ISYE 6201A,Q: Manufacturing Systems
Instructor: Spyros Reveliotis
Midterm Exam I
(Take Home)
Release Date: March 8, 2022
Due Date: March 11, 2022

Name: SOLUTIONS

Please, upload your responses to CANVAS as a pdf file named just by your last name.

Also, make sure to write clearly your complete name in your response document.

And if you create the pdf file by taking pictures of your solution write-up, please, try to control the size of the resulting pdf file by using the minimal resolution that will ensure a legible document.

More generally, please, keep the size of your response file as small as possible. Frequently, large files are not transmitted properly through email, and are not processed easily by the typical e-readers. So, I might have to ask for a smaller file if your file is too big. You should be able to obtain reasonably sized files by running your original file through Adobe Acrobat and (re-)saving it, before emailing it.

In addition, your response file must be properly paginated (sometimes, I have received such response files that, in Adobe Acrobat or any other pdf reader, open up as a very long page that is impossible to process). You can check this issue by running your file through Adobe Acrobat.

Finally, I remind you that you are expected to abide by the Georgia Tech Honor Code during this exam.

Answer the following questions **(8 points each)**:

1. Briefly explain how the current Information Technologies (IT) have helped modern corporations break the typical trade-off that exists between the strategic objectives of "responsiveness" and "cost leadership" that were discussed in the introductory lectures.

The typical tradeoff between responsiveness and cost leadership arises from the need to hedge against the uncertainty w.r.t. the expected demand and possibly other operational aspects of the company, by maintaining either extra inventory or extra processing capacity.

Modern IT have incurred a significant reduction of this uncertainty, and therefore, they have enabled the support of the same responsiveness with less buffering in terms of inventory or proc. capacity.

2. Provide some examples of how the notion of "market segmentation" has been implemented in academia.

Here are some such examples:

1) Coverage of the same topic in different ways and different profundity levels based on the maturity and/or the particular needs of the audience (undergraduate vs. graduate vs. short courses)

2) Deployment of overseas operations with the corresponding programs customized to the needs of the local communities.

3) Development of on-line programs with a different set of requirements and credit structure from the more traditional ones.

3 Consider a variation of the EOQ model discussed in class where any placed order of Q units does not incur a fixed ordering cost of A dollars, but it is shipped though a 3rd-party logistics provider who charges x dollars per unit. All the remaining operational assumptions and involved costs are similar to those of the EOQ model discussed in class. What is the optimal order size Q^* in this case? What are the practical implications of your result?

In this case, the cost associated with a replenishment order is not fixed (i.e., independent of the order size) but variable. Hence, using the notation that was introduced in the discussion of the EOQ model, we can write the corresponding total annual cost as a function of Q as:

$$TAC(Q) = (C+x)D + \frac{hQ}{2}$$

and for $Q \geq 0$, the above function is minimized at $\underline{Q=0}$!

A practical interpretation of this result is that when we do not have any fixed cost for replenishment, then we should just order on an as-needed (or <u>just-in-time</u>) basis.

In the context of replenishment that involves production activity, the above realization has culminated in a very extensive effort to reduce (or even eliminate) the involved set up costs, which define the corresponding fixed cost.

4. In an application of the news-vendor model, the purchasing price of a unit is \$20, the selling price is \$30, and any remaining units will be taken back by the supplier with a refund of \$10 per unit. Also, demand is normally distributed. Based on the provided information, we can conclude that the safety stock that is established by the optimal order size is equal to zero.

(a) TRUE (b) FALSE

Please, briefly explain your answer.

From the provided information we have:

$$C_s = 30-20 = 10 = 20-10 = C_o$$

and $C_s/(C_s+C_o) = 10/(10+10) = 0.5$

Hence,

$$G(Q^*) = 0.5 \Rightarrow \Phi\left(\frac{Q^*-\theta}{\sigma}\right) = 0.5 \Rightarrow$$

$$\Rightarrow \frac{Q^*-\theta}{\sigma} = z_{0.5} = \emptyset \Rightarrow Q^*-\theta = \emptyset$$

But $Q^*-\theta$ is the corresponding safety stock.

