ISYE 6201A,Q: Manufacturing Systems

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Midterm Exam I

March 11, 2019

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Discuss how the concept of "modularity" helps modern corporations support their strategic objective of mass-customization.

Modularity enables the development of different product configurations on the same basic platform, through the interchange of compatible standardized components.
This is the main mechanism behind combinatorial customization which is the main way that modern companies support mass-customization.

Modularity also permits the fast introduction of new product concepts through the reuse of existing parts and technologies.

2. What is the basic notion of "service level" in modern inventory control theory? Please, try to make your answer as general as possible.

Service level is a performance measure that characterizes how well an inventory system supports the experienced demand.
Given the randomness in this demand, typically such a measure is defined as the probability that the system will meet a certain performance target. Type I and Type II service levels discussed in class are specific (and most prominent) examples of this concept in the context of modern inventory control theory.

3. What is the meaning of the "robustness" of the EOQ formula? How can this robustness be useful in the management of the inventory systems that we have been considering in class?

The "robustness" of the EOQ formula implies that a deviation from the EOQ value will <u>not</u> increase very drastically the total annual cost of that item.

Knowing this result makes it easier to deviate from the EOQ value of any given item in order to satisfy additional constraints and/or objectives like capacity constraints or replenishment synchronization across a number of items.

4. Consider the inventory of a certain item that is managed according to the (Q, r) policy discussed in class. Furthermore, the lead-time demand for this item is normally distributed, and the reorder point r for this inventory system has been chosen so that it satisfies a target fill rate level a according to the approximating formula $S(Q, r) \approx 1 - \frac{B(r)}{Q}$.

If the variability of the experienced demand is increased, then, the necessary safety stock for supporting the target fill rate a under the new circumstances

i. will increase.

ii. will decrease.

iii. will remain the same.

iv. might increase or decrease.

Please, explain your answer.

We have: $S(Q,r) \simeq 1 - \frac{B(r)}{Q} =$ (for normally distr. demand)

$= 1 - \frac{\sigma \mathcal{L}\left(\frac{r-\theta}{\sigma}\right)}{Q} = a$ (for a given target level) $\Rightarrow$

$\Rightarrow \quad \mathcal{L}\left(\frac{r-\theta}{\sigma}\right) = \frac{(1-a)Q}{\sigma}$ (1)

If $\sigma \uparrow$ then the rhs of (1) will decrease, and since $\mathcal{L}(\;)$ is a decreasing function, $\frac{r-\theta}{\sigma} = x$ must increase.

Hence, $ss = r - \theta = x \cdot \sigma$ will increase (since σ is also increasing).

This result is qualitatively different from the corresponding result about the behavior of safety stock that we discussed in class because the ultimate objective of these adjustments is different: the objective of the in-class discussion was to control/minimize the accrued cost, while here the objective is to maintain the target fill rate.

5. While using the Wagner-Whitin algorithm in order to determine the optimal order plan for a certain item, we have found that the optimal solution for the two-period subproblem is to order separately for each of the two periods. From this information, we can infer that in the optimal solution for the overall problem that is defined by the entire planning horizon T, the demand for period 1 will be ordered separately from the demand of the other periods.

(a) TRUE (b) FALSE

Please, explain your answer.

This is an application of the "Planning Horizon" theorem discussed in class.

Problem 1 (30 points): A local store with an annual demand of 10,000 units for one of its main products is trying to select a supplier for this product by choosing among two different options. The first supplier (supplier A) quotes a purchasing price of \$15 per unit, and a contract of 50 free deliveries per year at the price of \$1,000 per year. The second supplier (supplier B) quotes a purchasing price of \$20 per unit and she will utilize a 3PL service provider for the deliveries, who charges a transactional (fixed) cost of \$15 per delivery and a variable cost of \$2 per delivered unit. The considered store computes its holding cost on the basis of an annual interest rate of 5%. Assuming that both suppliers can provide comparable quality of product and service, use the above information in order to determine the right supplier for this product.

For supplier A:

$$Q_A^* = \frac{10000}{50} = 200$$

and

$$TAC_A = 15 \times 10000 + 1000 + 15 \times 0.05 \times \frac{200}{2} =$$

$$= 151,075$$

For supplier B: $A = 15$; $C = 20 + 2 = 22$.

But then, the purchasing cost in TAC_B is

$$22 \times 10000 = 220,000 > 151,075$$

So, we should choose supplier A.

