ISYE 6201A,Q: Manufacturing Systems

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Midterm Exam II

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Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. A potential adversary effect of a high variability in the processing times of a given workstation is the destabilization of this workstation.

(a) TRUE (b) FALSE

Please, explain your answer, taking carefully into consideration the notion of "stability" of a manufacturing workstation that was defined in class.

The stability condition for a G/G/m queue is:

$r_a t_p \leq m$

where

- $r_a = 1/t_a$ = the average arrival rate
- t_p = the mean proc. time
- m = the number of (identical) servers at that workstation.

All the above ~~mom~~ quantities involve first moments while variability is essentially characterized by the second moments of the corresponding random variables.

2. The specification of the target throughput for a given workstation determines completely the degree of congestion in the operation of this workstation.

(a) TRUE (b) FALSE

Please, provide a thorough technical explanation of your answer. Also, please, make sure to define the "measures of congestion" that are relevant to the analysis of the above statement.

The congestion experienced at a given workstation is characterized by (i) the expected cycle time for parts going through this workstation and (ii) the average WIP concentrated at that workstation.

For a stable workstation modeled as a G/G/m queue,

$$CT_q = \frac{c_a^2 + c_e^2}{2} \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} t_e$$

and $u = r_a t_e$; $TH = r_a$

Also, $WIP_q = TH \cdot CT_q$

Hence, it is clear from the above factors that CT_q and WIP_q are impacted by many other factors besides r_a ($= TH$).

Similar remarks can be made for CT_q and WIP_q experienced at CONWIP workstations, if we consider the relevant theory developed in class for the computation of these quantities.

3. In class, we used the theoretical developments on the Asynchronous Transfer Line (ATL) model in order to argue the value of timely preventive maintenance in the operation of the contemporary production systems. Please, reproduce that argument.

Lack of timely preventive maintenance can result in unexpected preemptive outages that
(i) compromise the system availability, and therefore its effective production capacity since, ~~r_{max}~~ for any given workstation,

$$r_{max} = m/t_e = m/(t_p/A) = \frac{m}{t_p} A =$$

$$= r_{nominal} \cdot A ;$$

(ii) also increase the mean proc. times and the variability of these times, and thus the congestion experienced in the underlying system.

It is also true that the preventive maintenance itself requires some downtime for the various parts of the system going through this maintenance. But these downtimes can be much better controlled and planned in advance so that they have a limited impact on the system performance.

4. Consider a CONWIP line of five single-server stations where each station has the same processing time distribution. The line is operated with a constant WIP level of 20 parts, and the line throughput is estimated at 6 parts per hour. Based on this information, the average time of a part going through one of the line stations is

1. more than an hour.
2. less than an hour.
3. equal to an hour.
4. impossible to tell.

Please, explain your answer.

From Little's law, we get the total expected cycle time as:

$CT = WIP/TH = 20/6$ hrs

The "ring" structure of a CONWIP line together with the fact that all workstations have single servers with identical proc. time distributions imply that CT_i is identical across all workstations. Hence,

$CT_i = CT/5 = \frac{20/6}{5} = 4/6 = 2/3$ hrs.

5. Explain the notion of *"balancing"* and its significance in the context of the design of synchronous transfer lines.

A line is balanced if each of its workstations experiences the same workload, where the latter is defined by the total ~~utilization~~ expected amount of work per unit of time that takes place across all the servers of the workstation.

In the case of synchronous transfer lines, taking the line cycle time (i.e., the time between two consecutive advancements of the conveyor belt of the line), the above definition implies the equalization of the total amount of work assigned to each workstation.

As explained in class, such equalization would enable the attainment of the target throughput while minimizing the capacity losses across the line workstations, and therefore, it would also lead to the minimization of the required number of workstations.

Problem 1 (30 points): A certain operation at a manufacturing facility is supported by two different single-server workstations, $WS1$ and $WS2$, with the corresponding processing times distributed according to the normal distributions $N(10, 3^2)$ and$N(8, 3^2)$ (assume that these times are measured in minutes). Parts requesting the considered operation arrive according to a Poisson process with rate of 8 parts per hour, and they are diverted to workstations $WS1$ and $WS2$ with corresponding probabilities p and $1 - p$.

1. (10 pts) Determine the range of the values of p that lead to a stable operation for the two workstations.
2. (10 pts) Determine the value of p that balances the workload of the two workstations.
3. (10 pts) For the p value determined in part 2, compute the expected cycle time for a part that arrives at the considered facility in order to execute the considered operation.

Remark: The part routing scheme that is described in the above problem is known as "Bernoulli splitting" (with the corresponding routing probabilities). The Bernoulli splitting of an arrival process that is a Poisson process with rate λ, according to the routing probabilities p and $1 - p$, generates two Poisson processes with rates $p\lambda$ and $(1 - p)\lambda$.

