

ISYE 6201A,Q: Manufacturing Systems
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Midterm Exam I
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Name:

SOLUTIONS

Answer the following questions (8 points each):

1. In class we remarked that there is a trade-off between any pair of the three major strategic objectives that are pursued by modern corporations, in the sense that any effort to increase performance with respect to one of these objectives might lead to a compromise of the performance with respect to the other one. Explain how this trade-off is manifested between the two strategic objectives of (i) providing a highly responsive service to the arising demand and (ii) product differentiation.

For considerably differentiated / customized product, it is not pertinent to produce to stock. Typically you produce to order (or assemble to order from standardized components, which is the current practice for mass customization).

For very highly customized production, you might even have to design the product from scratch in order to meet the posed specs.

All these practices imply a non-zero lead time for any experienced unit of demand.

On the other hand, (most of) the demand of non-differentiated products can be met immediately from stock (i.e., with zero lead time).

2. One of the impacts of the current globalized economy is that the major multinational corporations can treat the entire world as one seamless market, something that leads to a major simplification of their production and distribution systems.

(a) YES

☒ (b) NO

Please, explain your answer.

Refer to the provided discussion on market segmentation.

3. Consider a rather odd instantiation of the news-vendor model where any excess quantity will be returned to and refunded by the supplier. Determine an optimized order size for this case, providing a complete mathematical explanation of your answer.

In this case, the overage cost $c_o = 0$ and the critical ratio $\frac{c_s}{c_o + c_s} = 1$. So, the optimal order size Q^* should satisfy $G(Q^*) = 1$.

Hence, for distribution $g(\cdot)$ with finite support over some interval $[a, b]$, we should set $Q^* = b$.

If the support of $g(\cdot)$ is infinite, we should set Q^* to the maximal possible deliverable value under the prevailing operational constraints; these could be available budgets, storage area, other supplier imposed constraints, etc.

4. Consider a basestock inventory system with basestock level R and suppose that at a certain time point t the number of the outstanding replenishment orders $O(t)$ (i.e., the replenishment orders that have been placed but not received yet) is equal to $R + 1$. Use this information in order to compute the amount of the backorders at time t , $BO(t)$. Please, provide a clear explanation for your answer.

From basestock theory, we have:

$$\forall t, \quad OHI(t) + O(t) - BO(t) = R \quad (1)$$

Since, $O(t) = R + 1$, (1) implies that

$$OHI(t) - BO(t) = -1 \quad (2)$$

Also, from the provided definitions:

$$\forall t, \quad OHI(t) \cdot BO(t) = 0 \quad (3)$$

$$\forall t, \quad OHI(t) \geq 0 ; \quad BO(t) \geq 0 \quad (4)$$

But then, (2), (3) & (4) imply that

$$OHI(t) = 0 \quad \wedge \quad BO(t) = 1$$

5. Consider an inventory control system with holding and backorder costs respectively equal to h and b dollars per unit and time unit. Replenishment orders are delivered by a third-party carrier at the cost of A dollars per unit delivered, and any other transactional costs are negligible. Replenishment orders are delivered over an average time length of two days. Given this information, which of the inventory control policies discussed in class would you consider as appropriate for this system? Please, justify your answer.

The above description implies that there is no fixed ordering cost involved in the considered operational regime; ~~but only~~ hence, from an economic standpoint we should focus on the management of the holding and the backorder costs. This realization implies that a basestock model could be adequate.

On the other hand, the basestock model assumes that ~~the~~ replenishment orders are triggered by the experienced demand, one unit at a time, and for fast moving items such an ordering scheme might not be feasible / efficient from a practical (operational) standpoint. In this case, we would need to supply a larger order size Q that would be suggested by the aforementioned practical considerations. On the other hand, Q cannot be computed from the EOQ formula since the assumed fixed ordering cost is equal to 0.

Problem 1 (20 points): A local store with an annual demand of 10,000 units for one of its main products is trying to select a supplier for this product by choosing among two different options. The first supplier (supplier A) quotes a purchasing price of \$10 per unit, and a contract of 10 free deliveries per year at the price of \$1,000 per year. The second supplier (supplier B) quotes a purchasing price of \$20 per unit and she will utilize a 3PL service provider for the deliveries, who charges a fixed cost of \$15 per delivery and a variable cost of \$2 per delivered unit. The considered store computes its holding cost on the basis of an annual interest rate of 5%. Assuming that both suppliers can provide comparable quality of product and service, use the above information in order to determine the right supplier for this product.