Problem 2 (30 points): A convenient store replenishes a particular brand of milk on a weekly basis. The store buys this product from a local supplier at the price of \$1.50 per one-gallon carton, and sells it at the price of \$3.00. Furthermore, any cartons left at the end of the week are taken back by the supplier at the price of \$0.75 to be used in the manufacture of some yogurt that is also produced by this company.

The daily demand for this item at the considered store is approximated quite accurately by a normal distribution with a mean of 20 cartons and a st. deviation of 5 cartons.

Determine an optimal order size for this product that must be placed every week by the considered store.

We have:

$$\left.\begin{array}{l} \text{purchasing price} = 1.50 \\ \text{selling} \quad " \quad = 3.00 \\ \text{salvage value} = 0.75 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} C_o = 1.50 - 0.75 = 0.75 \\ C_s = 3.00 - 1.50 = 1.50 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \text{critical ratio} = \frac{C_s}{C_o + C_s} = \frac{1.5}{0.75 + 1.5} = \frac{2}{3} \simeq 0.67$$

Assuming a 7-day operation for this store, the weekly demand is normally distributed with mean $\theta = 7 \times 20 = 140$ and variance $\sigma^2 = 7 \times 5^2 = 175 \Rightarrow \sigma \simeq 13.23$.

Then, from the main result of the newsvendor model, we must have:

$$G(x) = \Phi\left(\frac{x - 140}{13.23}\right) = 0.67 \Rightarrow \frac{x - 140}{13.23} = 0.44 \quad \text{(from the provided st. table)}$$

$$\Rightarrow x = 140 + 0.44 \times 13.23 = 145.82 \simeq 146 \text{ cartons.}$$

TABLE A–4
Normal Probability Distribution and Partial Expectations

Standardized Variate	Probabilities		Partial Expectations	
z	$F(z)$	$1 - F(z)$	$L(z)$	$L(-z)$
.00	.5000	.5000	.3989	.3989
.01	.5040	.4960	.3940	.4040
.02	.5080	.4920	.3890	.4090
.03	.5120	.4880	.3841	.4141
.04	.5160	.4840	.3793	.4193
.05	.5200	.4800	.3744	.4244
.06	.5239	.4761	.3697	.4297
.07	.5279	.4721	.3649	.4349
.08	.5319	.4681	.3602	.4402
.09	.5359	.4641	.3556	.4456
.10	.5398	.4602	.3509	.4509
.11	.5438	.4562	.3464	.4564
.12	.5478	.4522	.3418	.4618
.13	.5517	.4483	.3373	.4673
.14	.5557	.4443	.3328	.4728
.15	.5596	.4404	.3284	.4784
.16	.5636	.4364	.3240	.4840
.17	.5685	.4325	.3197	.4897
.18	.5714	.4286	.3154	.4954
.19	.5753	.4247	.3111	.5011
.20	.5793	.4207	.3069	.5069
.21	.5832	.4168	.3027	.5127
.22	.5871	.4129	.3027	.5186
.23	.5910	.4090	.2944	.5244
.24	.5948	.4052	.2904	.5304
.25	.5987	.4013	.2863	.5363
.26	.6026	.3974	.2824	.5424
.27	.6064	.3936	.2784	.5484
.28	.6103	.3897	.2745	.5545
.29	.6141	.3859	.2706	.5606
.30	.6179	.3821	.2668	.5668
.31	.6217	.3783	.2630	.5730
.32	.6255	.3745	.2592	.5792
.33	.6293	.3707	.2555	.5855
.34	.6331	.3669	.2518	.5918
.35	.6368	.3632	.2481	.5981
.36	.6406	.3594	.2445	.6045
.37	.6443	.3557	.2409	.6109
.38	.6480	.3520	.2374	.6174
.39	.6517	.3483	.2339	.6239
.40	.6554	.3446	.2304	.6304
.41	.6591	.3409	.2270	.6370
.42	.6628	.3372	.2236	.6436
.43	.6664	.3336	.2203	.6503
.44	.6700	.3300	.2169	.6569
.45	.6736	.3264	.2137	.6637
.46	.6772	.3228	.2104	.6704
.47	.6808	.3192	.2072	.6772
.48	.6844	.3156	.2040	.6840
.49	.6879	.3121	.2009	.6909
.50	.6915	.3085	.1978	.6978

TABLE A–4
(continued)

