ISyE 6201: Manufacturing Systems
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Solutions for Homework #3

Chapter 8

Problem 1

a. The mean is 5 and the variance is 0. The coefficient of variation is also zero. These process times could be from a highly automated machine dedicated to one product type.

b. The mean is 5, the standard deviation is 0.115 and the CV is 0.023. These process times might be from a machine that has some slight variability in process times.

c. The mean of these is 11.7 and the standard deviation is 14.22 so the CV is 1.22. The times appear to be from a highly regular machine that is subject to random outages.

d. The mean is 2. If this pattern repeats itself over the long run, the standard deviation will be 4 (otherwise, for these 10 observations it is 4.2). The CV will be 2.0. The pattern suggests a machine that processes a batch of 5 items before moving any of the parts.

Problem 3

a. The natural CV is 1.5/2 = 0.75.

b. The mean would be (60)(2) = 120 min. The variance would be $(60)(1.5)^2 = 135$. The CV will be CV = $\sqrt{135}/120 = 0.0968$

c. The availability will be $A = m_f/(m_f + m_r) = 60/(60+2) = 0.9677$. The effective mean will be $t_e = t_0/A = 120/0.9677 = 124$.

The effective SCV will be
$c_e^2 = c_0^2 + 2(1\text{-}A)Am_r/t_0 = 0.009375 + 2(1\text{-}0.9677)(0.9677)(120\text{min})/120 = 0.07181$.
So, $c_e = 0.268$.

Problem 5

In the tables below TH is the throughput that is given, t_e, is the mean effective process time, c_e^2 is the effective SCV of the process times, u is the utilization given by (TH)(t_e), CT_q is the expected time in queue given by

$$CT_q = \frac{(1+c_e^2)}{2}\frac{u}{(1-u)}t_e$$ for the single machine case and

$$CT_q = \frac{(1+c_e^2)}{2}\frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)}t_e$$ for the case with m machines.

Finally, CT is the sum of CT_q and t_e

a. In the first case, even though B has greater capacity, A has shorter cycle time since its SCV is much smaller.

Machine	A	B
TH	0.92	0.92
t_e	1	0.85
c_e^2	0.25	4
m	1	1
u	0.92	0.782
CT_q	7.1875	7.6227
CT	8.1875	8.4727

b. Doubling the arrival rate (TH) and the number of tools makes station A have a longer average cycle time than B.

Machine	A	B
TH	1.84	1.84
t_e	1	0.85
c_e^2	0.25	4
m	2	2
u	0.92	0.782
CT_q	3.4616	3.4125
CT	4.4616	4.2625

c. Note the large increase in cycle time with the modest increase in throughput as compare to (a).

Machine	A	B
TH	0.95	0.95
t_e	1	0.85
c_e^2	0.25	4
m	1	1
u	0.95	0.8075
CT_q	11.8750	8.9140
CT	12.8750	9.7640

d. We now consider Machine A only.
i) First we increase TH by 1% from 0.5. The increase in cycle time is less than one percent.
ii) Next we increase TH by 1% from 0.95. The increase in cycle time is almost 23%.

	(i)		(ii)	
Machine	A	A	A	A
TH	0.5	0.505	0.95	0.9595
t_e	1	1	1	1
c_e^2	0.25	0.25	0.25	0.25
m	1	1	1	1
u	0.5	0.505	0.95	0.9595
CT_q	0.6250	0.6376	11.8750	14.8071
CT	1.6250	1.6376	12.8750	15.8071
% Increase		0.7770		22.7736

Chapter 9

Problem 10

(a) If we reduce the buffer sizes, the number of jobs that balk will increase, thereby decreasing TH. The maximum WIP level also decreases as does cycle time. Cycle time goes down because it is a convex function of WIP.

(b) Reducing the variability should increase TH (slightly by reducing the amount of bakling and decrease CT.)

(c) If we unbalance the line without changing r_b we must add capacity to the other stations (otherwise a different station would become the bottleneck with a capacity lower than the current value of r_b). If we add capacity, we increase TH (slightly) and decrease CT (more significantly).

(d) The opposite of b.

(e) Decreasing the arrival rate decreases TH (obviously) and also decreases utilization which thereby decreases CT.

(f) If we decrease the variability enough we might see an increase in TH and a reduction in CT.

