HW 1 Solution

1. (20 pts)
D = 5000/yr, C = 600/unit, 1 year = 300 days, i = 0.06, A = 300
Current ordering amount Q = 200

(a) $T^* = \frac{Q}{D} \times 300\text{days} = \frac{200}{5000} \times 300 = 12 \text{ days}$

(b) Total(Holding + Setup) cost would be

$$TC = \frac{iC}{2}Q + \frac{D}{Q}A = \frac{0.06\times600}{2} \times 200 + \frac{5000}{200} \times 300 = 11{,}100/\text{yr}$$

(c) The optimum cost would be

$$\sqrt{2ADh} = \sqrt{2 \times 300 \times 5000 \times 0.06 \times 600} = 10392.30/\text{yr}$$

(d) T^* is 12 days. The closest power of two is 16 days(16/300 yr).

$$TC(16 \text{ days}) = \frac{TDh}{2} + \frac{A}{T} = \frac{\frac{16}{300}\times5000\times0.06\times600}{2} + \frac{300}{16} \times 300 = 10425/\text{yr}$$

The power of two on the other side of 12 days is 8 days(8/300 yr).

$$TC(8 \text{ days}) = \frac{TDh}{2} + \frac{A}{T} = \frac{\frac{8}{300}\times5000\times0.06\times600}{2} + \frac{300}{8} \times 300 = 13650/\text{yr}$$

2. (20 pts)
D = 200/month = 2400/yr, A = (100+55)*1.5 = 232.5
P = 50/hr = 50*6*20*12/yr = 72000/yr, i = 0.22, C = 2.50

(a) $Q^* = \sqrt{\frac{2AD}{h\left(1-\frac{D}{P}\right)}} = \sqrt{\frac{2\times232.5\times2400}{0.22\times2.50\times\left(1-\frac{2400}{72000}\right)}} = 1448.8 \cong 1449$

(b) $H = Q^*\left(1 - \frac{D}{P}\right) = 1449 \times \left(1 - \frac{2400}{72000}\right) = 1400.7 \cong 1401$

(c) $\frac{D}{P} = \frac{2400}{72000} = 0.0333 = 3.33\%$

3. (20 pts)

(a)

EOQ of A : $\sqrt{\frac{2\times100\times20000}{0.2\times2.5}} = 2828.42 \cong 2828 \rightarrow Q_A = 2828$

EOQ of B : $\sqrt{\frac{2\times100\times20000}{0.2\times2.4}} = 2886.75 \cong 2887 \rightarrow Q_B = 3000$

EOQ of C : $\sqrt{\frac{2\times100\times20000}{0.2\times2.3}} = 2948.84 \cong 2949 \rightarrow Q_C = 4000$

$$TC(A)=\frac{AD}{Q}+\frac{hQ}{2}+CD=100\times\frac{20000}{2828}+0.2\times2.5\times\frac{2828}{2}+2.5\times20000=51414.21$$
$$TC(B)=\frac{AD}{Q}+\frac{hQ}{2}+CD=100\times\frac{20000}{3000}+0.2\times2.4\times\frac{3000}{2}+2.4\times20000=49386.67$$
$$TC(C)=\frac{AD}{Q}+\frac{hQ}{2}+CD=100\times\frac{20000}{4000}+0.2\times2.3\times\frac{4000}{2}+2.3\times20000=47420$$

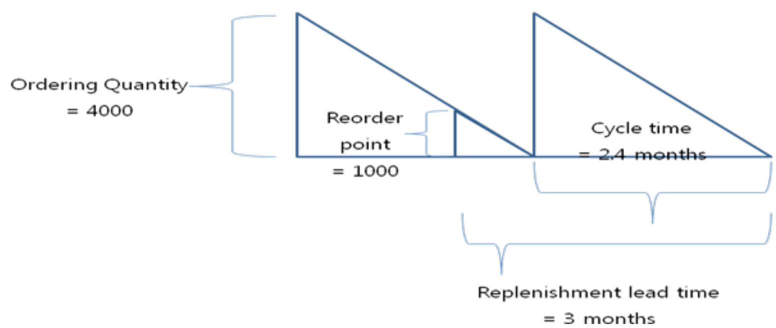
Therefore, optimal order quantity is 4000 with source C.

(b)

Holding + Setup cost = $100\times\frac{20000}{4000}+0.2\times2.3\times\frac{4000}{2}=1420$

(c)

Cycle Time = 4000/20000 = 0.2 year = 2.4 months.
Replenishment lead time = 3 months.
Reorder point = 3/2.4 X 4000 = 5000 → 1000 units is reorder point



It is interesting to interpret the above result for part (c) in terms of the definition of the Inventory Position IP(t) introduced in class during the discussion of the Stochastic Inventory Control theory. So, remember that

$$IP(t) = OHI(t) + O(t) - BO(t) \qquad (1)$$

where

- OHI(t) denotes the on-hand-inventory at time t;
- O(t) denotes the "pipeline" inventory at time t (i.e., material ordered but not received yet);
- BO(t) denotes the backorders at time t.

Also, let $Q_l = lD$ denote the demand experienced over a replenishment lead time interval *l*. In our case, this quantity is $Q_l = (3/12)\text{x}20000 = 5000$.

