ISYE 6201: Manufacturing Systems
Spring 2019
Instructor: Spyros Reveliotis
Final Exam
April 26, 2019

Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. Provide an intuitive explanation of the stability condition $\rho < 1$ for the G/G/1 queue.

We have that $\rho = \lambda t$, where:

* λ = arrival rate
* t = mean processing time

The product λt is the average workload that arrives at the station per unit of time.
Since the station has only one (nonfailing) server, this workload should not exceed the amount of work that can be done by this server in one time unit.
The strict inequality recognizes that with the exception of some very ideal conditions, the server utilization cannot be maintained at 100%.

2. While performing the mean value analysis for CONWIP lines that was presented in class on a 4-workstation CONWIP line with its WIP ceiling set to 10 jobs, we found that the average number of jobs and the expected cycle times per station are as follows:

Workstation	Average # of jobs	Expected Cycle Time (min)
1	2	10
2	4	20
3	3	12
4	1	7

Explain why, on the basis of the above numbers, we can infer that our application of the method to the considered line was erroneous.

For a CONWIP line, the workstation throughputs, as defined by the workstation WIP over the expected cycle time per visit at this workstation, should be constant. And this constant defines also the line throughput. But this rule is violated in the current case.

3. Consider an assembly line with a cycle time $c = 60$ secs and each workstation having an idle time of at least 7 secs during this cycle time. What is the maximum increase of throughput that can be supported by this line?

Please, provide your response as *a percentage of the current throughput* of the line, and show clearly all your computations.

Check Question # 4 in the final exam of 2018, that is posted at the course website.

4. What is the meaning of "demand chasing" in the context of the aggregate planning strategies that were discussed in class? Provide an example where the basic logic of this strategy could be naturally applicable.

"Demand chasing" is the pure aggregate planning strategy where the internal production capacity (typically defined by the regular labor time) is adjusted at every period of the planning horizon so that it supports exactly the production needs for that period.

Frequently, such a strategy will result in significant social pressures, but there are certain cases where this strategy constitutes a natural option. These cases are characterized by a very high seasonality and the additional fact that the workforce that will support this seasonal activity is available itself on the basis of this seasonal cycle; e.g. migrant workers that support the collection of crops during the pick season, or the staffing of many service facilities during the peak season (spring and summer season) at many Greek islands.

5. What is (i) the role, and (ii) the computational logic of the "Part Period Balancing" heuristic that is used in production planning?

"Part Period Balancing" is a popular lot sizing heuristic that is used in the context of MRP explosion.

This heuristic keeps adding the demand of subsequent periods during the creation of a new production lot, as long as the distance of the resulting set up and holding costs is reduced.

Problem 1 (20 points): A company procures five items, A, B, C, D and E from the same supplier, and it wants to use the "Power-of-2" order policy in order to synchronize its orders with respect to these items.

The purchasing price and the annual demand for these items is as follows:

Item	Purchasing Price ($)	Annual Demand (in 1000's)	$\tilde{T}_i$, $T^* \cdot 12$	$\hat{T}_i$
A	10	2	5.37	4
B	15	4	3.10	4
C	8	12	2.45	2
D	10	7	2.87	4
E	12	9	2.31	2

Also, the ordering cost for the company transactions with the corresponding supplier is 100$ per order, and the holding cost is computed based on an annual interest rate of 5%.

Please, address the following questions:

i. (10 pts) Apply the "Power-of-2" order policy to the above data in order to determine the replenishment periods for the different items, using the *month* as the basic time unit.

ii. (10 pts) Compute the difference in the total annual cost for these five items that is incurred by the considered ordering policy and the optimal ordering policy that utilizes the EOQ value for every item. In your calculation assume that items that have their orders synchronized share the same ordering cost.

