ISYE 6201A,Q: Manufacturing Systems
Instructor: Spyros Reveliotis
Final Exam
(Take Home)
Release Date: April 29, 2022
Due Date: May 1, 2022

Name: SOLUTION

Please, upload your responses to CANVAS as a pdf file named by your last and first name, in this order.

Also, make sure to write clearly your complete name in your response document.

And if you create the pdf file by taking pictures of your solution write-up, please, try to control the size of the resulting pdf file by using the minimal resolution that will ensure a legible document.

More generally, please, keep the size of your response file as small as possible. Frequently, large files are not transmitted properly through email, and are not processed easily by the typical e-readers. So, I might have to ask for a smaller file if your file is too big. You should be able to obtain reasonably sized files by running your original file through Adobe Acrobat and (re-)saving it, before emailing it.

In addition, your response file must be properly paginated (sometimes, I have received such response files that, in Adobe Acrobat or any other pdf reader, open up as a very long page that is impossible to process). You can check this issue by running your file through Adobe Acrobat.

Finally, I remind you that you are expected to abide by the Georgia Tech Honor Code during this exam.

Answer the following questions **(8 points each)**:

1. A stable single-server workstation is fed with parts according to a Poisson process with rate 10 hr^{-1}. Circle all the statements below that are true on the basis of this information.

i. The mean processing time of the workstation server is 6 min.

ii. The workstation throughput is 10 hr^{-1}.

iii. The processing capacity of this workstation is 10 hr^{-1}.

iv. The variance of the part inter-arrival times at this workstation is 0.1 hr^2.

Since the workstation is stable, its throughput is equal to the arrival rate.

However, the server processing capacity can be higher. For the same reason, we cannot infer the statement in (i).

Finally, the Poisson nature of the arrival process implies that the part inter-arrival times are exponentially distributed, and therefore,

$$SCV(T_a) = \frac{Var(T_a)}{E^2[T_a]} = 1 \Rightarrow$$

$$\Rightarrow Var[T_a] = E^2[T_a] = \left(\frac{1}{10} hr\right)^2 =$$

$$= 0.01 \; hr^2$$

2. For every G/G/1 queueing station, the probability that an arriving customer will find the server busy is equal to the server utilization.

(a) TRUE (b) FALSE

Please, briefly explain your answer.

Consider the example discussed in class of the single-server workstation with the inter-arrival and proc. times being deterministic and equal. In this case, the server utilization is 100%, but each arrival finds the server idle.

On the other hand, the considered statement is true for every M/G/1 queueing station, because of PASTA.

3. Briefly explain how pull manufacturing systems have raised the need for *proactive demand management*.

In order to deliver the desired throughput rate, pull manufacturing systems must operate in a "steady state" mode, and therefore, the control parameters of the system must be kept constant for significantly long periods. Characteristically, your textbook suggests keeping these parameters constant for at least one quarter.
Furthermore, in class it was demonstrated that even small adjustments of the target throughput require significant modifications of the WIP ceilings imposed on different parts of the line.

Proactive demand management recognizes the above limitation of pull manufacturing systems, and tries to develop strategies that will enable the line managers to accommodate any experienced variation in the demand, as well as operational contingencies of the line, while keeping the (nominal) capacity of the line constant to a specified value.

4. What is the essence of the aggregation that is performed in the context of the aggregate planning problem, and what are the main reasons for this aggregation?

Aggregate planning considers part families instead of SKUs. SKUs are grouped into families based on the commonality of their processing requirements and each family is represented by a fictitious product type. The demand of this type is the cumulative demand of all the constituent SKUs of the family. Also, the proc. time for producing a unit of this fictitious type is a weighted average of the corresponding proc. times of the family SKUs, with the weights being the percentages of the demand of each SKU in the total demand of the family.

This aggregation

(i) reduces the computational complexity of the corresponding planning task, and

(ii) renders the generated plans more robust to the forecasting error in the demand of the different SKUs.

5. Briefly explain the rationale for the lot-sizing heuristic of *Part-Period Balancing (PPB)* that is frequently used in the context of MRP explosion. What is the primary motivation of the logic implemented by this heuristic? Also, why this heuristic might be more preferable in the considered operational context than the Wagner-Whitin algorithm which is the optimal algorithm for the underlying lot-sizing problem?

