ISYE 6201: Manufacturing Systems
Spring 2018
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Final Exam
April 27, 2018

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. What is the basic definition of the notion of *"quality"* in the context of the operations of modern corporations? How does this concept become more operational by engineering practice?

At a very basic level, quality is defined by the extent ~~to which~~ that a product or service meets the customer needs.

The above definition is further operationalized through the definition of certain <u>specifications</u> for the considered product or service, and the eventual assessment of any given product unit or service instantiation against these specs.

2. What is the meaning of the *"Wagner-Whitin" property* in inventory control theory? What is its practical significance?

The Wagner-Whitin property applies to the case of uncapacitated dynamic lot sizing, and it ascertains that, when starting with zero initial inventory, every optimal plan must satisfy the condition $I_{t-1} \cdot \delta_t = \emptyset$, $\forall t$

In the above equation:

* I_{t-1} is the inventory carried to period t from period $t-1$, and
* δ_t is a binary variable indicating whether an order is placed at period t or not.

The practical significance of the W-W property is that it enables the development of solution approaches to the uncapacitated dynamic lot sizing problem that are computationally very efficient (of polynomial complexity).

3. Assuming that the part processing times for a batch of k parts are independent, identically distributed according to some general distribution, the variability of the processing time for the entire batch decreases as the batch size k increases.

(a) TRUE (b) FALSE

Please, explain your answer.

Under the aforestated assumption, it was shown in class that

$$c_b = \frac{c_p}{\sqrt{k}}$$

where c_b is the CV for the batch proc. times

and c_p " " " " " part " ".

4. Consider an assembly line with a cycle time $c = 60$ secs and each workstation having an idle time of at least 7 secs during this cycle time. What is the maximum increase of throughput that can be supported by this line?

Please, provide your response as *a percentage of the current throughput* of the line, and show clearly all your computations.

We know that for the considered lines, $TH = 1/c$, where c is the line cycle time. According to the above information, $c = 60$ secs, and it can be reduced to $60 - 7 = 53$ secs. This reduction will effect ~~a~~ relative increase of the line throughput equal to

$$\frac{\frac{1}{53} - \frac{1}{60}}{\frac{1}{60}} = \frac{60-53}{53} = \frac{60}{53} - 1 = 0.1321 = 13.21\%$$

5. Discuss the need for *aggregation* in aggregate planning.

Aggregation of the available data across product families

(i) helps with controlling the overall complexity of the underlying planning problem, and

(ii) it also introduces a "pooling" effect that helps controlling the noise in the available forecasts for the demand of the different SKUs in each family.

Problem 1 (20 points): A local store sells a particular model of fan, with almost all of its sales being made in the summer months. The store is to make a one-time purchase of these fans for the upcoming summer season at a cost of \$40 each, and it plans to sell each fan for \$60. Any fans unsold at the end of the summer will be marked down to \$29 and sold in a special fall sale (it is expected that all marked down fans will be sold during this sale). The store sales during the last 10 summers have been as follows: 30, 50, 30, 60, 10, 40, 30, 30, 20, 40.

i. (10 pts) Based on the provided data, construct an empirical probability distribution for the summer demand of the considered fan model.

ii. (10 pts) Use the distribution that you constructed in part (i) in order to determine an optimal order size for fans to be placed by the considered store for the upcoming summer season.

(i) Assuming that the summer demand can only take the values that show up in the provided data, the corresponding empirical distribution is characterized by the following pmf and cdf:

Value	PMF	CDF
10	0.1	0.1
20	0.1	0.2
30	0.4	0.6
40	0.2	0.8
50	0.1	0.9
60	0.1	1.0

(ii) From the provided data:

$$\left.\begin{aligned} C_s &= 60-40=20 \\ C_o &= 40-29=11 \end{aligned}\right\} \Rightarrow \quad CR = \frac{C_s}{C_o+C_s} = \frac{20}{11+20} = 0.645$$

Since we are dealing with a discrete distribution, the above value of the CR suggests an optimal order size of 40 units.

(We mentioned in class that for newsvendor problems with discrete demand X the optimal order size is the smallest value for X s.t. $G(X) \geq CR$, where $G(X)$ is the cdf of X.)

Problem 2 (20 points): Consider a single-server workstation where parts arrive for processing according to a Poisson process with rate $r_a = 15$ parts per hour. Part processing times at this workstation have a mean of 3 minutes and a standard deviation of 1 minute. Also, the station server experiences outages with an average duration of t_d minutes and mean-time-to-failure equal to 4 hrs. Both, the server downtime and uptime are exponentially distributed.

i. (10 pts) Compute the value of t_d so that the resulting server utilization (i.e., the percentage of time that the server is occupied by a part) is 90%.

ii. (5pts) What is the average cycle time for a part going through this station under the value of t_d that you computed in part (i)?

iii. (5pts) What is the average number of parts waiting for processing at this workstation?

