ISyE 6201: Manufacturing Systems
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Solutions for Homework #4

A. Question Set

Chapter 7

Question 2

Since $TH = \dfrac{WIP}{CT}$

one can have the same TH with high WIP levels and long cycle times or with low WIP levels and short cycle times. Obviously, the later is better since you have better control, less money tied up in inventory, shorter cycle times for better responsiveness, less reliance on forecasts, and better quality.

Question 3

The best case cycle time is given by

$$CT_{best} = \begin{cases} T_0 & \text{if } w \le w_0 \\ w/r_b & \text{if } w > w_0 \end{cases}$$

The corresponding throughput is obtained using Little's law.

$$TH_{best} = \begin{cases} w/T_0 & \text{if } w \le w_0 \\ r_b & \text{if } w > w_0 \end{cases}$$

The worst case cycle time is given by

$$CT_{worst} = wT_0$$

while the worst case throughput is given by

$$TH_{worst} = 1/T_0$$

Question 5

By the definition of the critical WIP, $W_0 = r_b T_0 = r_b \sum_{i=1}^{n} t_i$, where r_b denotes the bottleneck rate and t_i denotes the processing time at the i-th station of the line. Also, by the definition of the bottleneck rate r_b,

(i) $r_b = N_b/t_b$, where N_b is the number of machines at a bottleneck workstation and t_b is the processing time at that station;

(ii) $r_b \le r_i = N_i/t_i, \Leftrightarrow r_b.t_i \le N_i, \forall i=1,\dots,n$, where N_i and t_i are respectively the number of machines and the processing time at station i.

Hence, $W_0 = \sum_{i=1}^{n} r_b t_i \le \sum_{i=1}^{n} N_i$

More intuitively, W_0 denotes the *minimum* WIP that can ensure 100% utilization of the system bottlenecks, in a deterministic operational setting. Keeping a WIP which is larger than the total number of the machines in the system, will only increase the waiting time of the jobs, without enhancing the utilization of the system bottlenecks.

Finally, notice that $W_0 = \sum_{i=1}^{n} N_i$ only when $r_b.t_i = N_i, \forall i=1,\dots,n$, i.e., only when the line is balanced.

Chapter 18

Question 5

A paced assembly line does not have the problems of starving and blocking that an unpaced line would have because it *is* paced. The pacing mechanism is such that each station can perform its operation before the next unit arrives. This requires that the pacing be slower than the slowest station on the line. Consequently, the bottleneck of the line is the pacing mechanism itself.

Question 6

The conveyor time, *c*, should be greater than the maximum time assigned to any station to make sure that all stations are able to finish their task before the part is moved to the next station. If this were not the case, parts would move before they were completed and would disrupt the flow of the entire line.

CONWIP Flow Line Question

The critical WIP in this line is 3. The maximum TH that the line can achieve is TH_{best} = 2/(12/60) = 10 parts per hour, which is less than the targeted production rate of 12 parts per hour.

B. Problems

Chapter 7

Problem 5

a. r_b = 0.5 job/hr and T_0 = 5 hr

b. Adding another machine at station two makes the parameters

   r_b = 1 job/hr and T_0 = 5 hr

   so the bottleneck rate increases and the raw process time stays the same. This will decrease cycle time and increase throughput when WIP is greater than the critical WIP.

c. Speeding up station two makes the parameters

   r_b = 1 job/hr and T_0 = 4 hr

   and so it increases the bottleneck rate *and* decreases the raw process time. In this case, throughput will go up and cycle time will decrease regardless of the WIP level.

d. There is no change in the parameters from (a). Since there is no variability, there will be no change in performance.

e. This will make the parameters

$r_b = 0.5$ job/hr and $T_0 = 4.5$ hr

and it will reduce the cycle time and increase the throughput and, but only when WIP is below the critical WIP. So, in this (deterministic) case, it is true that saving an hour at a non-bottleneck does not help much.

