ISYE 6201: Manufacturing Systems
Spring 2007
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Midterm Exam II
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Name: SOLUTIONS

Answer the following questions **(10 points each)**:

1. A single-server station is operated at 75% of its effective capacity. If jobs arrive at this station according to a *Poisson* distribution, what is the probability that an arriving job will find the station server occupied? Briefly explain your answer.

The key observation for answering this question is the well-known PASTA effect, i.e., the fact that Poisson Arrivals See Time Averages".

Then, according to PASTA, the probability that an arrival will find the server occupied is equal to the probability that the server is occupied at any arbitrary point in time.
But this last quantity is equal to the effective utilization of the server, i.e, 0.75.

2. Briefly explain how the expected cycle time of a CONWIP-controlled flowline can be reduced by enhancing the *flexibility* of the line.

As discussed in class, enhancing the flexibility of the line can imply that the processing capacity that is available in the system becomes transferrable to the different stations on an "as needed" basis. Such a capability corresponds to an effective increase of the processing capacity at the different stations, and as shown in class, this last effect results in reduced WIP and cycle time.

Also, as suggested in your textbook, flexibility enhancement of the line can further imply variability reduction, e.g., through the reduction of the set up times required at the different stations. We know, however, that reducing $c_{e_j}^2$ will lead to a reduction of the cycle time experienced at station j.

3. Answer the following three questions: What is the practical meaning of the characterization of an asynchronous transfer line as *"stable"*? What is the mathematical condition for the stability of an asynchronous transfer line with single-server stations? What is an appropriate redefinition of the notion of stability for CONWIP-controlled production lines?

An ATL is stable if the applied feeding rate is within the production capacity of the line; i.e., the line can keep up with the workload fed to it, which further implies finite WIP accumulation and cycle times throughout the line operation.

Letting
- r_a = the line feeding rate
- r_{e_j} = the processing rate of station j
- $t_{e_j} = 1/r_{e_j}$ = the mean processing time at station j

we can express the mathematical condition for the line stability as follows:

$$\forall_j \quad r_a t_{e_j} < 1 \quad \Leftrightarrow$$

$$\forall_j \quad r_a < r_{e_j} \quad \Leftrightarrow$$

$$r_a < \min_j \{ r_{e_j} \}$$

In the case of CONWIP lines, stability implies that the target production rate, r_a, is less than or equal to the throughput $TH(W)$ that is attainable by the current line configuration.

4. Consider an asynchronous transfer line with two single-server stations. Which of the following conditions on the operation of station 1 will result in a low variability for the arrival process at station 2?

i. An arrival process with low-variability combined with a very high utilization.

ii. An arrival process with high variability combined a very high utilization.

iii. A low processing time variability combined with a very high utilization.

iv. A low processing time variability combined with a very low utilization.

Briefly explain your answer.

Under the stated conditions,

$$c_{a_2}^2 = c_{d_1}^2 \simeq u_1^2 c_{e_1}^2 + (1-u_1^2)\, c_{a_1}^2 \qquad (1)$$

But Eq. (1) implies that

- under (i), $c_{a_2}^2$ can be high if $c_{e_1}^2$ is high;
- under (ii) we can have the same effect as in (i) above;
- under (iv) $c_{a_2}^2$ can be high if $c_{a_1}^2$ is high.
- On the other hand (iii) works since
  (a) the first term in the rhs of (1) will be low since $c_{e_1}^2$ is low, and
  (b) the second term in the rhs of (1) becomes very small because of the factor $1-u_1^2$.

5. Explain why in a CONWIP line that operates at a WIP level W and satisfies the conditions characterizing the *"practical worst case"* defined in Chapter 7 of your textbook, the expected cycle time in the line is

$$CT(W) = T_0 + \frac{W-1}{r_b}$$

In the above expression T_0 denotes the line *raw* processing time and r_b denotes the *bottleneck* processing rate.

