Multihop MIMO Relay Networks with ARQ

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***Abstract*—A multiple antenna multihop relay network consisting of a source, a relay, and a destination node, is considered. The diversity-multiplexing-delay tradeoffs (DMDT) for various multihop ARQ protocols are obtained. It is shown that the tradeoff region is limited by the performance of the weakest link, and hence the optimal ARQ protocol should balance the link performances by allocating the ARQ rounds among all links. Based on this argument, a Variable Block-Length (VBL) ARQ protocol is proposed and its DMDT-optimality is shown.**[1]

I. Introduction

Multiple input-multiple output (MIMO) systems can provide increased data rates by creating multiple parallel channels and robustness against channel variations by increasing diversity. Relaying provides similar diversity and/or multiplexing gains by exploiting the resources of nearby terminals. To obtain the cooperative diversity provided by relaying, the destination combines the signals from the source and the relay to decode the underlying message [1], [2]. In hop-by-hop relaying, each terminal receives the signal only from the previous terminal in the route and hence, the relays are used for coverage extension rather than increasing diversity. Here we study a multihop MIMO relay system.

Another degree of freedom can be introduced by an automatic repeat request (ARQ) protocol for retransmissions. With the multihop ARQ protocol, the receiver at each hop feeds back to the transmitter a one-bit indicator on whether the message can be decoded or not. In case of a failure the transmitter sends additional parity bits until either successful reception or message expiration.

To characterize and compare the performances of various ARQ protocols, we use the diversity-multiplexing-delay tradeoff (DMDT) analysis. The diversity-multiplexing tradeoff (DMT) in a point-to-point MIMO system is introduced in [3]. Considering a third dimension of delay in this high SNR analysis, the DMDT analysis for a point-to-point MIMO system with ARQ is studied in [4], and the DMDT curve is shown to be the scaled version of the corresponding DMT curve without ARQ.

In this work, we extend the point-to-point DMDT analysis to multihop MIMO systems. In our model, there is no direct link between the source and the destination, and a relay enables the communication. We constrain the relay to half-duplex operation, that is, it cannot transmit and receive at the same time. Due to the half-duplex constraint, an important issue is the allocation of the transmit and listen times of the relay.

For cooperative relaying, a dynamic decode-and-forward (DDF) protocol is proposed and shown to dominate all existing protocols in terms of the DMT performance in [5]. In DDF, the relay listens to the source transmission until it can decode the message, and then starts transmitting jointly with the source. In a cooperative relay channel, the DMT performance of DDF falls short of the cut-set bound [5]; however, it is shown to achieve the optimal DMT performance in multihop MIMO networks in [6]. In this work, we extend the DMT analysis in [6] to include the delay dimension introduced by ARQ. A related work [7] considers the DMDT for cooperative transmission for single antenna terminals.

Suppose that a maximum of L ARQ rounds is allowed for the transmission of each message from the source to the destination. The DMDT analysis reveals that the system performance is limited by the weakest link. Hence the optimal ARQ protocol should allocate the number of ARQ rounds among the hops to balance their performances. We consider two types of ARQ protocols: fixed and adaptive. We further present two types of adaptive ARQ protocols: fixed block-length (FBL) ARQ and variable-block-length (VBL) ARQ. We study the DMDT under both the the long-term static and the short-term static channel assumptions [4]. The DMDT has closed-form expressions in some special cases, and can be cast into a convex optimization problem in general. We also prove that the VBL ARQ protocol achieves the optimal DMDT in multihop MIMO relay networks. The FBL ARQ may be a suboptimal approximation

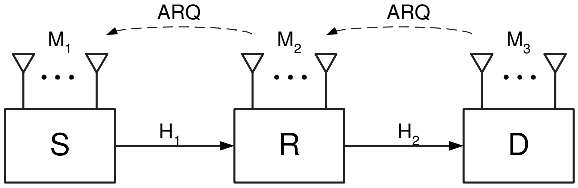

Fig. 1: The (M_1, M_2, M_3) multihop MIMO relay network with ARQ.

when a single terminal is allowed to transmit within each channel block. While we present our results for a single relay, extension to multiple relays is possible as in [6]. The rest of the paper is organized as follows. Section II and Section III introduce the system model and the ARQ protocol, respectively. Section IV presents our DMDT analysis for various ARQ protocols. Some illustrative numerical examples are shown in Section V. Finally Section VI concludes the paper.