Problem 6

When all jobs are processed before moving, we have the worst case performance with cycle time given by $CT = wT_0$ and $TH=1/T_0$.

a. In this new setting, the base parameters r_b and T_0 will be the same with those in Problem 1, for all four cases (a-d). However, the impact of the suggested changes on the system performance can be different, as discussed below.
b. Since T_0 remains the same as in (a), there is no change from (a) regarding cycle time and throughput. This is in contrast with Problem 1.
c. Speeding up station 2 reduces the raw process time and therefore reduces cycle time and increases throughput for all WIP levels.
d. If all the jobs are worked on by only one machine at station 1 (as assumed in the worst-case scenario, since the batch moves as a single block), there is absolutely no change in performance.
e. If station one is speeded up this will reduce the raw process time as in Problem 1 and so will improve performance for all WIP levels.

Problem 7

The practical worst case assumes exponential process times *and* a balanced line using only a single machine at each station.

a. Similar with Problem 2, the base parameters for the system do not change, but the observed performance sometimes does.

b. Throughput will increase and cycle times will decrease for *all WIP levels.*

c. Speeding up station 2 reduces the raw process time and therefore reduces cycle time and increases throughput for all WIP levels.

d. Adding a second machine at a non-bottleneck will improve throughput and reduce cycle times for all WIP levels above WIP = 1.

e. Speeding up station one will improve performance for all WIP levels.

Problem 10

$r_b = 2000$ per day = 125 per hour, and $T_0 = 0.5$ hr, $W_0 = 62.5$ cases, TH = 1700 cases/day = 106.25 per hour, CT = 3.5 hours.

a. Average WIP level = TH*CT = 106.25 cases per hour * 3.5 hours = 372 cases

b. At w = 372, we have $TH_{PWC}(372) = \frac{372}{62.5+372-1} \times 125$ cases/hour = 107.27 and $CT_{PWC} = 0.5 + \frac{372-1}{125} = 3.468$ hours, so we are roughly operating at the PWC level.

c. Throughput would increase (or at least not decrease), because bottleneck would be blocked/starved less. Unbalancing the PWC line causes it to perform better.

d. Throughput would increase (or at least not decrease), because bottleneck would be blocked/starved less. Replacing single machine stations with parallel machine stations in the PWC causes it to perform better. This is an example of performance improvement through flexibility enhancement.

e. Moving cases in batches would further inflate cycle time by adding "wait for batch" time, and increasing the variability experienced in the flow of the line.

Chapter 10

Problem 2

a. $\tilde{w} = 3\frac{u}{1-u} = 3\frac{0.9}{1-0.9} = 27$

b. $TH = \frac{w}{w+W_0-1} r_b = \frac{27}{27+3-1} 0.5 = 0.466$

It can be seen that the CONWIP line achieves a higher throughput for this same WIP level. This effect results from the fact that the push system may release work into the line when the queue is very long, causing congestion that inflates the cycle time and (by Little's law) reduces the throughput.

Problem 6

a. (i)

j	1		2		3		4			
te(j)	2.5		2		2		2			
ce(j)	1		1		1		1			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.500	0.294	2.000	0.235	2.000	0.235	2.000	0.235	8.500	0.118
2	3.235	0.608	2.471	0.464	2.471	0.464	2.471	0.464	10.647	0.188
3	4.019	0.942	2.928	0.686	2.928	0.686	2.928	0.686	12.804	0.234
4	4.854	1.297	3.372	0.901	3.372	0.901	3.372	0.901	14.971	0.267
5	5.743	1.674	3.802	1.109	3.802	1.109	3.802	1.109	17.149	0.292
6	6.686	2.075	4.217	1.308	4.217	1.308	4.217	1.308	19.337	0.310
7	7.686	2.498	4.617	1.501	4.617	1.501	4.617	1.501	21.537	0.325
8	8.745	2.946	5.001	1.685	5.001	1.685	5.001	1.685	23.749	0.337
9	9.865	3.418	5.369	1.861	5.369	1.861	5.369	1.861	25.973	0.347
10	11.046	3.916	5.721	2.028	5.721	2.028	5.721	2.028	28.209	0.354
11	12.289	4.438	6.056	2.187	6.056	2.187	6.056	2.187	30.458	0.361
12	13.596	4.986	6.374	2.338	6.374	2.338	6.374	2.338	32.719	0.367

