ISYE 6201: Manufacturing Systems
Spring 2008
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Midterm Exam I
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Name: SOLUTIONS

Answer the following questions (**10 points each**):

1. Briefly discuss how information technology (IT) has contributed to the expansion of the operational frontier of contemporary supply chains.

Information technology enables companies to have better visibility of their own operational status as well as of the events that take place across the entire supply chain.

As a result, companies can plan and coordinate their activity more effectively and with less disruption due to uncertainty / unexpected events.

When viewed from a strategic standpoint, this last effect further implies an ability to maintain a responsive operation while reducing at the same time the necessary buffers that protect against uncertainty.

But this simultaneous improvement of responsiveness and cost effectiveness (two strategic objectives that traditionally have been in conflict) essentially implies an expansion of the operational frontier. —

2. What are the basic types of *"buffering"* that can help a company accommodate the variability encountered in its operations?

The basic types of "buffer" that offer protection against variability are:

(i) excess inventory (safety stock)
(ii) " production capacity, and
(iii) " (inflated) lead times.

3. Discuss how you would redefine the notion of *service level* so that it applies to an item that is produced to order.

Service levels essentially measure the company's ability to meet the customer needs in a responsive and effective manner.

In a "produce-to-order" context, the above definition translates to an assessment of the company's ability to fill the customer demand within the time frame requested by the customer.

Some more concrete measures ~~for this~~ quantifying the above idea, are

(i) the probability that a certain order will be delivered no later than its due date;

(ii) the expected number of orders that are delivered no later than their due date during a certain business cycle.

4. Consider a spare part that constitutes a component of three different machines at a given shop floor. The inventory of this spare part is maintained according to a basestock policy with the reorder point r set equal to 5. If it is known that the current number of outstanding orders for this part is equal to 5, we can infer that the number of the machines that are down due to failure of this particular part, is equal to:

i. 0

ii. 1

iii. 3

iv. we cannot answer this question on the basis of the provided data.

Explain your answer.

Since the reorder point $r = 5$, the basestock level $R = 6$.

For the basestock model, we know that

$$\forall t,\ IP(t) = OHI(t) + O(t) - BO(t) = R \Leftrightarrow$$

$$\Leftrightarrow \forall t,\ OHI(t) - BO(t) = R - O(t)$$

Hence, in our case:

$$OHI - BO = 6 - 5 = 1 \Rightarrow \left\{ \begin{array}{l} OHI = 1 \\ BO = \emptyset \end{array} \right\} \Rightarrow$$

$\Rightarrow$ number of failing machines $= \emptyset$.

5. Consider the weekly demand for a bar soap that is experienced at two different locations of its supply chain:

i. the plant warehouse that stages the product as it comes out from the production line;

ii. a super market that conveys the product to the final customer.

Which of these two demand profiles is more likely to be *normally* distributed?

Explain your answer.

As discussed in class, the normality of the demand experienced by many inventory systems ~~results~~ stems from the fact that this demand is the cumulative result of the behavior of a (very) large number of similar customers that act independently (and therefore the Central Limit Theorem (CLT) applies).

Clearly, the above effect can be invoked naturally in the case of the supermarket.

On the other hand, the "customers" of the plant warehouse will be some (typically a few) distribution centers that interface this warehouse with the downstream part of the supply chain. Furthermore, the demand posed by each of these DC's to the considered warehouse will be strongly influenced by the inventory control policies followed by these DC's, making the behavior of each of these DC's quite idiosyncratic. All these factors imply a departure from the conditions stipulated by the CLT.

Remark: Notice that the remarks made in the above paragraph are also at the basis of the "Bullwhip effect".

Problem 1 (30 points): The inventory of a certain item is controlled by a periodic review policy with the review period set equal to one week. The item demand over an one-week interval has been found to be *uniformly* distributed between 300 and 600 units. Every week unmet demand is back-ordered, and at the end of the week we place a replenishment order that is adequate to cover the backorders of this week, and bring the inventory position at the beginning of the next week to some desired level S. The company accountants estimate that every unit of experienced backorder costs the company \$4.00 dollars, but also every unit of excess inventory experienced at the end of the week costs \$2.00.

i. (10 points) Compute a target level S that minimizes the expected weekly cost of the company.

ii. (10 points) What is the type-I service level that results from the target level S that you computed in part (i)?

iii. (10 points) Which of the following values is more likely to express the type-II service level that results from the target level S that you computed in part (i)?

   (a) 0.55
   (b) 0.67
   (c) 0.80
   (d) 1.00

   Justify your answer.

(i) As discussed in class, the analysis of this system can be decomposed on a period by period basis, with each period corresponding to a newsvendor model where: $C_u = b$ (unit backordering cost) and $C_o = h$ (unit holding cost per period). Hence

$$G(S^*) = \frac{b}{b+h} = \frac{4}{4+2} = \frac{2}{3} \Rightarrow S^* = 300 + (600-300)\frac{2}{3} =$$
$$= 500.$$

(ii) Type I service level = Prob (weekly demand $\leq S^*$) =
$= G(S^*) = 2/3$.