II. System Model

We consider a three node multihop relay network, denoted by (M_1, M_2, M_3), consisting of a source, a relay, and a destination with M_1, M_2, and M_3 antennas, respectively (see Fig. 1). At the source, the message is encoded by a space-time encoder and mapped into a sequence of L matrices, $\{\boldsymbol{X}_{1,l} \in \mathcal{C}^{M_1 \times T} : l = 1, \cdots, L\}$, where T is the block-length and L is the maximum number of ARQ rounds. We consider using the DDF protocol for transmission, which is described in more detail below. The decoded message at the relay is reencoded by a space-time encoder into a sequence of L matrices $\{\boldsymbol{X}_{2,l} \in \mathcal{C}^{M_2 \times T} : l = 1, \cdots, L\}$. The source-relay and the relay-destination channels are given by:

$$\boldsymbol{Y}_{i,l} = \sqrt{\frac{SNR}{M_i}} \boldsymbol{H}_{i,l} \boldsymbol{X}_{i,l} + \boldsymbol{W}_{i,l}, \quad 1 \leq l \leq L_i, \quad (1)$$

for $i = 1, 2$, respectively, L_i is the number of ARQ rounds used by link i, and $\boldsymbol{Y}_{i,l} \in \mathcal{C}^{M_{i+1} \times T}$, $i = 1, 2$, are the received signals at the relay and the destination, respectively, in the lth ARQ round. Channels are assumed to be frequency non-selective, block Rayleigh fading and independent of each other, i.e., the entries of the channel matrices $\boldsymbol{H}_{i,l} \in \mathcal{C}^{M_{i+1} \times M_i}$ are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance. The additive noise terms $\boldsymbol{W}_{i,l}$ are also i.i.d. complex Gaussian with zero mean and unit variance. There is no direct link from the source to the destination, and no feedback link from the destination to the source: forward and feedback links only exist between the source-relay and the relay-destination pairs.

Several other key assumptions in our model are as follows:

(i) The relay is half-duplex.
(ii) There is a constraint on the total number of ARQ rounds, $L_1 + L_2 = L$, or equivalently, a transmission time constraint of $t_1 + t_2 = LT$, where t_1 and t_2 are the number of channel uses of the two hops.
(iii) We assume short-term power constraints at the source and the relay for each block code, given by $\mathcal{E}\{\mathbf{tr}(\boldsymbol{X}_{i,l}^{\dagger} \boldsymbol{X}_{i,l})\} \leq M_i T$, $i = 1, 2$, where $\mathcal{E}\{\cdot\}$ denotes expectation, $\mathbf{tr}$ is matrix trace operator, and $\dagger$ denotes the Hermitian transpose.
(iv) We consider both the long-term static channel, in which $\boldsymbol{H}_{i,l} = \boldsymbol{H}_i$ for all l, i.e. the channel state, independent for different i, remains constant during all the ARQ rounds; and the short-term static channel, in which $\boldsymbol{H}_{i,l}$ are i.i.d. but not identical.

We remark that power control is not considered in our model as dictated by (iii). The goal is to isolate the ARQ gain from the power control gain.

III. Multihop ARQ Protocols

We consider several ARQ protocols and compare their DMDTs. These multihop ARQ protocols all use standard point-to-point ARQ retransmissions, and differ in the way they allocate the total number of ARQ rounds among the hops. To simplify the analysis, the ACK/NACK feedback is assumed to be error-free and zero-delay.

A. Fixed ARQ Procotols

One can allocate a fixed number of ARQ rounds L_1 and L_2 for each link, subject to $L_1 + L_2 = L$. The allocation is fixed regardless of the instantaneous link quality. In the kth round, the source transmits the block $\boldsymbol{X}_{1,k}$ to the relay. After receiving the whole block, the relay tries to decode the message. If the relay fails to decode, it sends a negative acknowledgement (NACK) message to the source, which triggers the source to transmit the next block of the current message $\boldsymbol{X}_{1,k+1}$. If the source receives a NACK at the end of block L_1, it moves on to the next message. On the other hand, if the relay succeeds in decoding in the kth round, $k \leq L_1$, it sends an acknowledgement message (ACK) to the source and starts transmitting $\boldsymbol{X}_{2,1}$ to the destination. A similar ARQ scheme is applied to the relay-destination hop. The relay keeps forwarding to the destination until either L_2 rounds of ARQ has been reached or the destination sends an ACK. Upon receiving an ACK signal from the destination, the relay sends a second ACK signal to the source. After receiving the first ACK from the relay, the source stops transmitting, and it starts the transmission of a new message if either it receives a second ACK from the relay, or L_2 ARQ rounds have been reached after the first ACK.

