ISYE 6201A,Q: Manufacturing Systems
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Midterm Exam I
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Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Discuss how the strategic concept of "product customization" is currently supported in some major industries, like automotive and computer manufacturing.

The main approach to mass-customization by many modern industries is "combinatorial customization" where the end product is assembled to order using standardized components that are produced to stock. Each of these components are offered in a number of options, and the entire product is modularized so that these various options are inter-changeable in the assembly that constitutes the end product.

2. Enumerate three distinct roles of inventory in modern production systems, and give a brief description for each of them.

Please, refer to the page in the introductory slides that discusses the notion of inventory.

3. What is the meaning of the "robustness" of the EOQ formula? How can this robustness be useful in the management of the inventory systems that we have been considering in class?

"Robustness" of the EOQ formula means that a deviation from the optimal order size, Q^*, will increase drastically the total annual cost.

This fact enables companies to deviate from Q^* when they select their order sizes w.r.t. different product, in an effort to address additional considerations (like buffer space restrictions, replenishment synchronization constraints across different items, etc) without having to worry too much about the impact of these deviations upon the operational costs that are captured by the "total annual cost" function that is used by the EOQ model.

4. Consider a (Q, r) inventory sysfem where the parameters Q and r are determined according to the approximating scheme that was presented in class during the discussion of the basic economic analysis of this model (i.e., Q is computed by the EOQ formula and r is computed from the critical ratio of $b/(h+b)$).

Suppose that the variability that is experienced in the lead-time demand of the corresponding product is increased, and, therefore, we update accordingly the model data and recompute the Q and r values as suggested above. Then, we can infer that the fill rate that will result from this updating

i. will be higher than the original fill rate.

ii. will be the same with the original fill rate.

iii. will be lower than the original fill rate.

iv. could be either higher or lower than the original fill rate.

Please, explain your answer.

$$S(Q,r) \simeq 1 - \frac{B(r)}{Q}$$

where $Q = \sqrt{\frac{2AD}{h}}$ and $G(r) = \frac{b}{h+b}$

In the considered scenario, Q will not be affected by the experienced change, but r will change because the distribution $G()$ will change. Furthermore, as demonstrated in class for the case of the normal distribution an increase of variability might lead either in an increase or decrease of r, with opposite effects for $B(r)$. Hence $S(Q,r)$ can increase or decrease.

5. Consider a pure inventory system with time-varying demand and no limits for the size of the placed orders, that is managed according to the rolling-horizon scheme discussed in class with a re-planning period of 1 week and a planning horizon of 5 weeks. The ordering cost for this system is \$100.00 per order and the unit holding cost per week is \$2.00. We are currently trying to determine our order for the next week, and we know that the demand for week 3 in the corresponding planning horizon is estimated to 60 units. From this information, we can infer that the order that we shall eventually place as a result of this planning activity cannot be more than the combined estimated demand for the next two weeks.

(a) YES (b) NO

Please, explain your answer.

The holding cost for carrying these 60 units of demand for week 3 by one week is

$60 \times 2 = 120 > 100$ which is the ordering cost.

Hence, in an optimized order plan, this quantity should be ordered in period 3.

Therefore any order placed for the next week (i.e., week #1 in our plan) cannot be larger than the total demand for weeks 1 and 2.

Problem 1 (30 points): A local store with an annual demand of 10,000 units for one of its main products is trying to select a supplier for this product by choosing among two different options. The first supplier (supplier A) quotes a purchasing price of \$15 per unit, and a contract of 50 free deliveries per year at the price of \$1,000 per year. The second supplier (supplier B) quotes a purchasing price of \$20 per unit and she will utilize a 3PL service provider for the deliveries, who charges a fixed cost of \$15 per delivery and a variable cost of \$2 per delivered unit. The considered store computes its holding cost on the basis of an annual interest rate of 5%. Assuming that both suppliers can provide comparable quality of product and service, use the above information in order to determine the right supplier for this product.

For supplier A:

$$Q_A^* = \frac{10000}{50} = 200$$

and $TAC_A = 15 \times 10000 + 1000 + 15 \times 0.05 \times \frac{200}{2} =$

$= 151{,}075$

For supplier B: $A = 15$; $C = 20 + 2 = 22$

$$Q_B^* = \sqrt{\frac{2AD}{c}} = \sqrt{\frac{2 \times 15 \times 10000}{0.05 \times 22}} = 522$$

$$TAC_B = 22 \times 10000 + 15 \frac{1000}{522} + 22 \times 0.05 \times \frac{522}{2} =$$

$= 220{,}574.456$

$\Rightarrow$ Choose A.

Problem 2 (30 points): Consider a basestock inventory system where daily demand follows a Poisson distribution with rate $\lambda = 0.05$ items per day. The order lead-time for the considered item is 10 days, since it has to be procured from abroad.

i. (15 pts) Determine a minimal basestock level that will ensure a fill rate of 95%.

   In your computations, use the Central Limit Theorem to get an approximation of the demand distribution over a lead-time interval. Also, remember that for the r.v. X that denotes the number of events that are generated by a Poisson process with rate λ over a time interval t, $E[X] = Var[X] = \lambda t$.

ii. (15 pts) Answer the question in part (i) in the case where the order lead-time is uniformly distributed between 7 and 10 days.

Let r.v.'s X_d and X_L denote, repectively, the daily demand and the demand over a lead-time interval.

(i) Then, for the first part of the problem:

$E[X_d] = 0.05 \times 1 = 0.05$

$Var[X_d] = 0.05 \times 1 = 0.05$

and since in this case lead time is deterministically equal to 10 days,

$$X_L = \sum_{i=1}^{10} X_d^{(i)}$$

Assuming independence of daily demands,

$$X_L \sim N(0.05 \times 10,\ 0.05 \times 10) = N(0.5,\ 0.5)$$

From the results for the fill rate of the base-stock model, we need:

$$\mathcal{G}(r) = \varphi\left(\frac{r-0.5}{\sqrt{0.5}}\right) = 0.95 \Rightarrow$$

$$\Rightarrow r = 0.5 + 1.65 \times \sqrt{0.5} = 1.67 \Rightarrow r = 2.$$

(ii) In this case, let r.v. L denote the random lead time.

Then $E[L] = \frac{7+10}{2} = 8.5$

$$E[L^2] = \frac{7^2+8^2+9^2+10^2}{4} = 73.5$$

$$Var[L] = E[L^2] - E^2[L] = 73.5 - 8.5^2 = 1.25$$

Also

$$X_L = \sum_{i=1}^{L} X_d^{(i)}$$

and as discussed in class,

$$E[X_L] = E[L] \cdot E[X_d] = 8.5 \times 0.05 = 0.425$$

$$Var[X_L] = Var[L] \cdot E^2[X_d] + E[L] \cdot Var[X_d] =$$

$$= 1.25 \times 0.05^2 + 8.5 \times 0.05 \simeq 0.428$$

Treating X_L again as a normally distributed r.v., we need:

$$\mathcal{G}(r) = \varphi\left(\frac{r-0.425}{\sqrt{0.428}}\right) = 0.95 \Rightarrow r = 0.425 + 1.65 \times \sqrt{0.428} \simeq$$

$$\simeq 1.5 \Rightarrow r = 2$$