Standardized Variate	Probabilities		Partial Expectations	
z	$F(z)$	$1 - F(z)$	$L(z)$	$L(-z)$
.51	.6950	.3050	.1947	.7047
.52	.6985	.3015	.1917	.7117
.53	.7019	.2981	.1887	.7187
.54	.7054	.2946	.1857	.7257
.55	.7088	.2912	.1828	.7328
.56	.7123	.2877	.1799	.7399
.57	.7157	.2843	.1771	.7471
.58	.7190	.2810	.1742	.7542
.59	.7224	.2776	.1714	.7614
.60	.7257	.2743	.1687	.7687
.61	.7291	.2709	.1659	.7759
.62	.7324	.2676	.1633	.7833
.63	.7357	.2643	.1606	.7906
.64	.7389	.2611	.1580	.7980
.65	.7422	.2578	.1554	.8054
.66	.7454	.2546	.1528	.8128
.67	.7486	.2514	.1503	.8203
.68	.7517	.2483	.1478	.8278
.69	.7549	.2451	.1453	.8353
.70	.7580	.2420	.1429	.8429
.71	.7611	.2389	.1405	.8505
.72	.7642	.2358	.1381	.8581
.73	.7673	.2327	.1358	.8658
.74	.7703	.2297	.1334	.8734
.75	.7733	.2267	.1312	.8812
.76	.7764	.2236	.1289	.8889
.77	.7793	.2207	.1267	.8967
.78	.7823	.2177	.1245	.9045
.79	.7852	.2148	.1223	.9123
.80	.7881	.2119	.1202	.9202
.81	.7910	.2090	.1181	.9281
.82	.7939	.2061	.1160	.9360
.83	.7967	.2033	.1140	.9440
.84	.7996	.2004	.1120	.9520
.85	.8023	.1977	.1100	.9600
.86	.8051	.1949	.1080	.9680
.87	.8067	.1922	.1061	.9761
.88	.8106	.1894	.1042	.9842
.89	.8133	.1867	.1023	.9923
.90	.8159	.1841	.1004	1.0004
.91	.8186	.1814	.0986	1.0086
.92	.8212	.1788	.0968	1.0168
.93	.8238	.1762	.0955	1.0250
.94	.8264	.1736	.0953	1.0330
.95	.8289	.1711	.0916	1.0416
.96	.8315	.1685	.0899	1.0499
.97	.8340	.1660	.0882	1.0582
.98	.8365	.1635	.0865	1.0665
.99	.8389	.1611	.0849	1.0749
1.00	.8413	.1587	.0833	1.0833
1.01	.8438	.1562	.0817	1.0917
1.02	.8461	.1539	.0802	1.1002
1.03	.8485	.1515	.0787	1.1087
1.04	.8508	.1492	.0772	1.1172
1.05	.8531	.1469	.0757	1.1257

(continued)

TABLE A–4
(continued)

Standardized Variate	Probabilities		Partial Expectations	
z	$F(z)$	$1 - F(z)$	$L(z)$	$L(-z)$
1.06	.8554	.1446	.0742	1.1342
1.07	.8577	.1423	.0728	1.1428
1.08	.8599	.1401	.0714	1.1514
1.09	.8621	.1379	.0700	1.1600
1.10	.8643	.1357	.0686	1.1686
1.11	.8665	.1335	.0673	1.1773
1.12	.8686	.1314	.0659	1.1859
1.13	.8708	.1292	.0646	1.1946
1.14	.8729	.1271	.0634	1.2034
1.15	.8749	.1251	.0621	1.2121
1.16	.8770	.1230	.0609	1.2209
1.17	.8790	.1210	.0596	1.2296
1.18	.8810	.1190	.0584	1.2384
1.19	.8830	.1170	.0573	1.2473
1.20	.8849	.1151	.0561	1.2561
1.21	.8869	.1131	.0550	1.2650
1.22	.8888	.1112	.0538	1.2738
1.23	.8907	.1093	.0527	1.2827
1.24	.8925	.1075	.0517	1.2917
1.25	.8943	.1057	.0506	1.3005
1.26	.8962	.1038	.0495	1.3095
1.27	.8980	.1020	.0485	1.3185
1.28	.8997	.1003	.0475	1.3275
1.29	.9015	.0985	.0465	1.3365
1.30	.9032	.0968	.0455	1.3455
1.31	.9049	.0951	.0446	1.3446
1.32	.9066	.0934	.0436	1.3636
1.33	.9082	.0918	.0427	1.3727
1.34	.9099	.0901	.0418	1.3818
1.35	.9115	.0885	.0409	1.3909
1.36	.9131	.0869	.0400	1.4000
1.37	.9147	.0853	.0392	1.4092
1.38	.9162	.0838	.0383	1.4183
1.39	.9177	.0823	.0375	1.4275
1.40	.9192	.0808	.0367	1.4367
1.41	.9207	.0793	.0359	1.4459
1.42	.9222	.0778	.0351	1.4551
1.43	.9236	.0764	.0343	1.4643
1.44	.9251	.0749	.0336	1.4736
1.45	.9265	.0735	.0328	1.4828
1.46	.9279	.0721	.0321	1.4921
1.47	.9292	.0708	.0314	1.5014
1.48	.9306	.0694	.0307	1.5107
1.49	.9319	.0681	.0300	1.5200
1.50	.9332	.0668	.0293	1.5293
1.51	.9345	.0655	.0286	1.5386
1.52	.9357	.0643	.0280	1.5480
1.53	.9370	.0630	.0274	1.5574
1.54	.9382	.0618	.0267	1.5667
1.55	.9394	.0606	.0261	1.5761
1.56	.9406	.0594	.0255	1.5855
1.57	.9418	.0582	.0249	1.5949
1.58	.9429	.0571	.0244	1.6044
1.59	.9441	.0559	.0238	1.6138