(i) First we compute the optimal replenishment period, T^*_i, for every item that is suggested by the EOQ formula. This computation can be organized as follows:

$$T^* = \frac{Q^*}{D} = \frac{1}{D}\sqrt{\frac{2AD}{iC}} = \sqrt{\frac{2A}{iCD}}$$

The above formula will give T^* in years. Since we want to measure this quantity in months, we have to multiply it by 12. The results of this computation is tabulated above. Next, we need to identify the "closest" power of 2 to each of these values. For this, we need to split the intervals (2, 4) and (4, 8) that contain all these values using the following "splitting" points: $2\sqrt{2} = 2.83$ and $4\sqrt{2} = 5.66$. The results are shown in the 2nd column above.

i.e, products C, E will be replenished every 2 months and products A, B and D every 4 months. Furthermore, the replenishment of these last three items will be ordered together with the replenishment of the first two (this is an additional advantage of the synchronization that is attained by the considered heuristic.

(ii) For this part, first we notice that the annual purchasing cost $C \cdot D$ is common for both cases, and since we interested in the difference of the total annual costs, it can be ignored.

For the case that we use T^* for each item the resulting (holding + ordering) annual costs can be computed from the formula: $A \times \frac{12}{\tilde{T}_i} + i C_i D_i \frac{\tilde{T}_i}{24}$, which accounts for the fact that $\tilde{T}_i$ is expressed in months. The resulting values are $< 447.21, 774.6, 979.8, 836.66, 1039.23 >$ with a total cost of 4077.50.

The annual holding costs that result when we order according to $\hat{T}_i$, are given by $i C_i D_i \hat{T}_i/24$; these values are: $< 166.67, 500.00, 400.00, 583.33, 450 >$ with a total of 2100.00.

Based on the above discussion, this alternative ordering scheme results in 6 orders per year. Hence, the annual ordering cost is $6 \times 100 = 600$. Finally, the cost difference is:

$$4077.50 - (2100 + 600) = 1377.50$$

This gain results from the control of the ordering cost that is attained by the second scheme.

Problem 2 (20 points): Consider a workstation where parts arrive according to a Poisson process with rate $\lambda = 25$ parts per hour, and are processed in an automated manner by a numerically controlled machine at that station.

The detailed processing of the parts by this machine is as follows: Parts are drawn automatically from the workstation buffer and they are loaded on the aforementioned machine that processes them for 2 minutes. However, there is a 0.2 probability that a part processing will fail, and as soon as such a failure is detected, the machine will reject that part, and the part will be scrapped. These failures can occur (and be detected) according to a uniform distribution over the nominal period of the part processing.

Please, answer the following questions:

i. (5 pts) What is the average effective processing time for a part and the coefficient of variation for these processing times?

ii. (5 pts) Show that the workstation operation is stable.

iii. (5 pts) Compute the effective throughput (i.e. the production rate of good parts) of this workstation.

iv. (5 pts) Compute the expected waiting time for a part that goes through this workstation and the average number of parts in the workstation buffer.

(i) Let T be a r.v. modeling the experienced proc. times at this station. Then

$$T \sim \begin{cases} 2 & \text{w.p. } 0.8 \\ U[0,2] & \text{w.p. } 0.2 \end{cases}$$

where $U[0,2]$ denotes a uniform distribution over the interval $[0,2]$.

Then, $t_e = E[T] = 0.8 \times 2 + 0.2 \times 1 = 1.8$ min

To get $CV[T]$, we work as follows:

Let T_1, T_2 be r.v.'s corresponding to each of the two branches that define T. Then, we know that:

$$E[T^2] = 0.8\, E[T_1^2] + 0.2\, E[T_2^2].$$

$E[T_1^2] = 4$

$E[T_2^2] = \frac{1}{2}\int_0^2 t^2 dt = \frac{1}{6}[t^3]_0^2 = \frac{8}{6} = \frac{4}{3} = 1.333$

Plugging the above two values into the expression for $E[T^2]$:

$E[T^2] = 0.8 \times 4 + 0.2 \times \frac{4}{3} = 3.467$

Then $Var[T] = E[T^2] - E^2[T] = 3.467 - 1.8^2 = 0.227$

and

$c_e = CV[T] = \sqrt{\frac{0.227}{1.8^2}} \simeq 0.265$

(ii) We have: $u = \lambda t_e = \frac{25}{60} \times 1.8 = 0.75 < 1 \Rightarrow$ stable.