The PPB heuristic recognizes that in the considered lot sizing problem, the key trade-off is between the resulting set up and holding costs, and therefore, it keeps adding periods in a created lot as long as the difference between these two costs, for the constructed lot, is decreased.

Frequently, this heuristic is preferred to the W-W algorithm, because it respects the W-W property of the optimal plans, usually returns a near-optimal plan with respect to the incurred costs, and at the same time, it does not experience the nervousness of the generated plans due to the uncertainty contained in the demand forecasts for more distant periods into the planning horizon. The W-W algorithm is susceptible to such nervousness.

Problem 1 (20 points): Consider a single-server manufacturing workstation which processes two different part types, 1 and 2. Parts of each type arrive according to independent Poisson processes with corresponding rates $r_{a_1} = 8\ hr^{-1}$ and $r_{a_2} = 5\ hr^{-1}$. The processing times for these two part types have means $t_{p_1} = 5\ min$ and $t_{p_2} = 3\ min$, and CV's $c_{p_1} = 0.6$ and $c_{p_2} = 0.7$. Arriving parts from both parts types join a common waiting queue and they are processed according to the FCFS policy.

Answer the following questions

i. (5 pts) Show that the considered workstation is stable.

ii. (10 pts) Perform a mean value analysis for each part type. In particular, for each part type, compute (i) the long-term throughput, (ii) the expected cycle time for a part of this type, and (iii) the expected number of parts from this type in the station, when the station is operated at steady-state.

iii. (5 pts) Also compute the following probabilities: (i) the probability that a part in the waiting queue is of type 1. (ii) The probability that, when the server is busy, the customer in service is type 1.

(i) The total expected workload arriving per hour at this workstation is

$$r_{a_1} \cdot t_{p_1} + r_{a_2} t_{p_2} = 8 \cdot \frac{5}{60} + 5 \cdot \frac{3}{60} \simeq 0.917 < 1.$$

Hence, the workstation is stable.

(ii) Since the workstation is stable, from the conservation of material we have

$$TH_1 = r_{a_1} = 8\ hr^{-1} \quad \text{and} \quad TH_2 = r_{a_2} = 5\ hr^{-1}.$$

Furthermore, the operation of this workstation can be depicted as follows:

Since the two arrival processes are independent, the combined arrival process that considers part arrivals irrespective of the part types, is also Poisson with rate $r_a = r_{a_1} + r_{a_2} = 8 + 5 = 13\ hr^{-1}$.

Furthermore, since all arriving parts are processed on a FCFS basis, we can perceive the considered workstation as an M/G/1 queue with the part proc. times modeled by the following r.v.:

$$T_e = \begin{cases} T_{p_1} & \text{w.p. } r_{a_1}/(r_{a_1}+r_{a_2}) = \frac{8}{13} \equiv q_1 \\ T_{p_2} & \text{w.p. } r_{a_2}/(r_{a_1}+r_{a_2}) = \frac{5}{13} \equiv q_2 \end{cases}$$

Hence,

$$E[T_e] = 5 \cdot \frac{8}{13} + 3\frac{5}{13} = \frac{55}{13} \simeq 4.23 \text{ min}$$

Also,

$$E[T_e^2] = q_1 E[T_{p_1}^2] + q_2 E[T_{p_2}^2] =$$

$$= q_1 (Var[T_{p_1}] + E^2[T_{p_1}]) + q_2 (Var[T_{p_2}] + E^2[T_{p_2}]) =$$

$$= q_1 (SCV[T_{p_1}] + 1) E^2[T_{p_1}] + q_2 (SCV[T_{p_2}] + 1) E^2[T_{p_2}] =$$

$$= \frac{8}{13}(0.6^2+1)\,5^2 + \frac{5}{13}(0.7^2+1)\cdot 3^2 \simeq 26.08 \text{ min}^2$$

