Homework 1 Solution
September 26, 2007

1. **(Ross 1.3)**
$S = \{(e_1, e_2, \cdots, e_n), n \geq 2\}$ where $e_i \in \{heads, tails\}$
In addition $e_n = e_{n-1} =$ heads and for i = 1,...,n-2 if $e_i =$ heads, then $e_{i+1} =$ tails.
$\Pr\{4\ tosses\} = \Pr\{(t,t,h,h)\} + \Pr\{(h,t,h,h)\} = 2[\frac{1}{2}]^4 = \frac{1}{8}$

2. **(Ross 1.4)**
(a) $F \bigcap E^c \bigcap G^c$
(b) $E \bigcap F \bigcap G^c$
(c) $E \bigcup F \bigcup G$
(d) $(E \bigcap F) \bigcup (E \bigcap G) \bigcup (F \bigcap G)$
(e) $E \bigcap F \bigcap G$
(f) $E^c \bigcap F^c \bigcap G^c$
(g) $(E \bigcup F)^c \bigcup (E \bigcup G)^c \bigcup (F \bigcup G)^c$
(h) $(E \bigcap F \bigcap G)^c$

Remark: In the rest of this document, to simplify notation, we shall omit $\bigcap$ in the provided expressions; thus, $E \bigcap F$ will be represented by EF.

3. **(Ross 1.8)**
$\Pr\{E \bigcup F\} = \Pr\{E\} + \Pr\{F\} - \Pr\{EF\} \leq 1$. Therefore, $\Pr\{EF\} \geq \Pr\{E\} + \Pr\{F\} - 1$.

For the provided numbers: 0.9 + 0.8 - 1 = 0.7.

4. **(Ross 1.12)**
The ample space will consist of n-dimensional vectors of possible events, n=1,2,..., such that the first n-1 entries are events other than E & F while the last entry is equal to either E or F.

$\Pr\{E\ occurs\ before\ F\} =$
$\sum_{n=1}^{\infty} \Pr\{E\ occurs\ before\ F \mid n\ trials\} \Pr\{n\ trials\} =$
$\sum_{n=1}^{\infty} \Pr\{outcome\ of\ nth\ trial\ is\ E \mid n\ trials\} \Pr\{n\ trials\} =$
$\sum_{n=1}^{\infty} \Pr\{E\}(1 - \Pr\{E\} - \Pr\{F\})^{n-1} =$
$\Pr\{E\} \frac{1}{1-(1-\Pr\{E\}-\Pr\{F\})} = \frac{\Pr\{E\}}{\Pr\{E\}+\Pr\{F\}}$.

5. **(Ross 1.20)**
Let E denote the desired event, D denote the event of all outcomes being different and F denote the event of all outcomes being the same. These events are not only mutually exclusive, but they also define the entire

sample space. Hence, 1 = P(E) + P(D) + P(F).
There are 216 possible outcomes (6^3). For D, there are 120 possible outcomes ($6 \times 5 \times 4$) and for F, only 6. Thus, $1 = P(E) + \frac{120}{216} + \frac{6}{216}$ and $P(E) = \frac{5}{12}$.

6. **(Ross 1.21)**
   Let C = the event that the chosen person is color blind.
   $P(Male|C) = \frac{P(C|Male)P(Male)}{P(C|Male)P(Male)+P(C|Female)P(Female)}$
   $= \frac{.05\times.5}{.05\times.5+.0025\times.5} = \frac{20}{21}$.

7. **(Ross 1.28)**
   Yes.
   $\Pr\{A|B\} > \Pr\{A\} \Leftrightarrow \Pr\{AB\} > \Pr\{A\}\Pr\{B\} \Leftrightarrow \Pr\{B|A\} > \Pr\{B\}$

8. **(Ross 1.29)**
   (a) 0
   (b) $\Pr\{E|F\} = P(EF)/P(F) = P(E)/P(F) \geq P(E) = 0.6$
   (c) $\Pr\{E|F\} = P(EF)/P(F) = P(F)/P(F) = 1$

9. **(Ross 1.35)**
   (a) $(1/2)^4 = 1/16$
   (b) $(1/2)^4 = 1/16$
   (c) 15/16, since the only way that pattern H, H, H, H can appear before pattern T, H, H, H is when the first 4 flips are all heads.

