ISYE 6201: Manufacturing Systems
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Midterm Exam II
April 11, 2013

Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. A manufacturing station fits the assumptions of a $G/G/1$ queueing station, and currently it is found to be unstable, i.e., it is not able to provide the desired target throughput. The station supervisor who had an one-day training in "factory physics" contests that the problem of the station should not be considered as a "capacity" problem but as a "variability reduction" problem.

Do you agree with this position?

(A) YES (B) NO

Explain your answer.

The stability condition for the G/G/1 queue is:

$$u = r_a t_p < 1$$

This condition involves only the mean of the proc. times at that station, ~~It has~~ and the rate of the job arrivals, ~~nothing~~ which is the inverse of the mean inter-arrival times. It has nothing to do with the second moments of the processing and inter-arrival times, that pertain to the notion of variability experienced at that station.

2. The propagation of variability in an asynchronous transfer line implies that stations further upstream (i.e., in the earlier stages) in the line will always have a more variable job arrival process, in terms of its inter-arrival times, compared to their downstream stations.

(A) TRUE (B) FALSE

Explain your answer.

From the provided formulae for C_d^2, that essentially describe the propagation of variability, we can see that ~~the~~ in the case of highly utilized stations, the variability in the arrival process that is experienced by its neighbering downstream station is primarily determined by the variability in the processing times of that station. But this is a local attribute of the station and has nothing to do with its placement in the line.

3. An effective way for reducing the mean cycle time, CT, in a CONWIP line is by . . .

* adding capacity to one of the stations, especially the "bottleneck" one(s).
* reducing the variability in the proc. times at some of the stations.
* introducing operational flexibility that would enable the better use of the system resources.

See your class notes for further elaboration on each of the above strategies.

4. Consider a single-server processing station with job arrival rate r_a, effective processing time t_e, and effective utilization $u = r_a t_e > 1$. What is the effective long-term throughput for this station? (especially consider the case where $t \to \infty$).

Explain your answer.

This station is ustable. Therefore, as $t \to \infty$, WIP $\to \infty$, which enable 100% utilization of the server and will lead to a

$TH = 1/te$.

5. What is the key element of the U-shaped layout that underlies most of the advantages that this layout enjoys in the contemporary lean manufacturing paradigm?

The proximity that it establishes among all the parts of the line.

Problem 1 (30 points): Consider a two-station asynchronous transfer line supporting the production of a certain item Y. The processing time of a single part of Y at the first station is distributed according to a general distribution with a mean of 1 min and st. dev of 0.25 minutes. On the other hand, the processing time of a single part of Y at the second station is distributed according to a general distribution with a mean of 1.25 min and st. dev of 0.5 minutes. Furthermore, Y is processed on this line in batches of 20 units per batch (this batch size defines, both, the processing and transfer batch size among the line stations and its ingress and egress points). Finally, batches are loaded on the line at a deterministic rate r_a. Please, answer the following questions:

i. (10 pts) What batch loading rate r_a will enable the line to produce at 85% of its production capacity?

ii. (10 pts) What is the expected cycle time for the parts moving through the line, if the line is operated at the rate that was computed in item (i) above?

iii. (10 pts) What is the average number of batches that are waiting in front of the second station?

First, let us compute $(t_{b_i}, c^2_{b_i})$, $i=1,2$

where t_{b_i} is the batch mean proc. time for station i, and $c^2_{b_i}$ is the corresponding SCV. We have:

$t_{b_1} = 20 \times 1 = 20$ min ; $t_{b_2} = 20 \times 1.25 = 25$ min

$$c^2_{b_1} = \frac{c^2_{p_1}}{\beta} = \frac{0.25^2}{20} = 3.125 \times 10^{-3}$$

$$c^2_{b_2} = \frac{c^2_{p_2}}{\beta} = \frac{(0.5/1.25)^2}{20} = 8 \times 10^{-3}$$

Next, we answer the problem questions:

(i) We need $u = r_a \times t_{b_2} = 0.85 \Rightarrow r_a = 0.034\ \text{min}^{-1}$

In the above computation we have taken into consideration the fact that $t_{b_2} > t_{b_1}$.

(ii)

$$CT_1 = \frac{c_{a_1}^2 + c_{b_1}^2}{2} \frac{u_1}{1-u_1} t_{b_1} + t_{b_1}$$

$$u_1 = 0.034 \times 20 = 0.68$$

$c_{a_1}^2 = 0$ (deterministic job release pattern) $\Rightarrow$

$$\Rightarrow CT_1 = \frac{0 + 3.125 \times 10^{-3}}{2} \frac{0.68}{1-0.68} 20 + 20 = 20.0664 \text{ min}$$

$$CT_2 = \frac{c_{a_2}^2 + c_{b_2}^2}{2} \frac{u_2}{1-u_2} t_{b_2} + t_{b_2}$$

$$c_{a_2}^2 = c_{d_1}^2 = u_{b_1}^2 c_{b_1}^2 + (1-u_{b_1}^2) c_{a_1}^2 = 0.68^2 \cdot 3.125 \times 10^{-3} = 1.445 \times 10^{-3}$$

$$u_2 = 0.85$$

$$\Rightarrow CT_2 = \frac{1.445 \times 10^{-3} + 8 \times 10^{-3}}{2} \frac{0.85}{1-0.85} 25 + 25 = 25.669 \text{ min}$$

Finally,

$$CT = CT_1 + CT_2 = 45.7354 \text{ min}$$

(iii) $WIP_{q_2} = WIP_2 - u_2 = r_a CT_2 - u_2 = 0.034 \times 25.669 - 0.85 =$

$$= 0.0227$$

Problem 2 (30 points): Consider a stable single-server manufacturing station which constitutes the first station of a longer production line. Jobs arrive at this station according to a Poisson process, and the server utilization is measured at 80% of its effective capacity. The server *nominal* (or *"natural"*) processing time is (deterministically) equal to 2 minutes and its current availability is 90%. The times between failures are exponentially distributed, and so are the experienced downtimes; the mean value for the latter is 30 minutes. Your task is to compute the following:

i. (10 pts) The throughput of this station in the operational regime that is described above.

ii. (10 pts) The expected cycle time CT for a job going through this station.

iii. (10 pts) The variability that is induced by this station to the rest of the production line, as measured by the CV of the inter-departure times.

(i) $t_e = t_o / A$;

$r_a t_e = u \Rightarrow r_a = u/t_e = \frac{uA}{t_o} = \frac{0.8 \times 0.9}{2} = 0.36 \text{min}^{-1}$

(ii) $CT = \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t_e + t_e$

$c_a^2 = 1$; $c_o^2 = 0$; $c_r^2 = 1$; $m_r = 30 \text{min}$

$c_e^2 = c_o^2 + (1 + c_r^2) A(1-A) \frac{m_r}{t_o} =$

$= 0 + (1+1)\, 0.9 \times 0.1 \times \frac{30}{2} = 2.7$

$\Rightarrow CT = \frac{1+2.7}{2} \frac{0.8}{1-0.8} 2.22 + 2.22 = 18.648 \text{ min}$

(iii) $c_d^2 = (1-u^2) c_a^2 + u^2 c_e^2 = (1-0.8^2) \cdot 1 + 0.8^2 \cdot 2.7 =$

$= 2.088 \Rightarrow c_d \simeq 1.445$