ISyE 6201A,Q: Manufacturing Systems
Instructor: Spyros Reveliotis
Spring 2019

Homework #2

Due Date: Monday, 3/4/19 for all students

Reading Assignment:
- Sections 2.4-2.5 and Appendices 2A and 2B from your textbook.
- I also suggest that you read Sections 17.5-17.8.

Problem set:

A. Solve Problems 12, 13 and 17 at the end of Chapter 2 of your textbook (the problem numbering refers to the 3rd edition of the book.)

B. In addition, solve the following two problems:

1. Consider your familiar newsboy, but assume that he sells N different newspapers. Each copy of newspaper i is bought at a cost c_i and it is sold at a price $p_i > c_i$. Furthermore, unsold copies of newspaper i are disposed to a recycling company at a value $s_i < c_i$ per copy. Daily demand for newspaper i can be approximated by a continuous random variable X_i with cdf $G_i(\ )$. Finally, a single copy of newspaper i weighs w_i lbs, and the newsboy cannot carry a weight greater than W.
i. Develop a methodology that will enable the newsboy to maximize his expected daily profit.
ii. Apply your methodology developed in Step (i) to the following problem instance:
In the table above, N stands for normal. Also, W=150 lbs.

Paper	Gi	ci ($)	pi ($)	si ($)	wi (lbs)
1	N(100,20)	0.25	1	0.1	0.5
2	N(75,10)	0.5	1.5	0.1	0.75
3	N(50,10)	0.7	2	0.1	1

Hint: Combine the Newsvendor theory with the theory of multi-product inventory control under resource allocation constraints. What is the Lagrangean function for this problem and what conditions does it imply for the optimal solution?

2. A company is faced with the strategic decision of determining the production capacity, in terms of pounds per month, that it should deploy at a new facility. The company wants to match production to demand in each month, to avoid shortages and inventory accumulation. In the contemplated operational regime, any excess demand that cannot be met through the planned capacity will have to be met through overtime, at the differential cost of $100 per pound. On the other hand, the accounting department estimates that under-utilized capacity translates to an opportunity cost of $200 per pound. If monthly demand is expected to be normally distributed with a mean of 100,000 pounds and a st. deviation of 5,000 pounds, what is an optimal selection of the plant capacity, based on the above information?

C. Extra credit (20 pts)

1. (5 pts) Prove that, as claimed in class,

$$\sum_{x=R}^{\infty}(x-R)p(x)=\Theta-\sum_{x=0}^{R-1}[1-G(x)]$$

where $p(x)$, $G(x)$ and Θ denote respectively the probability mass function, the cumulative distribution function and the mean value of some discrete distribution taking values over the set of nonnegative integers.

2. (5 pts) Let L, D, and X be random variables denoting respectively the replenishment lead time (in days), the daily demand, and the demand experienced over a lead time interval. Furthermore, assume that daily demands are independent, identically distributed (i.i.d.) random variables. Then, as it was shown in class,
$$E[X]=E[L]\cdot E[D]$$
Use a similar approach to show that
$$Var[X]=E[L]\cdot Var[D]+E[D]^2\cdot Var[L]$$

3. (10 pts) Consider the "newsvendor" problem and let *c, p* and *s* denote respectively the purchasing price, the selling price and the salvage value for the considered item. Also, let Q denote the order quantity and X denote the random demand to be experienced over the considered interval. In class, we determined Q by employing the following objective:
$$\min\ (c-s)E[\max\{Q-X,0\}]+(p-c)E[\max\{X-Q,0\}]$$
where (c-s) defines the "overage" unit cost, c_o, and (p-c) defines the "underage" or "shortage" unit cost, c_s.
Prove that the above problem is equivalent to the profit-maximizing objective:
$$\max\ pE[\min\{Q,X\}]+sE[\max\{Q-X,0\}]-cQ$$

Finally, those of you with the 2nd edition,, please, remember to consult the document with the errata regarding your textbook, that can be found at:

http://www.factoryphysics.com/documents/Errata_for_Second_Edition.pdf