**ISYE 6201: Manufacturing Systems**
**Spring 2007**
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**Midterm Exam I**
**March 1, 2007**

**Name:** SOLUTIONS

Answer the following questions **(10 points each)**:

**1.** A wholesaler applies the Wagner-Whitin (W-W) algorithm for the replenishment of one of her products, according to the following *"rolling-horizon"* policy: Towards the end of each week, the wholesaler develops an updated forecast of the expected demand for the next 8 weeks, she adjusts this forecast to account for the on-hand and any pipeline inventory, she uses the W-W algorithm on the adjusted demand vector in order to compute an optimized replenishment plan, and then, she executes the part of the plan corresponding to the very first period (i.e., she places a replenishment order equal to the order size suggested by the W-W plan for period 1). This order will be delivered in the early part of the next week. The order plan developed at the end of this week for the next 8 weeks, is as follows:

$$< 1000, 0, 0, 950, 0, 0, 500, 0 >$$

Hence, according to the above plan, the wholesaler must place an order of 1000 units, to be delivered early next week.

Assuming that the item, ordering and holding costs for the considered item do not vary with time, explain why the provision of additional information regarding the expected demand for weeks 9 and beyond, will not impact her current decision.

Based on the insights provided by the above case, can you suggest a *"natural"* length for the planning horizon that might be (more) appropriate for this setting?

Since the item, ordering and holding costs do not vary with time, the "rolling horizon" property of the W-W algorithm implies that the demand for period 9 will not be procured before period 7, and therefore the order plan will remain the same for the first six periods. And therefore the manager's decision, which is how much to order for the next week, will remain unchanged.

Based on the above remarks, a "natural" way to determine an appropriate length for the planning horizon in this setting, is to keep adding periods until you find a subproblem for which the demand for the last period is not procured in period 1.

**2.** Currently, the *only* reason that a distribution center might opt for a *periodic review* over a *continuous review* inventory control policy for certain items, is the very high cost of the monitoring function requested by continuous review policies.

(A) TRUE (B) FALSE

Briefly justify your answer.

This is not the only reason.
As discussed in class, frequently companies opt for a periodic review policy in an effort to synchronize replenishment across a set of products.

**3.** For an inventory system that is run by a $(Q, r)$ policy, a type-1 service level requirement of 0.9 will result in

Remark: for most commonly used distributions, the inequality given below will be strict and therefore (ii) will hold. However, a better phrasing would have been "a reorder point no lower than"

i. a smaller reorder point

ii. a higher reorder point

iii. the same reorder point

~~with~~ than a type-2 service level requirement of the same value.

Briefly justify your answer.

As discussed in class, type-1 service level is defined by the probability of experiencing no stockout during a replenishment cycle and therefore, for any given pair of $(Q,r)$, it is equal to $\text{Prob}(D_L \le r) = G(r)$.

On the other hand, type-2 service level is defined by the probability that any single unit of demand is met immediately from stock. And it was shown in class that for a pair $(Q,r)$ this service level is

$$S(Q,r) = \frac{1}{Q}\left[G(r) + \cdots + G(r+Q-1)\right]$$

But since $G(\,)$ is a cdf, $G(r) \le G(r+x)$ for $x > 0$. Hence,

$$S(Q,r) \ge \frac{1}{Q}\left[\underbrace{G(r) + G(r) + \cdots + G(r)}_{Q \text{ terms}}\right] = G(r)$$

This inequality implies that for a $(Q,r)$ such that $S(Q,r) = 0.9$, $G(r) \le 0.9$, and since $G(\,)$ is non-decreasing, the $r'$ for which $G(r') = 0.9$ will be greater than or equal to $r$.

**4.** Due to a highly politicized climate in the local community, the demand for the local newspaper has demonstrated a considerable increase. Mathematically, this fact is expressed as follows: Using the random variables $D^{cur}$ and $D^{past}$ to denote respectively the current and the past circulation of the paper, it will hold that $\forall a \geq 0,\ Prob(D^{cur} > a) > Prob(D^{past} > a)$.

Under the additional assumptions that (i) the relevant cost structure will not be affected by this increase in the demand, (ii) the local printer has ample production capacity, and (iii) the local publisher tries to operate optimally, use the newsvendor theory developed in class to explain that the publisher must now increase the produced number of copies.

Under the aforestated assumptions, we will have

$$G^{cur}(Q^{cur}) = \frac{c_s}{c_s + c_o} = G^{past}(Q^{past})$$

But

$$\frac{c_s}{c_s+c_o} = G^{past}(Q^{past}) = Prob(D^{past} \leq Q^{past}) >$$

$$> Prob(D^{cur} \leq Q^{past}) = G^{cur}(Q^{past}) \quad (1)$$

where the inequality holds from the given fact that

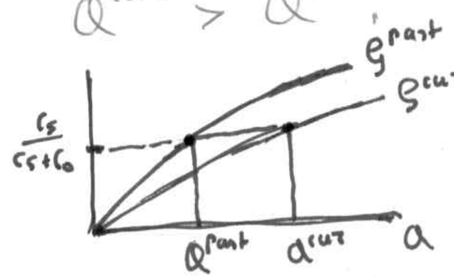
$$Prob(D^{cur} > Q^{past}) > Prob(D^{past} > Q^{past}) \iff$$

$$\iff 1 - Prob(D^{cur} \leq Q^{past}) > 1 - Prob(D^{past} \leq Q^{past}) \iff$$

$$\iff Prob(D^{past} \leq Q^{past}) > Prob(D^{cur} \leq Q^{past}).$$

Finally (1) combined with the monotonicity of $G^{cur}()$ implies that $Q^{cur} > Q^{past}$

Graphically:



**5.** Briefly explain what is the meaning of the *market segmentation* applied by modern corporations, and discuss what necessitates it, what are some of the challenges arising by it, and how the companies try to respond to these challenges.