G/G/1 station Problem:

$r_a = 0.1/\text{min}$

$c_a^2 = 0$

$u = r_a t_p = 0.95$

$t_p = 9.5/\text{min}$

$CT_q = \frac{c_a^2 + c_p^2}{2} \times \frac{u}{1-u} \times t_p = 45\ \text{min}$

$c_p^2 = \frac{45}{\frac{1}{2} \times \frac{0.95}{0.05} \times 9.5} = 0.4986$

$c_d^2 = u^2 c_p^2 + (1 - u^2) c_a^2 = 0.95^2 \times 0.4986$=0.45

Since it is a stable system,

$r_a = r_d$

Therefore,

$t_d = \frac{1}{r_a} = 10\ \text{min}$

$\sigma_d^2 = c_d^2 t_d^2 = 45$

Two-Station, Single-Machine Production Line Problem:

i. $A = \dfrac{m_f}{m_f + m_r}$, $A_1 = \dfrac{7}{7+1.5} = 0.824$, $A_2 = \dfrac{5}{5+0.5} = 0.909$

$t_e = \dfrac{t_0}{A}$, $t_{e1} = \dfrac{11}{0.824} = 13.36 \min = 0.223 hr$, $t_{e2} = \dfrac{11}{0.909} = 12.1 \min = 0.202 hr$

Station 1 is the effective bottleneck of the line.

ii. $u = r_a t_e$, $u_1 = \dfrac{35}{8} \cdot \dfrac{13.36}{60} = 0.974$, $u_2 = \dfrac{35}{8} \cdot \dfrac{12.1}{60} = 0.882$

The utilization is less than 1 at both stations, so the production line can sustain the production rate of 35 parts per 8-hour shift.

iii. For station 1, the SCV of the effective processing time is

$$c_e^2 = c_0^2 + (1 + c_r^2)(1 - A)A\frac{m_r}{t_0} = 0.5^2 + (1 + 0.75^2)(1 - 0.824)(0.824)\frac{1.5}{11/60} = 2.108$$

$$CT = \left(\frac{c_a^2 + c_e^2}{2} \cdot \frac{u}{1-u} + 1\right) t_e = 2 \text{ hr and } 7.815 \text{ min} = 2.13 \text{ hr}$$

$$\Rightarrow \left(\frac{0 + 2.108}{2} \cdot \frac{r_a \cdot 0.223}{1 - r_a \cdot 0.223} + 1\right) 0.223 = 2.13$$

$\Rightarrow r_a = 4$ parts per hour

The inter-release interval is 15 minutes.

iv. $CT_q = CT - t_e = 1.908 hr$, $WIP_q = TH \cdot CT_q = 4 \times 1.908 = 7.63$

v. Mean = 15 minutes = $\dfrac{1}{4}$ hrs

Variance = variance of inter-departure times

$$= c_{d1}^2 \cdot \left(\frac{1}{4}\right)^2 = \left(u_1^2 c_{e1}^2 + (1 - u_1^2) \cdot c_{a1}^2\right)\left(\frac{1}{4}\right)^2 = \left[(4 \cdot 0.223)^2 \cdot 2.108 + 0\right]\left(\frac{1}{4}\right)^2$$

= 0.1045 $(\text{hrs})^2$ = 376.2 $(\text{min})^2$

C. Extra Credit

1.

Let T_0 be the natural processing time

N be the number of failures that occur during the processing of a single part; since inter-failure times are exponentially distributed with mean equal to m_f, N follows a Poisson distribution with E(N) = var(N)= T_0/m_f

Y_j be the repair time of the j-th failure, j =1, 2, …, N, where E(Y_j) = m_r, var(Y_j)= σ_r^2

$f(\cdot)$ be the probability density function of T_0

T_e be the effective processing time. $T_e = T_0 + \sum_{j=1}^{n} Y_j$, and $E[T_e] = \frac{t_0}{A}$.

$$E[T_e^2] = \int_{x=0}^{\infty} \sum_{n=0}^{\infty} E\left(T_e^2 \mid T_0 = x, N = n\right) e^{-x/m_f} \frac{(x/m_f)^n}{n!} f(x)dx$$

$$= \int_{x=0}^{\infty} \sum_{n=0}^{\infty} E\left[(x + \sum_{j=1}^{n} Y_j)^2 \right] \cdot e^{-x/m_f} \frac{(x/m_f)^n}{n!} f(x)dx$$

$$= \int_{x=0}^{\infty} \sum_{n=0}^{\infty} E\left[x^2 + 2x\sum_{j=1}^{n} Y_j + (\sum_{j=1}^{n} Y_j)^2 \right] \cdot e^{-x/m_f} \frac{(x/m_f)^n}{n!} f(x)dx$$