Supplier A:

Obviously, the best order size for this case

$$\text{is } Q_A^* = 10,000 / 10 = 1,000 \text{ units}$$

Then, the resulting total annual cost is

$$\begin{aligned} TAC_A &= 10 \times 10,000 + 1,000 + 10 \times 0.05 \times \frac{10,000}{2} = \\ &= 101,250 \end{aligned}$$

Supplier B:

$$\text{In this case, } Q_B^* = \sqrt{\frac{2 \cdot 15 \cdot 10,000}{0.05 \times (20+2)}} \approx 522.233 \approx 522$$

and

$$\begin{aligned} TAC_B &= (20+2) \times 10,000 + 15 \frac{10,000}{522} + (20+2) \times 0.05 \times \frac{522}{2} = \\ &= 220,574.456 \end{aligned}$$

Hence, supplier A is the preferred deal.

Remark: Of course, for this problem it is rather easy to see that the effective unit cost of 22 \$/unit is too high when compared against the 10 \$/unit offered by supplier A. In particular, amortizing TAC_A over the 10,000 units of the demand we get: $101,250 / 10,000 = 10.125 < 22.00$

Problem 2 (40 points): A local supermarket buys the apples that it provides to its customers at \$1.5 per lb. and it sells them at \$2.5 per lb. Furthermore, the store applies a shelf-life policy of 3 days. At the end of the 3-day interval, any remaining apples are sold to a local cannery at a rate of \$.75 per lb, and the store is replenished with a new delivery. Finally, it is also estimated that the demand for apples in this supermarket over the 3-day replenishment interval is *uniformly* distributed with a mean of 500 lbs and a range of 200 lbs.

- (10 pts) Given the above information, what is the optimal replenishment order for this store?
- (10 pts) What is the probability that the store will not stock out during a replenishment cycle if it orders according to the policy that you determined in part (i)?
- (10 pts) What is the fill rate attained by the above policy?
- (10 pts) What is the *annual* expected profit w.r.t. to these apples for this supermarket?

$$(i) \quad C_o = 1.5 - 0.75 = 0.75 \quad ; \quad C_s = 2.5 - 1.5 = 1.0$$

$$\frac{C_s}{C_o + C_s} = \frac{1}{0.75 + 1} = \frac{1}{1.75}$$

$$\text{Hence, } \frac{Q^* - 400}{200} = \frac{1}{1.75} \Rightarrow Q^* = 514.286 \approx 514 \text{ lbs}$$

$$(ii) \quad P = \frac{C_s}{C_o + C_s} = \frac{1}{1.75} \approx 0.57 \quad \left(\text{More accurately: } \frac{514 - 400}{200} = 0.57 \right)$$

$$(iii) \quad \text{fill rate} = \frac{E[X_s]}{E[X]} = \frac{\int_{400}^{Q^*} x g(x) dx + Q^* [1 - g(Q^*)]}{500}$$

$$= \frac{1}{500} \left[\frac{1}{200} \int_{400}^{514} x dx + 514 [1 - g(514)] \right] =$$

$$= \frac{1}{500} \left[\frac{1}{400} (514^2 - 400^2) + 514(1 - 0.57) \right] =$$

$$\approx 0.963$$

(iv) Expected profit per cycle =

$$\begin{aligned}
 & \text{Expected sales} + \text{Expected salvage value} - \text{Expected cost} \\
 &= 2.5 \times \int_{400}^{600} \min\{x, 514\} \frac{1}{200} dx + \\
 & \quad + 0.75 \times \int_{400}^{600} \max\{514 - x\} \frac{1}{200} dx - \\
 & \quad - 1.5 \times 514 = \\
 &= \frac{2.5}{200} \left[\frac{1}{2} (514^2 - 400^2) + 514 (600 - 514) \right] + \\
 & \quad + \frac{0.75}{200} \left[514 (514 - 400) - \frac{1}{2} (514^2 - 400^2) \right] - \\
 & \quad - 771 = 1203.775 + 24.3675 - 771 = 457.1425
 \end{aligned}$$

Hence, the expected annual profit is:

$$\frac{365}{3} \times 457.1425 \approx 55,619$$