A correction to the RUN DAP for Conjunctive RAS presented in "Polynomial-Complexity Deadlock Avoidance Policies for Sequential Resource Allocation Systems", by Reveliotis, Lawley and Ferreira, IEEE TAC, vol. 42, pgs 1344-1357

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This technical correspondence first identifies a problem with the correctness of the RUN DAP for Conjunctive RAS, presented in [2], and it subsequently proceeds to the problem correction through an appropriate policy modification. More specifically, the first part of this note provides a counter-example to the policy correctness (i.e., deadlock-freedom of the controlled system), claimed in [2], while the second part introduces the proposed correction, and briefly argues the correctness of the modified DAP. A more detailed mathematical treatment of RUN DAP in the Petri net formalism, and its redefinition so that it covers the broader class of Conjunctive/Disjunctive RAS, can be found in [1]. For the sake of brevity, it is presumed in the following that the reader is familiar with the material of [2].

The counter-example Consider a conjunctive RAS with four resources, $\mathcal{R} = \{R_1, R_2, R_3, R_4\}$ and supporting a single job with process plan:

$$JT \ : \ \begin{bmatrix} 0 \\ 0 \\ \alpha \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \beta \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \gamma \\ \delta \\ 0 \\ \varepsilon \end{bmatrix} \tag{1}$$

Furthermore, suppose that the resource capacities satisfy the following inequalities:

$$C_1 \geq 2\gamma \tag{2}$$

$$2\delta \leq C_2 < \beta + \delta \quad (\Longrightarrow \beta > \delta) \tag{3}$$

$$C_3 \geq 2\alpha \tag{4}$$

$$C_4 \geq 2\varepsilon \tag{5}$$

The reader can verify that using the natural ordering of the system resources, i.e., $o(R_i) = i,\ i = 1, \ldots, 4$, the RUN definition of [2] (cf. pg. 1352) results in the following stage nominal requirements:

$$JT^{\mathcal{P}} \ : \ \begin{bmatrix} \gamma \\ \delta \\ \alpha \\ \varepsilon \end{bmatrix} \rightarrow \begin{bmatrix} \gamma \\ \beta \\ 0 \\ \varepsilon \end{bmatrix} \rightarrow \begin{bmatrix} \gamma \\ \delta \\ 0 \\ \varepsilon \end{bmatrix} \tag{6}$$

But then, it can be easily checked that, under the assumptions of Equations 2 - 5, state $s =< 2, 0, 0 >$ is a reachable induced-deadlock state, when the aforementioned RAS is controlled by the considered RUN implementation.

The policy correction The problem exposed in the above example is resolved, and a correct RUN definition for Conjunctive RAS is obtained, when Equation 10 in the original policy definition (cf. [2]), is modified as follows:

$$JT^{\mathcal{P}}_{ik}[q] = \begin{cases} \max\{JT_{il}[q] : \ JT_{il} \in \mathcal{N}_{ik}\} & \text{if} \ \ o(R_q) \ \geq \ \min\{o(R_j) : JT_{ik}[R_j] > 0\} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The reader can verify that the above modification resolves the induced deadlock constructed in the introductory example, since the new nominal stage requirements are:

$$JT^{\mathcal{P}} \ : \ \begin{bmatrix} 0 \\ 0 \\ \alpha \\ \varepsilon \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \beta \\ 0 \\ \varepsilon \end{bmatrix} \rightarrow \begin{bmatrix} \gamma \\ \delta \\ 0 \\ \varepsilon \end{bmatrix} \tag{8}$$

It is also noticed that the modification of Equation 7 incidentally enhances the policy flexibility. A proof for the correctness of the modified policy can still be based on the argument of Appendix B in [2], with R_{min}, in Case 3 of that argument, denoting a minimally ordered resource that has some of its units allocated, *under the resource reservation scheme established by the (modified) policy definition.*[1] Finally, as it was mentioned in the opening paragraph, a more detailed mathematical treatment of RUN DAP in the Petri net formalism, and in an extended form that covers the broader class of Conjunctive/Disjunctive RAS, can be found in [1].

References

[1] J. Park and S. A. Reveliotis. Deadlock avoidance in sequential resource allocation systems with multiple resource acquisitions and flexible routings. *IEEE Transactions on Automatic Control (to appear)*, 2001.

[1] It is this italicized part that was confounded in the original argument.

[2] S. A. Reveliotis, M. A. Lawley, and P. M. Ferreira. Polynomial complexity deadlock avoidance policies for sequential resource allocation systems. *IEEE Transactions on Automatic Control*, 42(10):1344–1357, 1997.