Then, $Var[T_e] = E[T_e^2] - E^2[T_e] = 26.08 - 4.23^2 \simeq 8.19 \text{ min}^2$

and $SCV[T_e] = Var[T_e] / E^2[T_e] = 8.19/4.23^2 \simeq 0.458$

So, $CT_q = \frac{1+SCV[T_e]}{2} \frac{u}{1-u} E[T_e] = \frac{1+0.458}{2} \frac{0.917}{1-0.917} \cdot 4.23 =$

$\simeq 34.1$ min

and $CT_1 = CT_q + t_{p_1} = 34.1 + 5 = 39.1$ min

$CT_2 = CT_q + t_{p_2} = 34.1 + 3 = 37.1$ min

Also, from Little's law:

$$WIP_1 = TH_1\, CT_1 = 8 \cdot \frac{39.1}{60} = 5.21$$

$$WIP_2 = TH_2\, CT_2 = 5 \cdot \frac{37.1}{60} = 3.09$$

(iii)

P[a waiting part in type 1] = P[corresp. arrival was type 1] = (from theory of Poisson process) $\frac{r_{a_1}}{r_{a_1}+r_{a_2}} = \frac{8}{13}$

P[part type 1 | server busy] =

P[server busy ∧ part type 1] / P[server busy] =

= (from PASTA) $\frac{r_{a_1} t_{p_1}}{r_{a_1} t_{p_1} + r_{a_2} t_{p_2}} = \frac{8 \cdot 5}{8 \cdot 5 + 5 \cdot 3} =$

$= \frac{40}{55} = \frac{8}{11}$

Problem 2 (20 points): Consider a CONWIP line with single-server stations that operates at 80% of its capacity. The current WIP in the line is set to 20 parts. Furthermore, it can be safely assumed that, at the current operational regime, the line cycle time CT varies linearly with the WIP. The bottleneck mean processing time t_b is equal to 15 minutes.

Answer the following questions:

i. (5 pts) What are the production capacity and the current throughput of this line? Please, provide your responses in parts per hour.

ii. (5 pts) What is the WIP level that will enable the line to operate at 90% of its capacity?

iii. (10 pts) What is the throughput and the expected cycle time of this line if we increase the WIP level to 30 parts?

(i) The line capacity is $C = r_b = 1/t_b = \frac{1}{15}\ min^{-1} =$

$= 60\ \frac{min}{hr}\ \frac{1}{15}\ min^{-1} = 4\ hr^{-1}$.

Hence, the current throughput is

$TH = 0.8\,C = 0.8 \cdot 4\ hr^{-1} = 3.2\ hr^{-1}$.

(ii) Let $W' = W + \Delta W$ and $\Delta W = x W$ for some x.

From the problem assumption, $CT' = CT + \Delta CT = CT + t_b \Delta W$

and from Little's law:

$$0.9\,C = TH' = \frac{W'}{CT'} = \frac{W + \Delta W}{CT + t_b \Delta W} = \frac{1 + \Delta W / W}{CT/W + t_b\, \Delta W / W} =$$

$$= \frac{1 + x}{\frac{1}{0.8C} + t_b x} \Rightarrow 0.9 \cdot 4 = \frac{1 + x}{\frac{1}{0.8 \cdot 4} + \frac{15}{60} x} \Rightarrow \underline{x = 1.25}$$

Finally, $W' = W + xW = W(1+1.25) =$

$= 2.25 \cdot 20 = 45.$

(iii) From the problem assumptions:

$\Delta CT = t_b \Delta W \Rightarrow \Delta CT = 15 \cdot 10 = 150 \text{ min} =$

$= 2.5 \text{ hr.}$

Hence, $CT' = \frac{W}{TH} + \Delta CT = \frac{20}{3.2} + 2.5 = 8.75 \text{ hr}$

and

$TH' = \frac{W'}{CT'} = \frac{30}{8.75} \approx 3.43 \text{ hr}^{-1}.$

Problem 3 (20 points): A local sports company will produce golf clubs on an assembly line, according to the eight operations that are listed in the following table:

Task	Req. Time (min)	Imm. Preds
1. Polish shaft	9	
2. Grind the shaft end	10	
3. Polish the club head	6	
4. Imprint number	4	3
5. Connect wood to shaft	5	1,2,4
6. Place and secure connecting pin	3	5
7. Place glue on other end of shaft	3	1
8. Set in grips and balance	9	6, 7

The above table also reports the required time for each task in minutes, and the immediate predecessors for each task.