(ii)

j	1		2		3		4			
te(j)	2		2		2.5		2			
ce(j)	1		1		1		1			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.000	0.235	2.000	0.235	2.500	0.294	2.000	0.235	8.500	0.118
2	2.471	0.464	2.471	0.464	3.235	0.608	2.471	0.464	10.647	0.188
3	2.928	0.686	2.928	0.686	4.019	0.942	2.928	0.686	12.804	0.234
4	3.372	0.901	3.372	0.901	4.854	1.297	3.372	0.901	14.971	0.267
5	3.802	1.109	3.802	1.109	5.743	1.674	3.802	1.109	17.149	0.292
6	4.217	1.308	4.217	1.308	6.686	2.075	4.217	1.308	19.337	0.310
7	4.617	1.501	4.617	1.501	7.686	2.498	4.617	1.501	21.537	0.325
8	5.001	1.685	5.001	1.685	8.745	2.946	5.001	1.685	23.749	0.337
9	5.369	1.861	5.369	1.861	9.865	3.418	5.369	1.861	25.973	0.347
10	5.721	2.028	5.721	2.028	11.046	3.916	5.721	2.028	28.209	0.354
11	6.056	2.187	6.056	2.187	12.289	4.438	6.056	2.187	30.458	0.361
12	6.374	2.338	6.374	2.338	13.596	4.986	6.374	2.338	32.719	0.367

Individual stations are affected by having bottleneck at station 1 or 3, but overall line performance is not.

b. (i)

j	1		2		3		4			
te(j)	2		2		2.5		2			
ce(j)	0.25		1		1		1			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.000	0.235	2.000	0.235	2.500	0.294	2.000	0.235	8.500	0.118
2	2.250	0.432	2.471	0.474	3.235	0.621	2.471	0.474	10.426	0.192
3	2.504	0.603	2.948	0.710	4.051	0.976	2.948	0.710	12.451	0.241
4	2.755	0.758	3.421	0.941	4.941	1.359	3.421	0.941	14.536	0.275
5	3.000	0.900	3.882	1.165	5.899	1.770	3.882	1.165	16.664	0.300
6	3.238	1.032	4.330	1.380	6.925	2.207	4.330	1.380	18.822	0.319
7	3.467	1.155	4.760	1.586	8.019	2.672	4.760	1.586	21.006	0.333
8	3.686	1.270	5.173	1.783	9.180	3.164	5.173	1.783	23.211	0.345
9	3.894	1.378	5.566	1.969	10.410	3.683	5.566	1.969	25.436	0.354
10	4.092	1.479	5.939	2.146	11.709	4.230	5.939	2.146	27.678	0.361
11	4.280	1.572	6.291	2.312	13.076	4.804	6.291	2.312	29.938	0.367
12	4.456	1.660	6.623	2.467	14.511	5.406	6.623	2.467	32.213	0.373

(ii)

j	1		2		3		4			
te(j)	2		2		2.5		2			
ce(j)	1		1		0.25		1			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.000	0.235	2.000	0.235	2.500	0.294	2.000	0.235	8.500	0.118
2	2.471	0.480	2.471	0.480	2.891	0.561	2.471	0.480	10.302	0.194
3	2.959	0.727	2.959	0.727	3.334	0.819	2.959	0.727	12.212	0.246
4	3.454	0.974	3.454	0.974	3.828	1.079	3.454	0.974	14.190	0.282
5	3.947	1.217	3.947	1.217	4.372	1.348	3.947	1.217	16.214	0.308
6	4.435	1.456	4.435	1.456	4.967	1.631	4.435	1.456	18.271	0.328
7	4.913	1.690	4.913	1.690	5.616	1.931	4.913	1.690	20.353	0.344
8	5.379	1.916	5.379	1.916	6.321	2.252	5.379	1.916	22.458	0.356
9	5.832	2.135	5.832	2.135	7.085	2.594	5.832	2.135	24.582	0.366
10	6.271	2.346	6.271	2.346	7.913	2.961	6.271	2.346	26.724	0.374
11	6.693	2.549	6.693	2.549	8.806	3.354	6.693	2.549	28.884	0.381
12	7.098	2.742	7.098	2.742	9.768	3.774	7.098	2.742	31.061	0.386