Since we want to have no shortages,

$$BO(t) = 0 \text{ for all } t \qquad (2)$$

Consider also the OHI(t) at any time t, and notice that at time t+*l*,

$$OHI(t+l) - BO(t+l) = OHI(t) + O(t) - Q_l \quad (3)$$

Furthermore, in the light of (1), Equation (3) becomes

$$OHI(t+l) - BO(t+l) = IP(t) - Q_l \quad (4)$$

Since t was chosen arbitrarily, Equation (4) implies that we shall have BO(t) = 0 for all t, as long as

$$IP(t) \geq Q_l \text{ at all } t \quad (5)$$

The condition of Equation (5) can be satisfied in a way that minimizes the incurred holding cost, by setting the reorder point with respect to the IP(t) signal equal to Q_l (since, in this case, every time that IP(t) gets to the Q_l level, we place a replenishment order and we increase IP(t) by Q_c).

Finally, the reorder point with respect to OHI(t) is provided from the reorder point with respect to IP(t) through (1), when noticing that BO(t) = 0.

The main lesson of the above discussion is that in the EOQ context, reorder points should be specified according to the formula

$$\mathbf{ROP} = l\,D$$

but with respect to the *inventory position*, and not the on-hand-inventory.

4. (20 pts)

Order quantity given data

Item	1	2	3
D	12,500	15,000	15,000
A	150	80	80
h	2.4	3.5	3
Unit. Stor(f_i)	5	4	4
EOQ	1250	828.0786712	894.427191
Stor. Need	6250	3312.314685	3577.708764
Total Storage	13140.02345		

Since total required storage area is over 6000 sq. ft., we need to adjust order quantities. We can find the optimal order quantities through the search process over the Lagrange multiplier λ, discussed in class, that computes the values $Q_i = \sqrt{\frac{2A_iD_i}{h_i+2\lambda f_i}}$ and checks whether they satisfy the resource constraint as equality.

After some search on the values of λ, we get : $\lambda^*=1.204799$

Item	1	2	3
D	12,500	15,000	15,000
A	150	80	80
h	2.4	3.5	3
Unit. Stor(f_i)	5	4	4
NEW Q	509.4621425	427.3999954	435.7723896
Stor. Need 2	2547.310713	1709.599982	1743.089559
Total Storage	6000.000253		

Thus, the optimized order quantities for item 1, 2, and 3 should be 509, 427, and 436, respectively. As discussed in class, $-\lambda^*$ denotes the derivative of the optimal cost with respect to the size of the storage area F, and therefore, we should not be willing to pay more than 1.2 dollars per extra sq. ft.

5. (20 pts)

Time	1	2	3	4	5	6
Demand	100	150	75	75	50	60
Inventory on hand	60					
Actual Demand	40	150	75	75	50	60

In the cost calculations provided in the above table, each cell (i,j), j ∈ {1,…,6}, i ∈ {1,…,j}, denotes the cost of the plan that produces the demand of period j at period i, while following an optimal production plan over the periods 1,…, i-1.

Time	1	2	3	4	5	6
Cost Calculation	80	192.5	305	473.75	623.75	848.75
		160	216.25	328.75	441.25	621.25
			240	296.25	371.25	506.25
				296.25	333.75	423.75
					376.25	421.25

						413.75
Order Quantity	40	225		125		60

Also, notice that according to the "Planning Horizon" theorem of the Wagner-Whitin algorithm, we could have skipped the calculation of all the cells in each column j ∈ {1,…,6}, that lies above the highlighted cell in column j-1, without compromising the identification of the optimal plan (i.e., in columns 3 and 4, we could have skipped the evaluation of their first cells, and in columns 5 and 6 we could have skipped the evaluation of the first three cells).

Finally, in the considered application context, we could have used the Planning Horizon theorem to incur even larger economies in the involved computations. The point is that, under the applied "rolling-horizon" scheme, all we really want to know is the size of the order that should be placed in the first period. Now, according to the Planning Horizon theorem, if the demand d_j at some period j>1 is ordered at period i>1, then the demand d_k for any other period k>j will also be ordered at a period i'>1. Hence, we can stop the entire computation as long as we find such a period j (in the considered example, we could have stopped at the second period – in practice, typically we want to complete the calculation since having the entire ordering plan gives us some visibility on how our future needs are going to shape up).

Extra Credit (25 pts)

We are given the following information, annual demand = 140 units, ordering cost = 30 per order, holding cost = 18%, and cost function

$$C(Q) = \begin{cases} 350Q & : 1 \le Q \le 25, \\ 8750 + 315(Q-25) & : 26 \le Q \le 50, \\ 16625 + 285(Q-50) & : 51 \le Q. \end{cases}$$

Then we have,

$$\frac{C(Q)}{Q} = \begin{cases} 350Q & : 1 \le Q \le 25, \\ 315 + \dfrac{875}{Q} & : 26 \le Q \le 50, \\ 285 + \dfrac{2375}{Q} & : 51 \le Q. \end{cases}$$

The total annual cost function, G(Q), that is implied by the above average unit costs, is given by:

$$G(Q) = \frac{DC(Q)}{Q} + \frac{AD}{Q} + \frac{h\left(\frac{C(Q)}{Q}\right)Q}{2}.$$

The Q's that minimize this last function for each of the three expressions of C(Q)/Q can be obtained by substituting each of these three expressions in G(Q) and computing the minimum of the resulting function. This procedure gives us:

$$Q(1) = 12$$
$$Q(2) = 67$$
$$Q(3) = 115.$$

Observing that Q(2) does not fall into the correct interval, we focus only on the total annual costs that are provided by Q(1) and Q(2):

$$G(Q(1)) = (350)(140) + \frac{(30)(140)}{12} + \frac{(0.18)(350)(12)}{2} = 49728$$
$$G(Q(3)) = \left(285 + \frac{2375}{115}\right)(140) + \frac{(30)(140)}{115} + \frac{(0.18)\left(285 + \frac{2375}{115}\right)(115)}{2} = 45991$$

Since Q = 115 results in a lower cost, company Y should use an order size of 115 units.