10. **(Ross 1.36)**
    P(Ball is black) = P(Ball is black | First box chosen)P(First box chosen) + P(Ball is black | Second box chosen)P(Second box chosen) = $\frac{1}{2} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2} = \frac{7}{12}$

11. **(Ross 1.37)**
    Let B_i be the event that box i is chosen, and W be the event marble is white.
    $\Pr\{B_1|W\} = \frac{\Pr\{W|B_1\}\Pr\{B_1\}}{\Pr\{W|B_1\}\Pr\{B_1\}+\Pr\{W|B_2\}\Pr\{B_2\}}$
    $= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5}$

12. **(Ross 1.39)**
    Let W = event woman resigns; A, B, C are events the person resigning works in store A,B,C, respectively.

$\Pr\{C|W\} = \frac{P(W|C)P(C)}{P(W|C)P(C)+P(W|B)P(B)+P(W|A)P(A)}$
$= \frac{.70\times\frac{100}{225}}{.70\times\frac{100}{225}+.60\times\frac{75}{225}+.50\times\frac{50}{225}}$

13. **(Ross 1.44)**
Let W = event white ball selected.
$\Pr\{T|W\} = \frac{P(W|T)P(T)}{P(W|T)P(T)+P(W|H)P(H)}$
$= \frac{\frac{1}{5}\cdot\frac{1}{2}}{\frac{1}{5}\cdot\frac{1}{2}+\frac{5}{12}\cdot\frac{1}{2}} = \frac{12}{37}$

14. **Prob 1.**
This experiment essentially corresponds to an unordered selection with replacement of three items out of a set with 6 possible types. Thus, applying the relevant result discussed in class, we get $\begin{pmatrix} n+k-1 \\ k \end{pmatrix} = \frac{(n+k-1)!}{k!(n-1)!} =$ $\begin{pmatrix} 6+3-1 \\ 3 \end{pmatrix} = 56.$

15. **Prob 2.**
Let iH = the event that i is Happy, i=1,2,3.

Then $P(1H\bigcap 2H\bigcap 3H) = P(2H\bigcap 3H|1H)P(1H)$
$= P(3H|1H\bigcap 2H)P(2H|1H)P(1H)$

However P(1H) = 1 (since she gets to choose first)

$P(2H|1H) = P(2H\bigcap 1A|1H) + P(2H\bigcap 1B|1H)$
$= P(2H|1H\bigcap 1A)P(1A|1H) + P(2H|1H\bigcap 1B)P(1B|1H)$
$= P(2H|1A)P(1A|1H) + P(2H|1B)P(1B|1H) = 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2} = 1$

$P(3H|1H\bigcap 2H) = P(3H\bigcap 1A|1H\bigcap 2H) + P(3H\bigcap 1B|1H\bigcap 2H)$
$= P(3H\bigcap 1A\bigcap 2B|1H\bigcap 2H)+P(3H\bigcap 1A\bigcap 2C|1H\bigcap 2H)+P(3H\bigcap 1B\bigcap 2C|1H\bigcap 2H)$
$= P(3H|1A\bigcap 2B)P(1A\bigcap 2B)+P(3H|1A\bigcap 2C)P(1A\bigcap 2C)+P(3H|1B\bigcap 2C)P(1B\bigcap 2C)$
$= P(3H|1A\bigcap 2B)P(1A)P(2B|1A) + P(3H|1A\bigcap 2C)P(1A)P(2C|1A) +$
$P(3H|1B\bigcap 2C)P(1B)P(2C|1B)$
$= 0 + 1\cdot\frac{1}{2}\cdot\frac{1}{2} + 1\cdot\frac{1}{2}\cdot 2$
$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$