$$= \int_{x=0}^{\infty} \sum_{n=0}^{\infty} \left[x^2 + 2xnE(Y_j) + n(n-1)E^2(Y_j) + nE(Y_j^2) \right] e^{-x/m_f} \frac{(x/m_f)^n}{n!} f(x)dx$$

$$= \int_{x=0}^{\infty} \left[x^2 + 2xE(N)E(Y_j) + E(N^2 - N)E^2(Y_j) + E(N)E(Y_j^2) \right] f(x)dx$$

$$= \int_{x=0}^{\infty} \left\{ x^2 + 2xE(N)m_r + [\text{var}(N) + E^2(N) - E(N)]m_r^2 + E(N)(m_r^2 + \sigma_r^2) \right\} f(x)dx$$

$$= \int_{x=0}^{\infty} \left[x^2 + 2\frac{x^2}{m_f} m_r + \left(\frac{x}{m_f}\right)^2 m_r^2 + \left(\frac{x}{m_f}\right)(m_r^2 + \sigma_r^2) \right] f(x)dx$$

$$= E(T_0^2)\left(\frac{m_f + m_r}{m_f}\right)^2 + (m_r^2 + \sigma_r^2)\frac{t_0}{m_f}$$

$$= (\sigma_0^2 + t_0^2)\left(\frac{1}{A}\right)^2 + (m_r^2 + \sigma_r^2)\frac{(1-A)t_0}{Am_r} \qquad \text{since } A = \frac{m_f}{m_f + m_r}$$

So we have equation (8.5): $\sigma_e^2 = E[T_e^2] - E^2[T_e] = \left(\frac{\sigma_0}{A}\right)^2 + (m_r^2 + \sigma_r^2)\frac{(1-A)t_0}{Am_r}$.

Taking $c_0^2 = t_0^2\sigma_0^2$ and $c_r^2 = m_r^2\sigma_r^2$, we can derive Equation (8.6):

$$c_e^2 = \frac{\sigma_e^2}{t_e^2} = \left(\frac{\sigma_0}{A}\right)^2 \left(\frac{1}{t_0 / A}\right)^2 + (m_r^2 + \sigma_r^2)\frac{(1-A)t_0}{Am_r}\left(\frac{1}{t_0 / A}\right)^2 = c_0^2 + (1+c_r^2)\frac{A(1-A)m_r}{t_0}$$

2.

$$\begin{aligned} E(T_{effective}) &= E(T_0 + T_{disruption}) = E(T_0) + E(T_{disruption}) \\ &= t + E\left[E\left(T_{disruption} \middle| \text{type 1 occures(may simultaneously)}\right)\right] \\ &= t + p_1 \times (1-p_2) \times \frac{1}{\mu_1} + p_2 \times (1-p_1) \times \frac{1}{\mu_2} \\ &\quad + p_1 \times p_2 \times E[\max(T_{type\,1}, T_{type\,2})] \end{aligned}$$

$$\begin{aligned} &E[\max(T_{type\,1}, T_{type\,2})] \\ &= E\left(\max(T_{type\,1}, T_{type\,2}) \middle| T_{type\,1} < T_{type\,2}\right) \times P(T_{type\,1} < T_{type\,2}) \\ &+ E\left(\max(T_{type\,1}, T_{type\,2}) \middle| T_{type\,1} > T_{type\,2}\right) \times P(T_{type\,1} > T_{type\,2}) \\ &= E\left(T_{type\,1} + (T_{type\,2} - T_{type\,1}) \middle| T_{type\,1} < T_{type\,2}\right) \times P(T_{type\,1} < T_{type\,2}) \\ &+ E\left(T_{type\,2} + (T_{type\,1} - T_{type\,2}) \middle| T_{type\,1} > T_{type\,2}\right) \times P(T_{type\,1} > T_{type\,2}) \\ &= \left(\frac{1}{\mu_1+\mu_2} + \frac{1}{\mu_2}\right) \times \frac{\mu_1}{\mu_1+\mu_2} + \left(\frac{1}{\mu_1+\mu_2} + \frac{1}{\mu_1}\right) \times \frac{\mu_2}{\mu_1+\mu_2} \end{aligned}$$

Plug in all the data,

$$E(T_{effective}) = 2 + 0.3 \times 0.8 \times 5 + 0.2 \times 0.7 \times 10 + 0.3 \times 0.2 \times \frac{35}{3} = 5.3 \text{ min}$$

Therefore, the effective processing capacity is 60/5.3 = 11.32 units per hour