Please c.f. the relevant slide in the introductory set of PowerPoint slides.

**Problem 1 (25 points):** A retailer tries to develop a continuous review inventory control policy for a certain item, and he currently entertains two possibilities: According to the first possibility, he will purchase the item at the cost of \$15.00 per unit and the order delivery will be supported by an external carrier at the cost of \$50.00 per delivery. In the second case, the retailer will procure the material from another distributor who will charge \$14.50 per unit and will also provide transportation at \$40.00 per delivery. However, due to limited transport capacity, this distributor will also impose a cap of 10 deliveries per year. Finally, it is also given that (i) the annual expected demand for this item is 10,000 units, (ii) holding cost is estimated on the basis of an interest rate of 10% on the item unit cost, and (iii) there is also a transactional cost of \$10.00 per order. Your task is to identify the best of the above two options, and the optimized order size.

Option 1:

$c^1 = 15; \quad A^1 = 50+10 = 60; \quad h^1 = 0.1 \times 15 = 1.5$

$$Q_1^* = \sqrt{\frac{2A^1D}{h^1}} = \sqrt{\frac{2 \cdot 60 \cdot 10000}{1.5}} = 894.43 \simeq 894$$

$$TAC(Q_1^*) = c^1 \cdot D + A^1 \cdot \frac{D}{Q_1^*} + h^1 \frac{Q_1^*}{2} =$$

$$= 15 \cdot 10000 + 60 \frac{10000}{894} + 1.5 \frac{894}{2} =$$

$$= 151\,341.641$$

Option 2:

$c^2 = 14.5, \quad A^2 = 40+10 = 50; \quad h^2 = 0.1 \cdot 14.5 = 1.45$

$$Q_2^* = \sqrt{\frac{2 \cdot A^2 \cdot D}{h^2}} = \sqrt{\frac{2 \cdot 50 \cdot 10000}{1.45}} = 830.45$$

However, we want: $\frac{D}{Q} \leq 10 \Leftrightarrow Q > \frac{D}{10} = \frac{10000}{10} = 1000$ $\Rightarrow$

$\Rightarrow Q_2^{best} = 1000.$

$$TAC(Q_2^{best}) = c^2 \cdot D + A^2 \frac{D}{Q_2^{best}} + h^2 \frac{Q_2^{best}}{2} =$$

$$= 14.5 \cdot 10000 + 50 \frac{10000}{1000} + 1.45 \frac{1000}{2} =$$

$$= 146225$$

$\Rightarrow$ Choose option 2 with order size $Q_2^{best} = 1000$.

**Problem 2 (25 points):** The local retailer of some luxury item faces a daily demand that is normally distributed with a mean value of 10 and a st. deviation of 3 units.

i. (15 pts) If the company procures this item from the local distribution center according to a $(Q, r)$ policy with $Q$ set equal to 50 units, and the order lead time is equal to 3 days, what is the minimum reorder point that will achieve a fill rate of 0.99? (*Hint:* In your calculations, use the approximation $S(Q,r) \approx 1 - \frac{B(r)}{Q}$. Also, notice that for normally distributed demand with mean $\theta$ and st. deviation $\sigma$, $B(r) = \sigma \cdot L(\frac{r-\theta}{\sigma})$, where $L()$ is the *loss function* of the standardized normal, that is provided in the attached table.)

ii. (10 pts) As an expression of goodwill to its customers, the company also offers a *"free rental"* for its backordered units, through a third party service provider. Assuming that the company pays \$50 per day, for any such free rental, what is the resulting (expected) cost per order to the company under the $(Q, r)$ policy that you derived in part (i)? (*Hint:* In your calculations, assume that $B(Q,r) \approx B(r)$ and ... think a little ...:)

(i) First we need to characterize the distribution of the demand to be experienced over a period equal to lead time. Assuming independence of the daily demands, this is normally distributed with mean $\theta = 3 \cdot 10 = 30$, and variance $3 \cdot 3^2 \Rightarrow$ st. dev $= \sigma = \sqrt{3} \cdot 3 \simeq 5.196$.

Then $S(Q,r) \simeq 1 - \frac{B(r)}{Q} = a \Leftrightarrow B(r) = (1-a)Q \Leftrightarrow$

$\Leftrightarrow \sigma L\left(\frac{r-\theta}{\sigma}\right) = (1-a)Q \Leftrightarrow L\left(\frac{r-\theta}{\sigma}\right) = (1-a)\frac{Q}{\sigma} =$

$= (1-0.99) \cdot \frac{50}{5.196} = 0.0962$ and from the provided tables, $\frac{r-\theta}{\sigma} = 0.93 \Leftrightarrow$

$r = \theta + 0.93 \cdot \sigma = 30 + 0.93 \cdot 5.196 = 34.8 \simeq 35$

(ii) We can see from above that $B(r,Q) \simeq B(r) = (1-a)Q =$
$= (1-0.99) \cdot 50 = 0.5$. Then, from Little's law:

Average delay per order $= \frac{\text{Average backorder level}}{\text{order arrival rate}} =$

$= \frac{0.5}{10} = 0.05$ days $\Rightarrow$ Average cost per order $= 0.05 \times 50 = 2.5$. (\$/order)