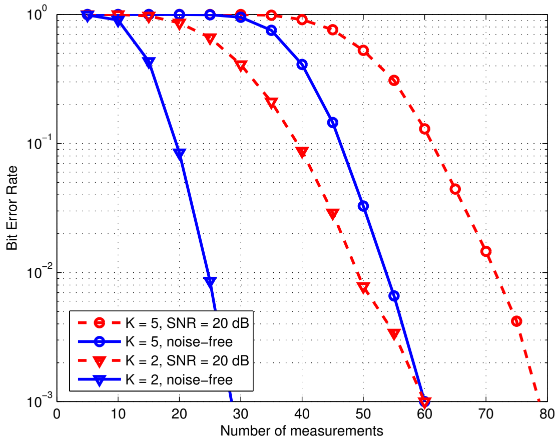
These expressions for worst-case and average coherence imply that we can find an $\boldsymbol{M}$ such that the subsampled Gabor frame satisfies the coherence property (6).

5. NUMERICAL RESULTS

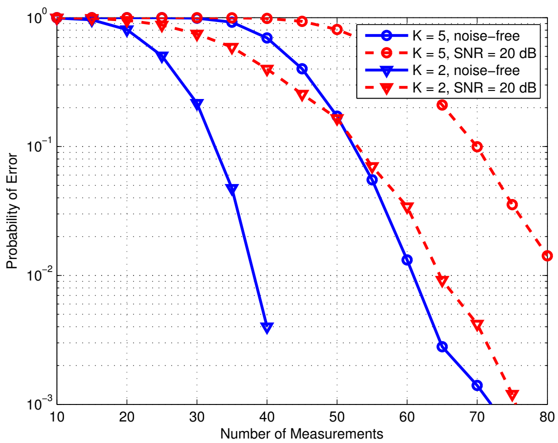
We first consider when each circulant matrix in the Gabor frame supports only one user. This corresponds to the maximum delay the algorithm can work in the asynchronous case. Let the seed vector for the Gabor frame be either an Alltop sequence with length $P = 127$, or a random uniform vector with length $P = 128$. The channel gain and transmitted power is assumed to be known, and with $R_{mm} = 1$ for all $m = 1, \cdots, N(\tau+1)$.

The active users are selected first by uniformly choosing at random from 1 to P, and then, for each active user, the delay is chosen uniformly at random. First, we fix the number of active users, namely $K = 2$ or $K = 5$, and apply the Matching Pursuit Decoder described in Algorithm 1 for noise-free case or noisy case where the AWGN noise is SNR = 20dB per measurement. The partial DFT matrix is applied with randomly selected rows and the number of Monto Carlo runs is 10,000. Fig. 1 shows the MUD error rate with respect to the number of measurements. We attribute superior performance of the Gabor frame determined by the Alltop sequence to optimal coherence. A different perspective is found in Fig. 2 which describes the phase transition for detection of K users using M measurements.

Finally, we consider when the maximum delay is relatively small, for example $\tau = 16$ when $P = 128$ for a random Gabor frame. We transmit the first sequence within the block of the circulant matrix, resulting in a total number of $P^2/\tau = 1024$ users, and Fig. 3 shows the MUD error rate with respect to the number of random measurements.



(a) Alltop Gabor Frame



(b) Random Gabor Frame

Fig. 1: Multi-user detection error rate with respect to the number of measurements using a Gabor frame generated from (a) an Alltop sequence with length $P = 127$, and (b) a random uniform vector with length $P = 128$ for different active users and SNR, where the maximum chip delay is P.

6. CONCLUSIONS

We have provided a theoretical analysis of a new architecture for compressive demodulation of mutually interfering signals. The advantage over standard MUD is that, while only using a number of samples on the order of $K \log N(\tau + 1)$, we can detect N active users with a maximum discrete delay τ, compared to $N(\tau+1)$ samples for standard MUD. The architecture also supports blind MUD by assigning multiple waveforms to a given user and transmitting information by the choice of waveform.

7. REFERENCES

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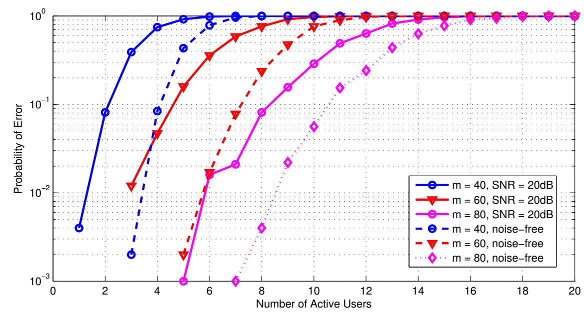

Fig. 2: Multi-user detection error rate with respect to the number of active users using an Alltop Gabor frame with $P = 127$ for fixed number of measurements.

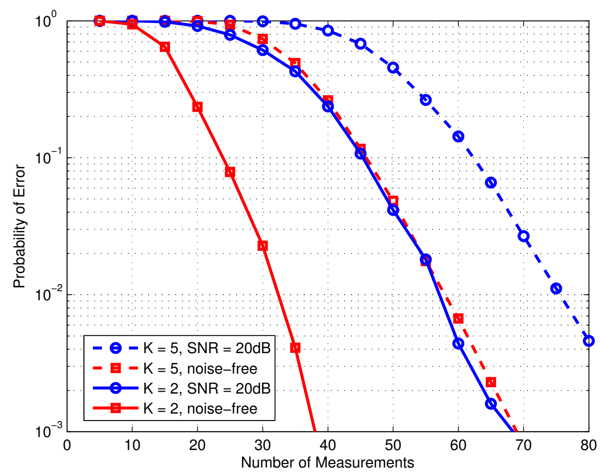

Fig. 3: MUD error rate as a function of the number K of active users for a fixed number of measurements M.