5. Summarize brielfy the main reasoning that provides the analytical characterization of the average inventory level, average backorder level, and the fill rate of the (Q, r) model.

1) By defining the reorder point r w.r.t. the inventory position $IP(t)$, we can see that (possibly with the exception of the initial replenishment cycle), $IP(t) \in \{r+1, \ldots, r+Q\}$.

2) Furthermore, as claimed in class, as $t \to \infty$, $IP(t)$ takes its possible values with equal probability $1/Q$.

3) Finally, we also know that, for any t, $IP(t)$ is the available inventory for meeting the experienced demand ~~over~~ the next l time units, where l is the replenishment lead times. In particular, $IP(t)$ together with the corresponding demand determine $OHI(t+l)$, $BO(t+l)$, and the fill rate over this time interval.

4) For a fixed $IP(t)$ level R, over t, the corresponding performance indices are those of the basestock model with basestock level R. Now that $IP(t)$ varies as discussed above, the resulting performance is the average of the performances of the corresponding basestock models.

Problem 1 (30 points): Three different items, A, B and C, are replenished by the same supplier. The cost of receiving a single shipment from this supplier is \$50, while holding cost is determined by an annual interest rate of 10%.

The monthly demand and the purchasing price for these three items are as follows:

Item	Monthly Demand (units)	Purchasing price (\$/unit)
1	100	25.0
2	200	50.0
3	150	40.0

i. (10 pts) Determine the optimal order sizes for these three items and the resulting total annual cost for the company (summed across all three items) when the inventories of these items are controlled independently from each other, according to the EOQ model.

ii. (20 pts) Also, determine the order sizes for these three items and the resulting total annual cost for the company when replenishment orders are coordinated according to the heuristic of the "Power-of-2-order-intervals". In the involved computation, assume that the considered company operates every day of the calendar year, and use the day as the basic time unit for the employed grid of the valid replenishment periods. Also assume that synchronized shipments incur a single delivery cost. What are the annual savings for the company that result from the effected synchronization of its replenishments?

(i) Using the EOQ formula $\sqrt{\frac{2AD}{iC}}$, and rounding to the nearest integer, we find

$Q_1^* = 219$; $Q_2^* = 219$; $Q_3^* = 212$

And from $TAC(Q) = C \cdot D + A\, D/Q + iC\, Q/2$, we also have:

$$\left.\begin{aligned} TAC_1(Q_1^*) &= 30547.72 \\ TAC_2(Q_2^*) &= 121095.45 \\ TAC_3(Q_3^*) &= 72848.53 \end{aligned}\right\} \Rightarrow \sum_{i=1}^{3} TAC_i(Q_i^*) = 224491.70$$

(ii) First we compute the replenishment period T_i^* that result from Q_i^*, using the formula:

$$T_i^* = \text{round}\left(\frac{Q_i^*}{D} \cdot 365\right)$$

In the above formula D is the annual demand, and the multiplication with 365 converts the result into days. We get

$$T_1^* = 67; \quad T_2^* = 33; \quad T_3^* = 43$$

Next, we refer each of the above values to a power of 2, according to the logic discussed in class:

$$T_1^* \in (2^6 = 64, 2^7 = 128) \text{ and } 2^6\sqrt{2} \cong 91$$

Hence, $T_1 \rightarrow 64$

$$T_2^* \in (2^5 = 32, 2^6 = 64) \text{ and } 2^5\sqrt{2} \cong 45 \Rightarrow$$

$$T_2 \Rightarrow 32$$

$$T_3^* \in (2^5 = 32, 2^6 = 64) \text{ and } 2^5\sqrt{2} \cong 45 \Rightarrow$$

$$T_3 \Rightarrow 32$$

So, items B and C will be replenished every 32 days, and at every second replenishment of these two items, there will also be a replenishment of item A.