(iii) We know that for these systems,
Type II service level for a certain selection of S is greater than Type I service level, for the same S.
This remark excludes options (a) and (b).
On the other hand, $S^* < 600$, and therefore, Type II service level cannot be 1.00, either.
Hence, a plausible guess is (c).

Remark: It turns out that the Type II service level resulting from S^*_0 is equal to $\simeq 0.97$ (This value can be computed through either of the two approaches presented in class. Some of you calculated the above value and ~~were lead~~ to as a result, you suggested (d) as a correct response. I accepted this response as correct, but what I was really looking for, was some reasoning along the line described above.

Also, it is interesting to juxtappose the two values for type I and type II service levels, to see how much more stringent is the type I requirement.

Problem 2 (20 points): A club with 200 members is organizing a celebrating event and it plans to offer to each attending member a souvenir cup with the club logo imprinted on it. The production of these cups has been outsourced to a printing company and the club has added 7 dollars to its ticket price to cover this expense. The club has asked its members to RSVP no later than 5 days before the event, and it plans to place the exact order to the printing company once the RSVP period is over.

However, currently we are 8 days before the event, and the printing company, in an effort to balance its expected demand, has called the club management and has offered to produce any order placed at this point at the reduced price of 5 dollars per cup. (Of course, the club management can still place an order for additional cups at a later point, at the original price of 7 dollars per cup.)

Given that the number of tickets currently sold is 130, and, according to the club management, each of the remaining patrons has a 70% chance to attend the event, determine whether the club management should place an order at this reduced rate or not, and in case of a positive answer, what should be the optimal order size.

Hint: In your calculations, approximate the distribution of the unknown component of the demand for cups by a *normal* distribution, with mean and variance obtained through application of the following *Central Limit Theorem*:

Theorem: Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed random variables, each with its mean μ and variance σ^2. Then, as $n \to \infty$, the distribution of $\sum_{i=1}^{n} X_i$ tends to a normal distribution with mean $n\mu$ and variance $n\sigma^2$.

Also, the table for the standard normal distribution that you might need for your calculations, is provided at the end of this document.

Clearly, in this case,

$C_o = 5$ (since any undisposed cup will not be covered by a ticket and it has no salvage value);

$C_u = 2$ (since any additionally needed cups will have to be procured at the cost of \$7.00, instead of \$5.00)

Also, the club will definitely have to order the 130 cups for its members that have already bought their tickets, while each of the remaining patrons essentially constitutes a Bernoulli r.v. with "success" probability $p = 0.7$. Hence, this second component of the demand follows a Binomial distribution with parameters $n = 70$ and $p = 0.7$.

To approximate this distribution by a normal, we apply the CLT, as suggested by the problem, writing this second component of the demand as

$$D = \sum_{i=1}^{70} X_i$$

where each X_i is the Bernoulli r.v. mentioned above.

Hence

$$D \overset{\sim}{\to} N(\mu_D, \sigma_D^2)$$

where

$$\mu_D = 70 \cdot E[X_i] = 70 \cdot p = 70 \times 0.7 = 49$$

$$\sigma_D^2 = 70 \cdot Var[X_i] = 70 \cdot p(1-p) = 70 \times 0.7 \times 0.3 = 14.7$$

and $\sigma_D = \sqrt{\sigma_D^2} \simeq 3.834$.

(Remark: $Var[X_i]$ can be easily obtain by:

$Var[X_i] = E[X_i^2] - E[X_i]^2$ when noticing that

$E[X_i^2] = p \cdot 1 - (1-p) \cdot 0 = p$ and therefore, $Var[X_i] = p - p^2$)

Letting r^* denote the optimal order size (corresponding to demand generated by the 70 members who have not RSVPed yet, we have from the newvendor theory, that

$$G(r^*) = \frac{c_u}{c_o + c_u} \iff \varphi\left(\frac{D - \mu_D}{\sigma_D} \leq \frac{r^* - \mu_D}{\sigma_D} \right) = \frac{2}{5+2} \iff$$

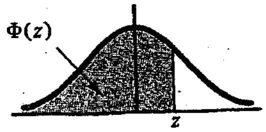
$$\iff r^* = \mu_D + z_{2/7} \cdot \sigma_D \implies \text{(from the attached table)}$$

$$\Rightarrow r^* = 49 - 0.565 \times 3.834 = 46.834 \simeq 47$$

Hence, the total number of cups to be ordered at the reduced price of \$5.00 is equal to $130 + 47 = 177$.

<u>Remark:</u> In the above calculation, it is also interesting to notice the agreement between the mean and the variance obtained for the approximating normal distribution, and the corresponding values for the Binomial distribution that models the exact demand.

TABLE A.1 VALUES OF THE STANDARD NORMAL PROBABILITY DISTRIBUTION FUNCTION



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE A.1 *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998