Please, answer the following questions:

i. (3 pts) Draw the precedence diagram representing the task precedence constraints that are specified by the above table, and determine the successor set S_i for each task $i = 1, \ldots, 8$, according to the definition of this set that was provided during the coverage of the ALB problem.

ii. (3 pts) What is the *maximal* throughput that can be supported by this line?

iii. (3 pts) What is the *cycle time* for this assembly line that will attain a throughput of 5 clubs per hour?

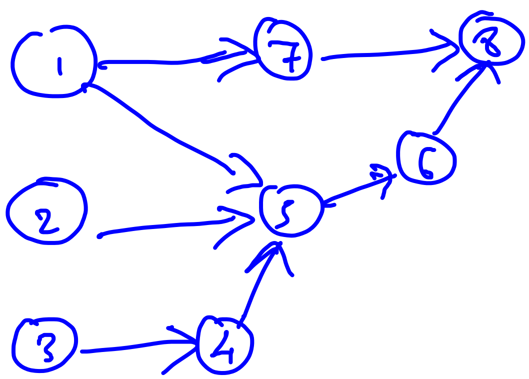
iv. (3 pts) What is a lower bound to the required number of workstations for this assembly line that is implied by the cycle time that you computed in item (iii) above?

v. (3pts) Assuming that there is an assembly line that can deliver the desired throughput with the number of workstations that you computed in part (iv), what is the total idle time per cycle, across all the workstations, in this line?

vi. (5 pts) Argue that $|S_i|$ (i.e., the cardinality of the task successor set S_i) is another index that can be used in the position of the Positional

Weights PW_i in the corresponding heuristic for the ALB problem presented in class, and develop a design for the considered assembly line *using this modified heuristic* and the target throughput specified in part (iii).

(i)



From this diagram we have that:

$S_8 = \{8\}$

$S_7 = \{7\} \cup S_8 = \{7, 8\}$

$S_6 = \{6\} \cup S_8 = \{6, 8\}$

$S_5 = \{5\} \cup S_6 = \{5, 6, 8\}$

$S_4 = \{4\} \cup S_5 = \{4, 5, 6, 8\}$

$S_3 = \{3\} \cup S_4 = \{3, 4, 5, 6, 8\}$

$S_2 = \{2\} \cup S_5 = \{2, 5, 6, 8\}$

$S_1 = \{1\} \cup S_5 \cup S_7 = \{1, 5, 6, 7, 8\}$

(ii) $TH_{max} = \frac{1}{\max\{t_i\}} = \frac{1}{10} = 0.1 \text{ min}^{-1} = 6 \text{ hr}^{-1}$

(iii) $c = \frac{1}{TH} = \frac{1}{5 \text{ hr}^{-1}} = 0.2 \text{ hr} = 12 \text{ min}$

(iv) $N_{min} = \left\lceil \frac{\sum_i t_i}{c} \right\rceil = \left\lceil \frac{49}{12} \right\rceil = 5$

(v) Total idle time per cycle =

$= N_{min} \cdot c - \sum_i t_i = 5 \cdot 12 - 49 = 11 \text{ min}$

(vi) As discussed in class, for any pair of tasks (i, j) with i being a predecessor of j, $S_i \supset S_j$, and the inclusion is strict since $i \in S_i \wedge i \notin S_j$. Furthermore, this inclusion implies $|S_i| > |S_j|$. Hence, ordering tasks in decreasing cardinality of their successor sets respects the precedence constraints among them.

From part (i) we have:

task	1	2	3	4	5	6	7	8
$\lvert S_i \rvert$	5	4	5	4	3	2	2	1

and the corresponding task list is

1, 3, 2, 4, 5, 6, 7, 8

Setting $c = 12$ min and working with the above list as in the case of the RPW heuristic, we get:

WS	WS2	WS3	WS4	WS5
1	3, 4	2	5, 6, 7	8
~~12~~	~~12~~	~~12~~	~~12~~	~~12~~
3	~~6~~	2	~~7~~	3
	2		~~4~~	
			1	