Thanks to this coordination of the item replenishments, the total annual ordering cost for all these three items is equal to $\$50 \times \frac{365}{32} = \$570.31$

Also, the order sizes Q_i corresponding to the determined T_i can be computed by

round $\left(\frac{T_i}{365} \times D \right)$, obtaining

$Q_1 = 210$; $Q_2 = 210$; $Q_3 = 158$

Then, the annual purchasing plus holding cost for item i is:

$$X_i = C_i D_i + i C_i \frac{Q_i}{2}$$

Hence

$X_1 = 30262.5$; $X_2 = 120525$; $X_3 = 72316$

Finally, the total annual cost in this case is

$$570.31 + \sum_{i=1}^{3} X_i = 223673.81$$

We can see that we have a cost reduction of

$224491.7 - 223673.81 = 817.89$ \$/yr.

While the previous computations correspond to a routine implementation of the employed heuristic, in the considered scenario we can do even better when noticing that we can also replenish item 1 every 32 days instead of 64. This decision will not impact the experienced ordering cost, but it will reduce the holding cost of this item by a factor of 1/2, since the new order size for this item will be

$$Q_1' = \text{round}\left(\frac{T_1/2}{365} \times D\right)$$

I took both answers as correct in the grading of this problem.

Problem 2 (30 points): Consider a hot-dog seller who runs her stand 12 hours every day. She gets a new batch of hot-dogs at the end of every second day, and due to quality considerations, at that point she must dispose any remaining hot dogs of the previous batch (she gives these unsold hot dogs to a local food bank).

The seller purchases the hot dogs at the price of \$1.00 and sells them at the price of \$2.50. Also, while she is open, customers arrive according to a Poisson process with rate 20 customers per hour.

Finally, her stand cannot store more than 450 hot dogs.

Please, answer the following questions:

i. (10 pts) What is an optimized replenishment batch for this operation?

ii. (10 pts) What is the expected profit per batch?

iii. (10 pts) What is the probability that there will be unsold hot dogs in any given batch, if the seller operates according to your findings in part (i) above?

Remark: Remember that for arrivals occurring according to a Poisson process with rate λ, the number of arrivals occurring in an interval $[0, t]$ is distributed according to a Poisson distribution with parameter λt. Also, notice that this distribution is a discrete distribution. Finally, see Theorem 1 in item 2 of the list of course materials (`https://www2.isye.gatech.edu/~spyros/courses/IE6201/Spring-22/course_materials.html`) for the determination of the optimal order size in a news-vendor model with discrete demand (I also talked about this issue when I presented the news-vendor example on the mug ordering for the club celebration).

(i) Over a 2-day period, the hot-dog seller is open for $t = 2 \times 12 = 24$ hours.
Furthermore, since during this time the customers arrive according to a Poisson process with rate $\lambda = 20\ hr^{-1}$, the 2-day demand experienced by this seller follows a Poisson distribution with mean $\lambda t = 480$.

Also, from the provided data,

$C_s = 2.5 - 1 = 1.5$ (\$/unit)

$C_o = 1.0$ (\$/unit)

and $C_s/(C_o + C_s) = \frac{1.5}{1+1.5} = 0.6$.

Since in this case the demand is a discrete r.v., the optimal order size Q^* is the smallest value s.t. $G(Q^*) \geq 0.6$, and $G(\cdot)$ is the cdf for Poisson (480). Using the formula for this cdf, it can be checked that $Q^* = 485$.

Considering also that the stand capacity is a 450 hot dogs, the optimized order is $Q = 450$.

cii) We have

$$E[\text{Profit}] = E[\text{Revenue}] - \text{Purchasing Cost}$$

$$\text{Purchasing cost} = \$1 \cdot 450 = 450$$

$$E[\text{Revenue}] = 2.5\left[\sum_{x=0}^{450} x \cdot \frac{e^{-480} \cdot 480^x}{x!} + 450\,[1 - G(450)]\right] \simeq 2.5\,[38.79 + 450\,(1 - 0.088)) \simeq 1122.94$$

Hence, $E[\text{profit}] = 1122.94 - 450 = 672.94$

(iii) Letting X denote the 2-day demand, we want to compute $P(X < 450) = P(X \leq 449) = G(449) \approx 0.081$

<u>Remark</u>: In the above calculations, the involved cdf and pmf values for the Poisson(480) distr. have been obtained from Excel.