TABLE A–4
(continued)

Standardized Variate z	Probabilities F(z)	1 − F(z)	Partial Expectations L(z)	L(−z)
1.60	.9460	.0540	.0232	1.6232
1.61	.9463	.0537	.0227	1.6327
1.62	.9474	.0526	.0222	1.6422
1.63	.9484	.0516	.0216	1.6516
1.64	.9495	.0505	.0211	1.6611
1.65	.9505	.0495	.0206	1.6706
1.66	.9515	.0485	.0201	1.6801
1.67	.9525	.0475	.0197	1.6897
1.68	.9535	.0465	.0192	1.6992
1.69	.9545	.0455	.0187	1.7087
1.70	.9554	.0446	.0183	1.7183
1.71	.9564	.0436	.0178	1.7278
1.72	.9573	.0427	.0174	1.7374
1.73	.9582	.0418	.0170	1.7470
1.74	.9591	.0409	.0166	1.7566
1.75	.9599	.0401	.0162	1.7662
1.76	.9608	.0392	.0158	1.7558
1.77	.9616	.0384	.0154	1.7854
1.78	.9625	.0375	.0150	1.7950
1.79	.9633	.0367	.0146	1.8046
1.80	.9641	.0359	.0143	1.8143
1.81	.9649	.0351	.0139	1.8239
1.82	.9656	.0344	.0136	1.8436
1.83	.9664	.0336	.0132	1.8432
1.84	.9671	.0329	.0129	1.8529
1.85	.9678	.0322	.0126	1.8626
1.86	.9685	.0314	.0123	1.8723
1.87	.9693	.0307	.0119	1.8819
1.88	.9699	.0301	.0116	1.8916
1.89	.9706	.0294	.0113	1.9013
1.90	.9713	.0287	.0111	1.9111
1.91	.9719	.0281	.0108	1.9208
1.92	.9726	.0274	.0105	1.9305
1.93	.9732	.0268	.0102	1.9402
1.94	.9738	.0262	.0100	1.9500
1.95	.9744	.0256	.0097	1.9597
1.96	.9750	.0250	.0094	1.9694
1.97	.9756	.0244	.0092	1.9792
1.98	.9761	.0239	.0090	1.9890
1.99	.9767	.0233	.0087	1.9987
2.00	.9772	.0228	.0085	2.0085
2.01	.9778	.0222	.0083	2.0183
2.02	.9783	.0217	.0080	2.0280
2.03	.9788	.0212	.0078	2.0378
2.04	.9793	.0207	.0076	2.0476
2.05	.9798	.0202	.0074	2.0574
2.06	.9803	.0197	.0072	2.0672
2.07	.9808	.0192	.0072	2.0770
2.08	.9812	.0188	.0068	2.0868
2.09	.9817	.0183	.0066	2.0966
2.10	.9821	.0179	.0065	2.1065
2.11	.9826	.0174	.0063	2.1163
2.12	.9830	.0170	.0061	2.1261
2.13	.9834	.0166	.0060	2.1360
2.14	.9838	.0162	.0058	2.1458

(continued)

TABLE A–4
(*continued*)