B. Adaptive ARQ Protocols

One can also allocate a variable number of ARQ rounds for each hop, adapted to the instantaneous link quality. The following two adaptive ARQ protocols are designed depending on whether the relay can start transmitting only at the beginning of a block or at any time within a block.

1) Fixed Block-Length (FBL) ARQ Protocol: One type of adaptive ARQ protocol is the *Fixed Block-Length (FBL) ARQ protocol*, in which the relay can start forwarding only at the beginning of a channel block. In FBL ARQ, the source-relay link can use up to $L-1$ rounds for retransmission. If the relay succeeds in decoding after the τth round, $\tau \leq L-1$, then it can use $L-\tau$ ARQ rounds to transmit the message to the destination. Note that τ depends on the instantaneous source-relay link quality. To be more specific, in the first hop, the source retransmits each time it receives a NACK from the relay. Once the relay is able to decode, it starts transmitting at the beginning of the next channel block. In the second hop, the relay retransmits each time it receives an NACK from the destination. Once the destination is able to decode (or fail to decode in $L-\tau$ rounds), it sends an ACK (or NACK) to the relay and now the relay sends an ACK to the source to initiate the transmission of a new message.

2) Variable Block-Length (VBL) ARQ Protocol: Another type of adaptive ARQ protocol is the *Variable Block-Length (VBL) ARQ protocol*, in which the relay can start forwarding at any time within a block.

IV. DMDT Analysis

The DMDT characterizes the tradeoff among the data rate (the multiplexing gain), the reliability (the diversity gain), and the time diversity gain (the allowed amount of delay or total number of ARQ rounds). We consider a family of space-time codes $\mathcal{C}(\rho)$ indexed by their operating SNR ρ, which has rate $R(\rho)$ and error probability $P_e(\rho)$. For this family, the multiplexing gain r and the diversity gain d are defined by $r = \lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho}$, and $d = -\lim_{\rho\to\infty} \frac{\log P_e(\rho)}{\log\rho}$. It is shown in [3] that the error probability is dominated by the information outage probability $P_{out}(\rho)$ when the block length is sufficiently long. For each r, define $f_{M_1,M_2}(r)$ as the supremum of the diversity gain d over all families of codes for a MIMO system with M_1 transmit and M_2 receiver antennas. The characterization of the diversity-multiplexing tradeoff (DMT) $f_{M_1,M_2}(r)$ for a point-to-point MIMO system is given by the following theorem [3].

Theorem 1. *For a sufficiently long block-length, $L \geq M_1+M_2-1$, the diversity-multiplexing tradeoff (DMT) $f_{M_1,M_2}(r)$ for a MIMO system with M_1 transmit and M_2 receive antennas is given by the piece-wise linear function connecting the points $(r,(M_1-r)(M_2-r))$, for $r = 0,\cdots,\min(M_1,M_2)$.*

The DMT for MIMO ARQ system is defined similarly in terms of the effective multiplexing rate $r_e = \lim_{\rho\to\infty}\frac{\eta(\rho)}{\log\rho}$ where $\eta(\rho)$ is the throughput of the system. For half-duplex channel, using renewal theory we have $r_e = \frac{r}{2}$.

A. Long-Term Static Channel

First, we analyze the DMDT of the aforementioned ARQ protocols for the long-term static channel model. The proofs for all the theorems below can be found in [8].