It is more effective (i.e., it improves TH more) to reduce variability at bottleneck.

c. (i)

j	1		2		3		4			
te(j)	2		0.25		2.5		2			
ce(j)	1		1		1		1			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.000	0.296	0.250	0.037	2.500	0.370	2.000	0.296	6.750	0.148
2	2.593	0.585	0.259	0.058	3.426	0.772	2.593	0.585	8.870	0.225
3	3.169	0.862	0.265	0.072	4.431	1.205	3.169	0.862	11.034	0.272
4	3.723	1.126	0.268	0.081	5.512	1.667	3.723	1.126	13.226	0.302
5	4.252	1.377	0.270	0.088	6.667	2.159	4.252	1.377	15.442	0.324
6	4.754	1.614	0.272	0.092	7.897	2.681	4.754	1.614	17.676	0.339
7	5.227	1.836	0.273	0.096	9.202	3.232	5.227	1.836	19.929	0.351
8	5.672	2.044	0.274	0.099	10.580	3.813	5.672	2.044	22.198	0.360
9	6.088	2.238	0.275	0.101	12.032	4.423	6.088	2.238	24.484	0.368
10	6.476	2.418	0.275	0.103	13.558	5.062	6.476	2.418	26.785	0.373
11	6.836	2.584	0.276	0.104	15.154	5.728	6.836	2.584	29.101	0.378
12	7.168	2.736	0.276	0.105	16.820	6.422	7.168	2.736	31.432	0.382

(ii)

j	1		2		3		4			
te(j)	2		2		2.5		2			
ce(j)	0.5		0.5		1		0.5			
w	CT1(w)	WIP1(w)	CT2(w)	WIP2(w)	CT3(w)	WIP3(w)	CT4(w)	WIP4(w)	CT(w)	TH(w)
0		0		0		0		0		0
1	2.000	0.235	2.000	0.235	2.500	0.294	2.000	0.235	8.500	0.118
2	2.294	0.453	2.294	0.453	3.235	0.640	2.294	0.453	10.118	0.198
3	2.610	0.656	2.610	0.656	4.099	1.031	2.610	0.656	11.930	0.251
4	2.936	0.846	2.936	0.846	5.077	1.463	2.936	0.846	13.884	0.288
5	3.259	1.023	3.259	1.023	6.157	1.932	3.259	1.023	15.935	0.314
6	3.575	1.188	3.575	1.188	7.330	2.436	3.575	1.188	18.054	0.332
7	3.878	1.342	3.878	1.342	8.590	2.973	3.878	1.342	20.222	0.346
8	4.165	1.486	4.165	1.486	9.933	3.543	4.165	1.486	22.429	0.357
9	4.436	1.619	4.436	1.619	11.358	4.144	4.436	1.619	24.666	0.365
10	4.690	1.742	4.690	1.742	12.860	4.775	4.690	1.742	26.930	0.371
11	4.926	1.855	4.926	1.855	14.438	5.436	4.926	1.855	29.217	0.376
12	5.145	1.958	5.145	1.958	16.090	6.125	5.145	1.958	31.524	0.381

Speeding up non-bottleneck here is better than reducing variability at nonbottlenecks. Note that while this will often be the case, it will not always occur. Depending on the specifics of the problem, including the costs, reducing variability can be more attractive than increasing capacity.

Problem 27 on ALB

a)

Task	Positional Weight
1	100
2	94

3	46
4	43
5	37
6	47
7	23
8	20
9	29
10	16
11	16
12	20
13	12
14	8
15	5

Ranking: 1-2-6-3-4-5-9-7-8-12-10-11-13-14-15

b) 100/30 = 3.33 implying a minimum of 4 stations would be required

Station	1	2	3	4
Tasks	1,2,6,3	4,5,7,9	8,12,10,11	13,14,15
Idle Time	2	1	2	15

c) Start with C=25. A perfect balance would require four stations. Unfortunately this is not possible.
For C=26 we do find the following 4-station balance.

Station	1	2	3	4
Tasks	1,2,3,5	4,6,7,8	9,11,12	10,13,14,15
Idle Time	0	0	1	3