Standardized Variate	Probabilities		Partial Expectations	
z	$F(z)$	$1 - F(z)$	$L(z)$	$L(-z)$
2.15	.9842	.0158	.0056	2.1556
2.16	.9846	.0154	.0055	2.1655
2.17	.9850	.0150	.0053	2.1753
2.18	.9854	.0146	.0052	2.1852
2.19	.9857	.0143	.0050	2.1950
2.20	.9861	.0139	.0049	2.2049
2.21	.9864	.0136	.0048	2.2148
2.22	.9868	.0132	.0046	2.2246
2.23	.9871	.0129	.0045	2.2345
2.24	.9875	.0125	.0044	2.2444
2.25	.9878	.0122	.0042	2.2542
2.26	.9881	.0119	.0041	2.2641
2.27	.9884	.0116	.0040	2.2740
2.28	.9887	.0113	.0039	2.2839
2.29	.9890	.0110	.0038	2.2938
2.30	.9893	.0107	.0037	2.3037
2.31	.9896	.0104	.0036	2.3136
2.32	.9898	.0102	.0035	2.3235
2.33	.9901	.0099	.0034	2.3334
2.34	.9904	.0096	.0033	2.3433
2.35	.9906	.0094	.0032	2.3532
2.36	.9909	.0091	.0031	2.3631
2.37	.9911	.0089	.0030	2.3730
2.38	.9913	.0087	.0029	2.3829
2.39	.9916	.0084	.0028	2.3928
2.40	.9918	.0082	.0027	2.4027
2.41	.9920	.0080	.0026	2.4126
2.42	.9922	.0078	.0026	2.4226
2.43	.9925	.0075	.0025	2.4325
2.44	.9927	.0073	.0024	2.4424
2.45	.9929	.0071	.0023	2.4523
2.46	.9931	.0069	.0023	2.4623
2.47	.9932	.0068	.0022	2.4722
2.48	.9934	.0066	.0021	2.4821
2.49	.9936	.0064	.0021	2.4921
2.50	.9938	.0062	.0020	2.5020
2.51	.9940	.0060	.0019	2.5119
2.52	.9941	.0059	.0019	2.5219
2.53	.9943	.0057	.0018	2.5318
2.54	.9945	.0055	.0018	2.5418
2.55	.9946	.0054	.0017	2.5517
2.56	.9948	.0052	.0017	2.5617
2.57	.9949	.0051	.0016	2.5716
2.58	.9951	.0049	.0016	2.5816
2.59	.9952	.0048	.0015	2.5915
2.60	.9953	.0047	.0015	2.6015
2.61	.9955	.0045	.0014	2.6114
2.62	.9956	.0044	.0014	2.6214
2.63	.9957	.0043	.0013	2.6313
2.64	.9959	.0041	.0013	2.6413
2.65	.9960	.0040	.0012	2.6512
2.66	.9961	.0039	.0012	2.6612
2.67	.9962	.0038	.0012	2.6712
2.68	.9963	.0037	.0011	2.6811
2.69	.9964	.0036	.0011	2.6911
2.70	.9965	.0035	.0011	2.7011

TABLE A–4
(concluded)

Standardized Variate	Probabilities		Partial Expectations	
z	$F(z)$	$1 - F(z)$	$L(z)$	$L(-z)$
2.71	.9966	.0034	.0010	2.7110
2.72	.9967	.0033	.0010	2.7210
2.73	.9968	.0032	.0010	2.7310
2.74	.9969	.0031	.0009	2.7409
2.75	.9970	.0030	.0009	2.7509
2.76	.9971	.0029	.0009	2.7609
2.77	.9972	.0028	.0008	2.7708
2.78	.9973	.0027	.0008	2.7808
2.79	.9974	.0026	.0008	2.7908
2.80	.9974	.0026	.0008	2.8008
2.81	.9975	.0025	.0007	2.8107
2.82	.9976	.0024	.0007	2.8207
2.83	.9977	.0023	.0007	2.8307
2.84	.9977	.0023	.0007	2.8407
2.85	.9978	.0022	.0006	2.8506
2.86	.9979	.0021	.0006	2.8606
2.87	.9979	.0021	.0006	2.8706
2.88	.9980	.0020	.0006	2.8806
2.89	.9981	.0019	.0006	2.8906
2.90	.9981	.0019	.0005	2.9005
2.91	.9982	.0018	.0005	2.9105
2.92	.9982	.0018	.0005	2.9205
2.93	.9983	.0017	.0005	2.9305
2.94	.9984	.0016	.0005	2.9405
2.95	.9984	.0016	.0005	2.9505
2.96	.9985	.0015	.0004	2.9604
2.97	.9985	.0015	.0004	2.9704
2.98	.9986	.0014	.0004	2.9804
2.99	.9986	.0014	.0004	2.9904
3.00	.9986	.0014	.0004	3.0004

Source: R. G. Brown, *Decision Rules for Inventory Management* (Hinsdale, IL.: Dryden, 1967). Adapted from Table VI, pp. 95–103.