Theorem 2. *The DMDT of the fixed ARQ protocol in a two-hop MIMO relay network with long-term static channel, L_1 ARQ rounds on the first hop and $L-L_1$ ARQ rounds on the second hop, is given by*

$$d_F(r_e, L_1, L) = \min\left\{ f_{M_1,M_2}\left(\frac{2r_e}{L_1}\right), f_{M_2,M_3}\left(\frac{2r_e}{L-L_1}\right)\right\}.$$

Theorem 2 shows that the system performance is limited by the weakest link. This implies that the optimal choice of the L_is should equalize the DMTs such that $f_{M_1,M_2}\left(\frac{2r_e}{L_1}\right) = f_{M_2,M_3}\left(\frac{2r_e}{L-L_1}\right)$. When the solution is not an integer, we consider the two integers closest to the solution, and choose the one with the higher diversity gain.

For some special cases, there are closed form solutions captured in the following corollary.

Corollary 3. *For an $(M_1,1,M_3)$ system, letting $A = \frac{M_1}{M_3}$ and $L_1(r_e) = \beta L$, w hen $A \neq 1$, the optimal ARQ allocation is found by solving $\beta = \frac{A-1+(A+1)2r_e\pm\sqrt{[2A+(1+A^2)](2r_e-1)^2-4A}}{2(A-1)}$, where the sign is chosen so that $\beta \in (0,1)$. When $A=1$, we have $\beta = \frac{1}{2}$. If the solution is not an integer, we check the two closest integers for the highest diversity gain.*

Similarly, for an (M,M_2,M) system, the optimal number of rounds for the fixed ARQ protocol is given by $L_1 = L_2 = \frac{1}{2}L$, independent of the multiplexing gain. If the solution is not an integer, we check the two closest integers for the highest diversity gain.

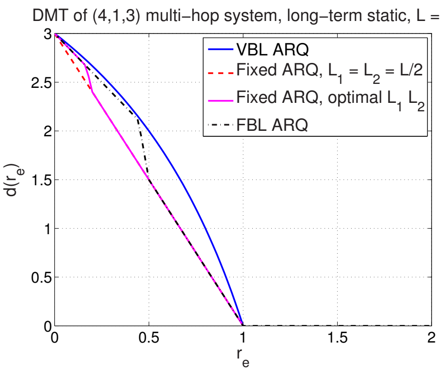

Fig. 2: The DMDT of a $(4, 1, 3)$ system with $L = 4$.

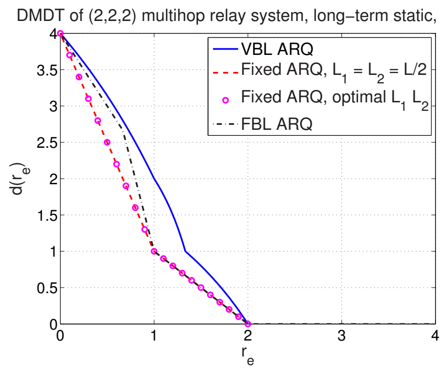

Fig. 3: The DMDT of a $(2, 2, 2)$ system with $L = 4$.

The DMDT of the FBL ARQ protocol is also a piecewise linear function as stated in the next theorem.

Theorem 4. *The DMDT of the FBL ARQ protocol in a two-hop MIMO relay network with a long-term static channel is given by*

$$d_{FBL}(r_e, L) = \min_{l=2,\cdots,L-1} \left\{ f_{M_1,M_2}\left(\frac{2r_e}{L-1}\right), f_{M_1,M_2}\left(\frac{2r_e}{l-1}\right) + f_{M_2,M_3}\left(\frac{2r_e}{L-l}\right), f_{M_2,M_3}\left(\frac{2r_e}{L-1}\right) \right\}.$$

In particular, we have a closed-form DMDT expression for an $(M_1, 1, M_3)$ system.

Corollary 5. *The DMDT of the FBL ARQ protocol for an $(M_1, 1, M_3)$ system is given by*

$$d_{FBL}(r_e, L) = \min_{l=2,\cdots,L-1} \left\{ M_1\left(1 - \frac{2r_e}{L-1}\right)^+, M_1\left(1 - \frac{2r_e}{l-1}\right)^+ + M_2\left(1 - \frac{2r_e}{L-l}\right)^+, M_2\left(1 - \frac{2r_e}{L-1}\right)^+ \right\}.$$

Next, we characterize the DMDT of the VBL ARQ protocol. The DMDT of the VBL ARQ protocol under the long-term static channel assumption is similar to the DMT of DDF without ARQ given in [6], with proper scaling of the multiplexing gain.

