

# ISyE7201: Production and Service System Engineering

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## Homework #5

Due Date: April 18, 2016

**A. Reading assignment:** This homework focuses on the theory of the Jackson and Gordon-Newell networks, the M/G/1 queue, the M/G/1 queue with priority disciplines, and the application of these models to production and service systems.

The solution of this homework can be based on the relevant material presented in class, but you can also refer to the following sources: For queueing networks, Sections 4.1 - 4.3 in your textbook, but I also recommend that you read the rest of the sections in Chapter 4. The material covered in M/G/1 queue is in Section 5.1 of your textbook, primarily in Sections 5.1.1-5.1.4. Finally, priority queues are addressed in Section 3.4 of your textbook primarily in the context of Markovian queueing stations. The class presentation on this subject generalizes and extends some of these developments.

### B. Problems:

**a.** Solve Problems 8.11, 8.14 and 8.15 from the textbook by Cassandras and Lafortune.

**b.** Solve Problems 3.28, 3.33, 3.35 and 3.36 from your textbook.

**c.** In class it was suggested that the computation of the probability generating function for the steady state distribution  $\pi$  for the number of customers in a stable M/G/1 queue enables another approach for developing the PK formulae characterizing the steady-state performance of this queue in terms of expected number of customers in the system, mean time in system, mean number of customers waiting in the queue and the expected waiting time in queue. Provide the details for this development, i.e., provide the complete derivation of the PK formulae starting from the formula

$$\Pi(z) = \frac{(1 - \rho)(1 - z)K(z)}{K(z) - z}$$

that characterizes the probability generating function of the steady state distribution  $\pi$  of the number of customers in a stable M/G/1 queue.

**d.** Consider a workstation that produces a final product by fastening together two major sub-assemblies. Jobs arriving at this workstation consist of kits containing one unit from each of the two sub-assemblies, and if both parts are in good order, the fastening operation can be performed at an average time of  $t = 2min$ . However, each of the two parts in a kit can also be defective, with corresponding probabilities  $p_1 = 0.3$  and  $p_2 = 0.2$ . A defective part must go through some additional rework that occurs locally and requires an exponentially distributed time; the corresponding processing rates are  $r_1 = 0.2min^{-1}$  and  $r_2 = 0.1min^{-1}$ . Part failures are independent from each other, and in the case that both parts in a kit are defective, the necessary reworks take place simultaneously. Use the above information in order to determine the effective processing capacity of this station. Express your result in product units per hour.

**e.** A service station is processing the commingled stream of two part types, each of which arrives according to a Poisson process with rate  $\lambda_i$ . Parts are processed on a FCFS basis, and the expected service time for either part is equal to  $t_p$  time units. But when switching from one part type to the other, there is an additional deterministic set-up time equal to  $t_s$  time units. Provide the stability condition for this station; your response must be expressed in terms of the data set provided above.

**f.** The single server of a considered workstation can experience two types of failure. Both types of failure can occur only when the server is operational, they occur independently from each other, and their occurrences follow Poisson processes with corresponding rates  $\lambda_i$ ,  $i = 1, 2$ . Mean times to repair for these two types of failure are respectively  $t_i$ ,  $i = 1, 2$ . We define the *availability*  $A$  of this server as the percentage of time that the server is functional (in the presence of jobs that request its processing).

Answer the following questions:

1. What is the availability of the considered server?
2. Assuming that both types of failure are non-destructive (i.e., the server resumes the processing of the running jobs once it is repaired) and the "nominal" processing times (i.e., the times that are required for the processing of the parts without accounting for the downtimes due to failure) for this server are uniformly distributed over the interval  $[a, b]$ , what is the expected number of failures that take place during the processing of a single part?
3. Finally, assuming that the jobs requesting processing at this work-

station arrive according to a Poisson process with rate  $r_a$ , compute the maximum value for  $r_a$  that will lead to a stable operation of the considered workstation. The requested  $r_a$  is the *effective processing capacity* of this workstation.