(i) We have: $A = \dfrac{4\times 60}{4\times 60 + t_d}$

$t_e = t_o/A = \dfrac{3}{240}(240+t_d)$

$u = r_a t_e = \dfrac{15}{60}\cdot\dfrac{3}{240}(240+t_d) = 0.9 \Rightarrow t_d = 48$ min.

Also, $A = \dfrac{240}{240+48} = 0.8\bar{3}$ and $t_e = \dfrac{3}{240}(240+48) = 3.6$ min

(ii) $c_o^2 = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$; $c_e^2 = c_o^2 + (1+c_r^2)A(1-A)\dfrac{m_r}{t_o} =$

$= \dfrac{1}{9} + (1+1)\,0.833(1-0.833)\dfrac{48}{3} = 4.563$

$CT = \dfrac{c_a^2+c_e^2}{2}\,\dfrac{u}{1-u}\,t_e + t_e = \dfrac{1+4.563}{2}\,\dfrac{0.9}{1-0.9}\,3.6 + 3.6 =$

$= 93.72$ min.

(iii) $WIP_q = r_a \cdot CT_q = \dfrac{15}{60}\times 90.12 = 22.53$

Problem 3 (20 points): An assembly process involves the following ten atomic tasks with the corresponding processing times and precedence constraints being reported in the following table:

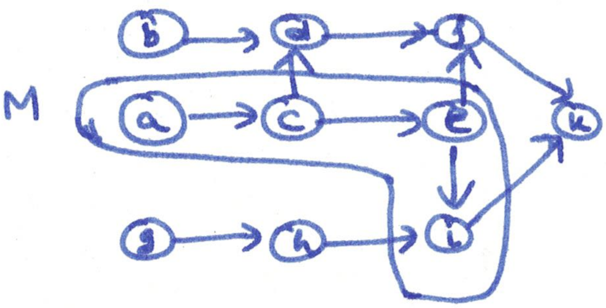
Task	Proc. Time (secs)	Imm. Predecessors
a	10	-
b	8	-
c	11	a
d	7	c, b
e	5	c
f	15	d, e
g	11	-
h	15	g
i	10	e, h
k	5	f, i

Furthermore, tasks a and i have to be performed at the same workstation since they need to share some equipment.

1. (10 pts) What is the maximal possible hourly throughput that can be attained for this assembly process? Please, explain your answer.
2. (10 pts) Adapt the RPW heuristic to the considered case, in order to design a production line for this assembly process with a production rate of 60 parts per hour.

Hint: Consider carefully the implications of the new requirement of having tasks a and i assigned to the same station for the overall design process. It might be pertinent to define a new "macro-task" to accommodate this new situation.

The precedence diagram for this problem is as follows:

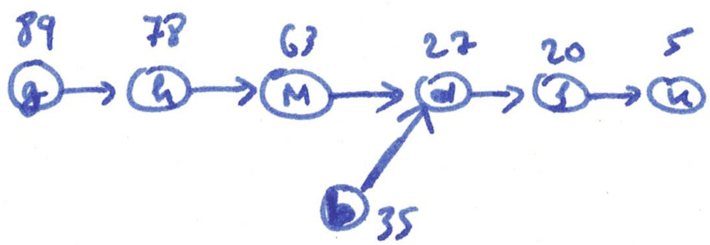


From this diagram it can be seen that the workstation that will perform tasks a and i must also perform tasks c and e. So, we can combine these four tasks (a, c, e, i) into a "macro-task" M. Obviously, the required proc. time for this new task M is $t_M = t_a + t_c + t_e + t_i =$
$= 10 + 11 + 5 + 10 = 36$ secs.

1. Hence, $C_{min} = 36\,sec \Rightarrow TH_{max} = \frac{1}{C_{min}} = \frac{1}{36\,secs} =$

$$= 0.028\ sec^{-1} = \frac{1}{36\,sec}\ 3600\ \frac{secs}{hr} = 100\ hr^{-1}$$

2. With the introduction of the macro-task M, the task precedence diagram can be redrawn as follows:



The above figure also shows the PWs for the various tasks. Hence, the corresponding ordered list is

< g, h, M, b, d, f, k >

For the target throughput $c = 1\text{ min} = 60$ secs.

Applying the RPW heuristic with these data, we will get

	WS1	WS2	WS3
	g, h, b	M, d, f	k
Idle time	26	2	55

A better balanced line is:

	WS1	WS2	WS3
	g, h, b	M	d, f, k
Idle time:	26	24	33