1. The stability condition for the two workstations are:

$$\begin{cases} WS1: & \frac{8}{60} p \times 10 < 1 \\ WS2: & \frac{8}{60}(1-p) \times 8 < 1 \end{cases} \Rightarrow \begin{cases} p < 60/80 = 3/4 = 0.75 \\ (1-p) < \frac{60}{64} \Rightarrow p > \frac{4}{64} = \frac{1}{16} = 0.0625 \end{cases}$$

2. According to the discussion in the response of Question #5 in the previous page, the requested balancing of the workload in the considered case, implies the equalization of the utilization of the two workstations; i.e., we need:

$$\frac{8}{60} p \times 10 = \frac{8}{60}(1-p) \times 8 \Rightarrow 18p = 8 \Rightarrow p = 4/9 \approx 0.444$$

Then, $u_1 = u_2 = \frac{8}{10} \frac{4}{9} 10 = 16/27 \approx 0.592$

3. For the values obtained in part 2, we get:

$$CT_1 = \frac{1 + 9/10^2}{2} \; \frac{16/27}{1 - 16/27} \; 10 + 10 \simeq 17.927 \text{ min}$$

$$CT_2 = \frac{1 + 9/8^2}{2} \; \frac{16/27}{1 - 16/27} \; 8 + 8 \simeq 14.636 \text{ min}$$

In the above expressions we have considered that the total arrival stream is Poisson, and the Bernoulli splitting of such a Poisson stream generates two Poisson substreams, hence $c_{a_1} = c_{a_2} = 1$.

Finally, this splitting also implies that the expected cycle time for any given arrival is:

$$CT = p\, CT_1 + (1-p)\, CT_2 =$$

$$= \frac{4}{9}\, 17.927 + \frac{5}{9}\, 14.636 \simeq 16.1 \text{ min}$$

Problem 2 (30 points): An assembly process involves the following ten atomic tasks with the corresponding processing times and precedence constraints being reported in the following table:

Task	Proc. Time (secs)	Imm. Predecessors
a	10	-
b	8	-
c	12	a
d	7	c, b
e	5	c
f	15	d, e
g	11	-
h	15	g
i	10	e, h
k	5	f, i

1. (10 pts) What is the maximal possible hourly throughput that can be attained for this assembly process? Please, explain your answer.

2. (15 pts) In class we discussed the Ranked Positional Weight (RPW) heuristic as a design procedure for the assembly line balancing problem. Consider a variation of the RPW where tasks are ordered in decreasing cardinality of their successor sets S_i. Argue that this ordering also respects the precedence constraints of the tasks, and apply the resulting heuristic in order to design a line for the above assembly process that possesses a production rate of 60 parts per hour. Please, show clearly all the computations of your algorithm.

3. (5 pts) Provide a lower bound for the number of workstations that are necessary for the line designed in part #2 above. What are the implications of this bound for the line that you derived in part #2?

1. Since $TH = 1/c$, the maximum throughput is determined by the minimum possible cycle time. The assumed indivisibility of the tasks across the line workstations implies that $C_{min} = \max_i \{t_i\} = 15$ secs. Hence,

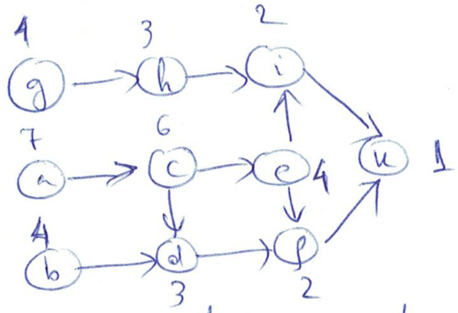
$$TH_{max} = 1/15\text{ secs} = \frac{3600 \text{ secs/hr}}{15 \text{ sec}} = 240 \text{ hr}^{-1}$$

2. From the class discussion,

$$S_i = \{i\} \cup \{S_j : j \text{ an immediate successor of } i\}$$

From this definition it follows that if task k is a predecessor (not necessarily immediate) of task ℓ, then $S_k \supset S_\ell$ (since $k \in S_k$ but $k \notin S_\ell$ and every element of S_ℓ must belong in S_k)

Hence, $Card(S_k) > Card(S_\ell)$ and ordering tasks in decreasing cardinality of their successor sets respects the imposed precedence constraints. So, a heuristic similar to the RPW where the PW's are replaced by $Card(S_i)$ is a correct heuristic for the ALB problem. To apply this heuristic to the considered problem, first we develop the corresponding precedence graph:



From this graph we get the $Card(S_i)$ for each task i annotated next to the task node. Hence, the corresponding task list is as follows: (see next page).

a, c, b, e, g, d, h, f, i, k

from this list and the fact that the line cycle time corresponding to the target throughput is

$$C = \frac{1}{60}\,\cancel{\text{hr}} = \frac{3600}{60}\,\text{sec} = 60\ \text{sec}$$

working as in the case of the RPW heuristic we obtain the following workstations:

WS1	WS2
a, c, b, e, g, d	h, f, i, k
idleness: 7 sec per cycle	idleness: 15 sec per cycle

3. A lower bound for the required workstations is:

$$\underline{N} = \lceil 98/60 \rceil = 2$$

since $\sum_i t_i = 98$ sec

This lower bound implies that the design obtained in part 2 is "optimal" in the sense that it minimizes the number of the employed workstations. Also, we can see that the obtained line is fairly well-balanced and the attained utilizations at each station quite high: $\frac{60-7}{60} \approx 0.88$ for WS1 and $\frac{60-15}{60} = 0.75$ for WS2.