(iii) Since a part is scrapped with prob. 0.2,

$TH = \lambda \times 0.8 = 25 \times 0.8 = 20 \text{ hr}^{-1}$.

(iv) $CT_q = \frac{1+c_e^2}{2}\frac{u}{1-u}t_e = \frac{1+0.265^2}{2}\frac{0.75}{1-0.75} \cdot 1.8 = 2.89 \text{ min}$

$W_q = \lambda \cdot CT_q = \frac{25}{60} \cdot 2.89 \approx 1.20$

Problem 3 (20 points): A local sports company will produce golf clubs on an assembly line, according to the eight operations that are listed in the following table:

Task	Req. Time (min)	Imm. Preds
1. Polish shaft	12	
2. Grind the shaft end	14	
3. Polish the club head	6	
4. Imprint number	4	3
5. Connect wood to shaft	6	1,2,4
6. Place and secure connecting pin	3	5
7. Place glue on other end of shaft	3	1
8. Set in grips and balance	12	6, 7

The above table also reports the required time for each task in minutes, and the immediate predecessors for each task.

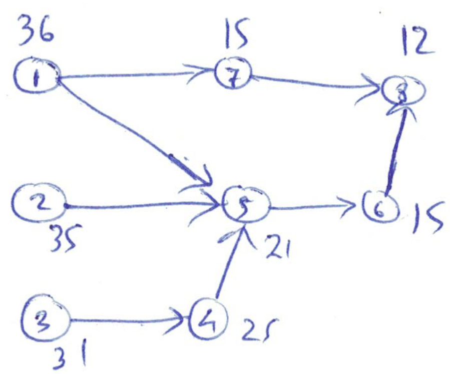
Please, answer the following questions:

i. (5 pts) Draw the precedence diagram that represents the task precedence constraints that are specified by the above table.

ii. (5 pts) What is the maximal throughput that can be supported by this line?

iii. (10 pts) Use the heuristic of the "ranked positional weights" in order to design an assembly line that will deliver the throughput that you computed in part (ii) above.

(i)



(ii) We know that $TH = 1/C$, where C is the line cycle time. Hence, throughput is maximized by minimizing C, and since tasks are indivisible, the minimum C in this case is 14 min, which is the $\max_i t_i$. Then $TH_{max} = \frac{1}{14\,min} = 0.07\,min^{-1} = 4.29\,hr^{-1}$

(iii) First we compute the positional weight for each task. This computation can be tabulated as follows:

Task	Successors	$PW = \sum_{\text{successors}} t_i$
1	1, 5, 6, 7, 8	$12+6+3+3+12=36$
2	2, 5, 6, 8	$14+6+3+12=35$
3	3, 4, 5, 6, 8	$6+4+6+3+12=31$
4	4, 5, 6, 8	$4+6+3+12=25$
5	5, 6, 8	$6+3+12=21$
6	6, 8	$3+12=15$
7	7, 8	$3+12=15$
8	8	12

Ordering the tasks in decreasing PW, we get the following list:

$$\langle 1, 2, 3, 4, 5, 6, 7, 8 \rangle$$

Finally, we distribute these tasks to the line workstations using a cycle time $c = 14$ min, by drawing from the above list; the resulting line is as follows:

W1	W2	W3	W4	W5
$\boxed{1}$	$\boxed{2}$	$\boxed{3,4,7}$	$\boxed{5,6}$	$\boxed{8}$
$\cancel{14}$	$\cancel{14}$	$\cancel{14}$	$\cancel{14}$	$\cancel{14}$
2	0	$\cancel{8}$	$\cancel{8}$	2
		$\cancel{4}$	5	
		1		

Also, a slightly better balanced line can be obtained by moving task 7 to W4.

Finally, a lower bound for the necessary workstations for this line is $\left\lceil \frac{\sum_i t_i}{c} \right\rceil = \left\lceil \frac{60}{14} \right\rceil = \lceil 4.286 \rceil = 5$; hence, we have achieved the minimum.