ISYE 6201: Manufacturing Systems
Spring 2017
Instructor: Spyros Reveliotis
Midterm Exam I
March 15, 2017

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Provide at least three reasons that may force a company to maintain inventory with respect to a particular item.

Please, refer to slide #21 in the set of slides entitled "Course Introduction".

2. What are the main types of *"buffering"* that can used by a company in an effort to deal with the variability experienced in the demand of its product?

The company can buffer against this variability by using

(i) either a "material" buffer, i.e., carrying a certain amount of inventory,

(ii) or a "capacity" buffer, i.e., maintaining a certain level of excess capacity.

Otherwise, the company will have to quote long delivery lead times.

3. What is the meaning of *"safety stock"* in the context of the basestock model? Under what circumstances can this safety stock assume a negative value?

The technical definition of safety stock in the context of the basestock model is

$$ss = r - \theta$$

where

* r : the applying reorder point
* θ : the expected demand over a lead time period.

In other words, the safety stock characterizes the deviation of the reorder point from the mean lead time demand.

The safety stock will be negative if $r < \theta$.

In case of lead time demand that is symmetrically distributed around its mean, the above inequality will hold if the holding cost is higher than the backorder cost. (C.f. the relevant discussion about normally distributed demand.

4. A fashion house is currently placing an order for one of its top selling items for the upcoming winter season. The item is purchased at the price of \$75.00 and it is sold at the price of \$110.00. Any remaining items at the end of the season are expected to be sold at the marked down price of \$45.00. On the other hand, the company can place expedited orders during the season for additional product units at the price of \$95.00 (and in this way, it can satisfy its entire demand during the season). Assuming that (a) the season demand for this item is contemplated to have a *uniform* distribution in the range of 1000 to 2000 units, (b) for reasons of customer loyalty and satisfaction, the company policy is to try to meet the entire demand, and (c) the values quoted above define the key cost structure experienced by the company with respect to the considered item, we can conclude that an optimal size for this initial order is equal to

i. 0 units

ii. 1400 units

iii. 1600 units

iv. 2000 units

Briefly justify your answer.

We have:

$C_o = 75 - 45 = 30$

$C_s = 95 - 75 = 20$ (because we shall need to buy any missing units at the higher price of \$95 and thus will reduce the corresponding profit by \$20.)

But then:

$$\frac{Q^* - 1000}{1000} = \frac{20}{20+30} = 0.4 \Rightarrow Q^* = 1000 + 1000 \times 0.4 = 1400.$$

5. Briefly discuss how the modern information technology (IT) can be a *strategic* asset for contemporary organizations.

IT can help modern corporations establish better visibility with respect to their own operations as well as their operational environment (on, both, the supply and the demand sides), and therefore they can be more responsive with less "buffer" (in the spirit discussed in Question #2). In other words, with effective use of IT, these corporations can break the traditional trade-off that exists between responsiveness and cost leadership.

Problem 1 (30 points): The inventory for a particular item is replenished on a weekly basis. The size of the order placed for each week can be arbitrarily large, and the delivery of every placed order costs \$50.00. On the other hand, the inventory holding cost that results from transferring one unit of inventory from one week to the next is equal to \$2.00. Furthermore, the considered item has a *"shelf-life"* of only three weeks, i.e., any product units that remain in the company stock for more than three weeks must be discarded. Finally, the company places its orders on the basis of a 4-week planning horizon, the forecasted demand for the next four weeks is respectively 20, 20, 30, and 25 units, and the current on-hand inventory is 10 units. Based on this information, determine an optimized size for the order that must be placed by the company for the upcoming week.

We have: $A = \$50.00$; $h = \$2.00$

shelf-life = 3 weeks $\quad D = \langle 20, 20, 30, 25 \rangle \Big\} \Rightarrow$

$I_0 = 10$

$\Rightarrow \quad D_{eff} = \langle 10, 20, 30, 25 \rangle$.

Shelf-life of 3 weeks implies that we cannot carry any inventory for 4 weeks or more.

Application of the DP algorithm for dynamic lot sizing for this problem gives:

$$Z_1^* = 50$$

$$Z_2^* = \min \begin{cases} 50 + 2 \cdot 20 = 90 \Leftarrow \\ 50 + 50 = 100 \end{cases}$$

$$Z_3^* = \min \begin{cases} 50 + 2 \cdot 20 + 4 \cdot 30 = 210 \\ 50 + 50 + 2 \cdot 30 = 160 \\ 90 + 50 = 140 \Leftarrow \end{cases}$$

$$Z_4 = \min \begin{cases} 90 + 50 + 2 \cdot 25 = 190 \\ 140 + 50 = 190 \end{cases}$$

Hence, we have two optimal plans:

$\langle 30, 0, 55, 0 \rangle$ and $\langle 30, 0, 30, 25 \rangle$

Both suggest an initial order of 30 units.

Problem 2 (30 points): The inventory of a certain item is controlled by a periodic review policy with the review period being equal to one week. The item demand over an one-week interval has been found to be *uniformly* distributed between 400 and 600 units. Every week unmet demand is back-ordered, and at the end of the week a replenishment order is placed which is adequate to cover the backorders of this week, and bring the inventory position at the beginning of the next week to some desired level S.

i. (15 points) Compute the target level S so that the probability of experiencing no backorders in any single week is equal to 80%.

ii. (15 points) What is the fill-rate that results from the target level S that you computed in part (i)?

(i) $g(S) = 0.8 \Rightarrow \frac{S-400}{200} = 0.8 \Rightarrow$

$\Rightarrow S = 400 + 200 \times 0.8 = 560$

(ii) Using the formulae developed in class, we have:

Expected weekly demand $= \frac{400+600}{2} = 500.$

Expected weekly demand met from stock =

$= \int_{400}^{560} x \frac{1}{200} dx + 560 [1 - 0.8] =$

$= \frac{1}{400} [560^2 - 400^2] + 560 \times 0.2 = 496$

Hence, fill rate $= \frac{496}{500} = 0.992$