Remember that the conditions defining the PWC are:

(i) Single-server stations
(ii) Exp. processing times
(iii) a balanced line

Because of (ii), the arrival theorem of Gordon-Newell networks applies exactly. Also the symmetry of the line implies that the average number of customers encountered by a new arrival at any station j is equal to $\frac{W-1}{N}$, where N denotes the number of stations in the line. Finally, the memoryless property of the exp. distribution implies that the total expected processing time for the aforementioned customer is equal to $\frac{W-1}{N} t$, where t denotes the mean processing time at any of the line stations. But then:

$$CT_j(W) = t + \frac{W-1}{N} t \quad \text{and}$$

$$CT(W) = \sum_j CT_j(W) = N \cdot t + (W-1)t =$$

$$= T_0 + \frac{W-1}{r_b}$$

where $* \; T_0 = N \cdot t$ = raw process time for the line
$* \; r_b = 1/t$ = bottleneck rate " " "

Problem 1 (20 points): Consider a $G/G/1$ station operated at 95% of its effective processing capacity. The station is fed with parts at a deterministically paced rate of one part per 10 minutes and the average waiting time experienced by a part before it enters the server is equal to 45 minutes. Use the above information in order to compute the *mean* and the *variance* of the part inter-departure times.

(i) $t_{d_1} = t_{a_1} = 10 \text{ min}$ (since the line is stable)

(ii) $u_1 = r_{a_1} t_{e_1} = 0.95 \Rightarrow 6 \cdot t_{e_1} = 0.95 \Rightarrow$

$\Rightarrow t_{e_1} = 0.1583 \text{ hrs} = 9.5 \text{ min}$

$$CT_{q_1} = \frac{c_{a_1}^2 + c_{e_1}^2}{2} \frac{u_1}{1-u_1} t_{e_1} \Rightarrow$$

$$\frac{0 + c_{e_1}^2}{2} \frac{0.95}{1-0.95} 9.5 = 45 \Rightarrow$$

$\Rightarrow c_{e_1}^2 = 0.498$

$$c_{d_1}^2 = u_1^2 c_{e_1}^2 + (1 - u_1^2) c_{a_1}^2 \Rightarrow$$

$\Rightarrow c_{d_1}^2 = 0.95^2 \cdot 0.498 \simeq 0.45 \Rightarrow$

$$\Rightarrow \frac{\sigma_{d_1}^2}{t_{d_1}^2} = 0.45 \Rightarrow \sigma_{d_1}^2 = 0.45 \cdot (10 \text{ min})^2 =$$

$$= 45 \text{ min}^2.$$

Problem 2 (30 points): A small assembly line consists of five tasks, a, b, c, d and e, with the following processing time and precedence requirements:

Task	Proc. Time (secs)	Immediate predecessors
a	10	-
b	15	-
c	20	a, b
d	30	c
e	20	c

The target production rate is 120 units per hour.

i. (10 pts) What is the *maximum cycle time*, c, for this line? (*Hint:* Remember that in the ALB context, the terms "cycle time" implies the time between two consecutive advancements of the line.)

ii. (10 pts) Compute a lower bound for the number of workstations that are necessary to support the production rate indicated in part (i).

iii. (10 pts) The following configuration has been proposed as a possible set-up for this line

$$W1(a,b) \to W2(c) \to W3(e) \to W4(d)$$

where $Wi(\cdot)$ indicates the i-th workstation in the line, and the elements in parentheses indicate its assigned tasks. Is this a *feasible* configuration? Is it *optimal*?

(i) $c = 1/TH = \frac{60}{120} = 0.5\,\text{min} = 30\,\text{sec}$

(ii) $N = \left\lceil \frac{\sum_i t_i}{c} \right\rceil = \left\lceil \frac{95}{30} \right\rceil = \lceil 3.167 \rceil = 4$

(iii) We can check that under the proposed solution

(a) each workstation has an assigned workload less than or equal to c;

(b) Precedence constraints are observed $\Rightarrow$

$\Rightarrow$ feasible.

Also since the number of the employed workstations is equal to the lower bound obtained in (ii), we can say that this solution is optimal according to the basic definition of the problem.