Theorem 6. *The DMDT of a VBL ARQ protocol for an (M_1, M_2, M_3) system, in the long-term static channel, is given by*

$$d_{VBL}(r_e, L) = \inf_{(\alpha_1,\alpha_2)\in\mathcal{O}} \sum_{i=1}^{2} \sum_{j=1}^{M_i^*} (2j - 1 + |M_i - M_{i+1}|)\alpha_{i,j},$$

where $M_i^* \triangleq \min\{M_i, M_{i+1}\}$, α_{ij} are the exponents of the channel matrix eigenvalues, and

$$\mathcal{O} \triangleq \left\{ (\alpha_1, \alpha_2) \in \mathcal{R}^{M_1^*} \times \mathcal{R}^{M_2^*} : \alpha_{i,1} \geq \cdots, \geq \alpha_{i,M_i^*} \geq 0, \frac{2r_e}{L} > \frac{S_1(\alpha_1)S_2(\alpha_2)}{S_1(\alpha_1) + S_2(\alpha_2)} \right\}$$

where $S_i(\alpha_i) \triangleq \sum_{j=1}^{M_i^*}(1 - \alpha_{i,j})^+$.

Corollary 7. (1) *The DMDT of a VBL ARQ protocol in an $(M_1, 1, M_3)$ system with a long-term static channel is given by*

$$d_{VBL}(r_e, L) = \begin{cases} \min\{M_1, M_3\}\frac{1-4r_e/L}{1-2r_e/L}, & 0 \leq 2r_e \leq L/2; \\ 0, & \text{otherwise.} \end{cases}$$

(2) *The DMDT of a VBL ARQ protocol in a $(1, M_2, 1)$ two-hop MIMO relay network with a long-term static channel is given by $d_{VBL}(r_e, L) = M_2\frac{1-4r_e/L}{1-2r_e/L}$ for $0 \leq 2r_e \leq L/2$, and 0 otherwise.*

(3) *The DMDT of a VBL ARQ protocol in a (2, 2, 2) two-hop MIMO relay network with a long-term static channel is given by*

$$d_{VBL}(r_e, L) = \begin{cases} \frac{2(4-10r_e/L)}{2-2r_e/L}, & 0 \leq 2r_e \leq L/2; \\ \frac{3-8r_e/L}{1-2r_e/L}, & L/2 \leq 2r_e \leq 2/3L; \\ \frac{4(1-2r_e/L)}{2-2r_e/L}, & 2/3L \leq 2r_e \leq L. \end{cases} \quad (2)$$

B. Short-Term Static Channel

Compared to the fixed ARQ, the adaptive ARQ protocols are more flexible, and hence, achieve better DMDT performance. The DMDT of the FBL ARQ under the short-term static channel model is given as follows.

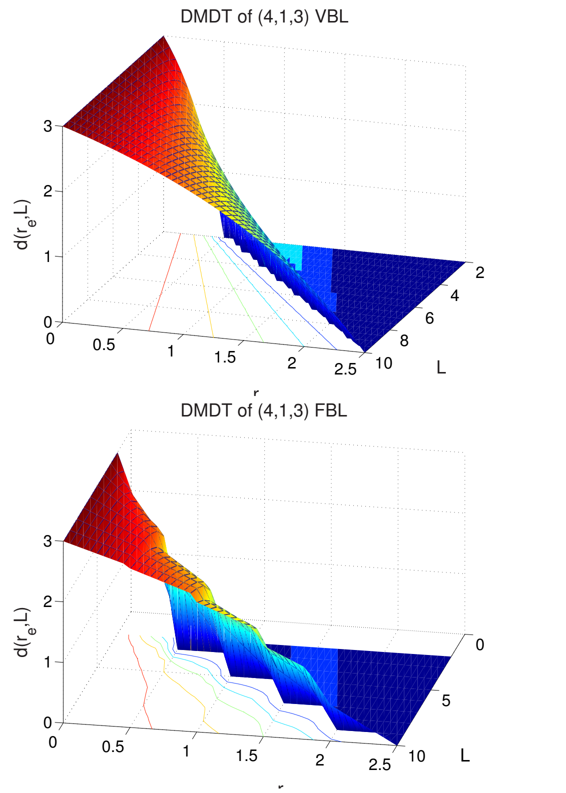

Fig. 4: DMDT of a (4,1,3) system with the VBL ARQ protocol (left), and the FBL ARQ protocol (right).

Theorem 8. *The DMDT of a FBL ARQ protocol in an (M_1, M_2, M_3) system with a short-term static channel is given by*

$$d_{FBL}(r_e) = \min_{k=2,\cdots,L-1} \left\{ (L-1) f_{M_1,M_2}\left(\frac{2r_e}{L-1}\right), (k-1) f_{M_1,M_2}\left(\frac{2r_e}{k-1}\right) + (L-k) f_{M_2,M_3}\left(\frac{2r_e}{L-k}\right), (L-1) f_{M_2,M_3}\left(\frac{2r_e}{L-1}\right) \right\}.$$

Then one can derive a similar corollary to Corollary 5 for Theorem 8.

Comparing the above result with that of the long-term static channel model in Theorem 4, we note that each part of the piece-wise linear DMT function is multiplied by a gain factor due to the time diversity.

We do not have a closed-form expression for the DMDT of the VBL ARQ protocol in the short-term static channel, since a closed-form solution for the number of ARQ blocks (τ) after which the relay decodes, is random rather than deterministic as in the previous case. Here the accumulated mutual information at the relay is a random walk with a positive drift. Hence the stopping time τ when this random walk hits the threshold R is random.

Theorem 9. *The DMDT of the VBL ARQ protocol in a two-hop MIMO relay network in the short-term static channel is*

$$d_{VBL}(r_e, L) = \inf_{(\alpha_1,\alpha_2)\in\mathcal{O}} \sum_{i=1}^{2} \sum_{j=1}^{M_i^*} \sum_{l=1}^{L} (2j - 1 + |M_i - M_{i+1}|)\, \alpha_{ij}^l,$$

where α_{ij}^ls are the exponents of the channel matrix eigenvalues in the lth channel use. The feasible domain is defined as

$$\begin{aligned} \mathcal{O}_1 &= \Bigg\{ (\alpha_1, \alpha_2) : \sum_{l=1}^{L-\lfloor t_1 \rfloor - 1} S_2(\alpha_2^l) + \\ &\quad (1-\gamma) S_2(\alpha_2^{L-\lfloor t_1 \rfloor}) < 2r_e, \Bigg\}, \\ \mathcal{O}_2 &= \left\{ (\alpha_1, \alpha_2) : \alpha_{i,1}^l \geq \cdots, \geq \alpha_{i,M_i^*}^l \geq 0, \forall l \right\} \end{aligned} \quad (3)$$

where

$$t_1 \doteq \inf \left\{ t \in \mathbb{R} : \sum_{i=1}^{\lfloor t \rfloor} S_1(\alpha_1^l) + (t - \lfloor t \rfloor) S_1(\alpha_1^{\lfloor t \rfloor + 1}) = 2r_e \right\}$$

and $\gamma = t_1 - \lfloor t_1 \rfloor \in (0,1)$ are determined from $\sum_{l=1}^{\lfloor t_1 \rfloor} S_1(\alpha_1^l) + \gamma S_1(\alpha_1^{\lfloor t_1 \rfloor + 1}) = 2r_e$ for a given r, while

$$\mathcal{O} \triangleq (\mathcal{R}^{M_1^* L} \times \mathcal{R}^{M_2^* L}) \cap (\mathcal{O}_1 \cap \mathcal{O}_2), \quad (4)$$

and $S_i(\alpha_i^l) = \sum_{j=1}^{M_i^*} (1 - \alpha_{i,j}^l)^+$.

Theorem 10. *The VBL ARQ achieves the optimal DMDT in the multihop MIMO relay network with ARQ, in both the long-term and short-term static channel models.*

V. Numerical Examples

We first present a set of examples for the long-term static channel model. Consider the $(4, 1, 3)$ system and a maximum of $L = 4$ ARQ rounds. In Fig. 2 we show the DMDT of the fixed ARQ scheme with $L_1 = L_2 = 2$ and with the optimal L_1, L_2 given in Corollary 3, as well as the DMDT of FBL and VBL ARQs. The DMDT of the FBL ARQ is a piece-wise approximation to that of the VBL ARQ, which is the optimal DMDT. Similarly, the DMDTs of all the ARQ protocols in a $(2, 2, 2)$ system are shown in Fig. 3. Fig. 4 shows the three dimensional DMDT surfaces of the VBL and the FBL ARQs, respectively, for a $(4, 1, 3)$ system. Fig. 5 shows the cross sections of the surfaces in Fig. 4 at $L = 2$ and $L = 10$. Note that the DMDT curve not only "stretches" as L increases, like in the point-to-point

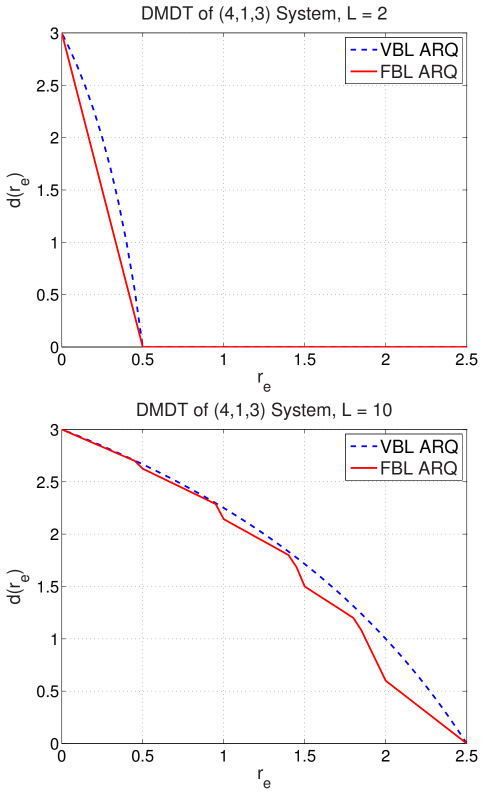

Fig. 5: The slices of the DMDT surfaces in Figure 4 at $L = 1$ (left), and $L = 10$ (right).

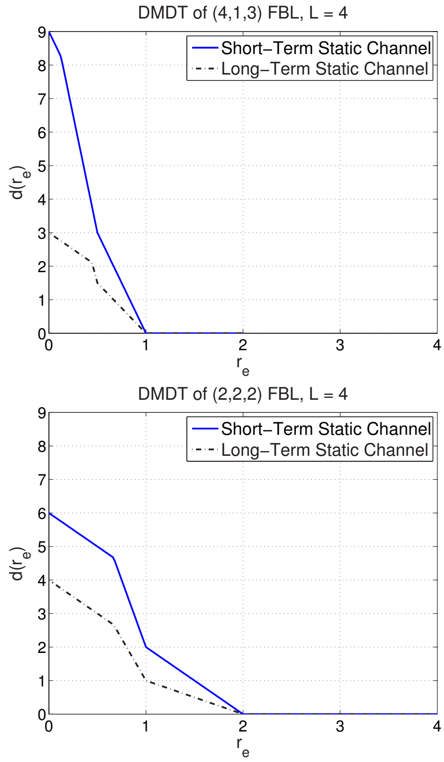

Fig. 6: The DMDT for a (4,1,3) system in the long-term static channel (left), and in the short-term static channel (right), with $L = 4$.

MIMO system [4], but also has more pieces for a larger L. As L increases, the DMDT of FBL ARQ becomes a better approximation to the DMDT of the VBL ARQ.

The DMDT of a $(4, 1, 3)$ system under the short-term static channel model for the FBL ARQ is shown in Fig. 6. Also note that the DMDT in the short-term static channel is not necessarily that of the long-term static channel multiplied by L. In this respect, the results in the multihop MIMO relay network differs from those in the point-to-point MIMO channel [4].

VI. Conclusions

We have derived the diversity-multiplexing-delay tradeoff (DMDT) for multihop MIMO relay networks. We have considered both the long-term and short-term static channel models. We have derived closed-form expressions for the DMDT in some special cases, and showed some illustrative numerical examples. Finally, we have proved that the VBL ARQ protocol achieves the optimal DMDT under both channel models. Our results, presented here for a single relay case, can be extended to the multiple relay scenario as well.

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