ISYE 6201: Manufacturing Systems
Spring 2007
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Final Exam
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Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Given a CONWIP line with one station that presents especially long outages in its operation, the disruptive effect of this station to the throughput of the line can be mitigated by placing this station towards the end of the line.

(a) TRUE (b) FALSE

Briefly justify your answer.

As remarked in class, the dynamics of a CONWIP line are characterized by a circular topology, and therefore, every position in the line is equivalent in terms of its impact on the line dynamics.

Cf. also the remarks after the "Corollary" on Variability Placement in pg. 305 of your textbook.

2. The *unavailability* of a processing station due to various outages is 20%. Furthermore, jobs are processed at this station in batches with an average size of 20 jobs per batch, and the average set up time experienced between two consecutive batches is equal to 10 minutes. If the station *nominal* processing time per job is equal to 1 minute, then, its *effective* mean processing time per job is

i. 2 minutes

ii. 1.75 minutes

iii. 1.875 minutes

iv. none of the above

Briefly explain your answer.

The 20% unavailability of the station due to outages implies that the effective processing time of any job, once loaded on the station server, is equal to $t_{e_1} = \frac{t_0}{A} = \frac{1}{0.8} = 1.25$ min

This value subsequently must be adjusted in order to account also for the effect of set up times. Amortizing the set up time of 10 min over the 20 jobs of the batch implies the inflation of t_{e_1} to

$$t_{e_2} = t_{e_1} + 10/20 = 1.25 + 0.5 = 1.75 \text{ min}$$

3. Provide at least two reasons that might necessitate the *batching* of the net requirements with respect to a particular item into larger production lots.

a) The need to control the capacity losses due to setup times

b) The need to attain a high utilization for equipment units like fermentors and furnaces, that necessitate parallel processing of many units.

c) In case of external procurement, the need to control the ordering and transportation costs

d) etc.

4. Write the *workforce balance* equation to be employed in a Linear Programming formulation for the aggregate planning problem, assuming that

i. at the end of every period, 10% of the active workforce will leave voluntarily the company for other jobs, and

ii. workers hired in period t will have to undergo 2 periods of training before starting working in the company's operations.

In your formulation, please, use the following notation:

- W_t: number of workers engaged in the company operations at period t
- H_t: number of workers hired at the beginning of period t
- F_t: number of workers laid off at the end of period t

$\forall t, \quad W_t = 0.9\, W_{t-1} + H_{t-2} - F_{t-1}$

5. Increasing the amount of the available past observations during the implementation of a time-series forecasting model will enhance the quality of the resulting model.

(a) TRUE (b) FALSE

Briefly justify your answer.

This is a rather open ended question:
If the observed process is stable, and the trends that are present in the past data are expected to persist in the future, the the provision of more data can enhance the quality of the future forecasts.
In case, however, that the observed quantity changes its behavior, in terms of the involved trends and/or their parameterization, the inclusion of more past observations can compromise the ability of the model to adjust to the new developments.

Problem 1 (20 points): A local boutique sells a large number of white dress shirts. The shirts, which bear the store label, are manufactured in Italy, and the store owner has made the following statement: "I want to be sure that I never run out of dress shirts. I always try to keep at least two months' supply in stock. When my inventory drops below that level, I order another two-month supply. I have been using this method for 20 years, and it works."

The shirts cost \$6 each and sell for \$15. The cost of processing an order and receiving new goods amounts to \$80, and it takes three weeks to receive a shipment. Monthly demand is approximately normally distributed with mean 120 and standard deviation 32. Assume a 20 percent annual interest rate for computing the holding cost, and answer the following:

i. What is the value of the order quantity, Q, and the reorder point, r, that are used by this store to control its inventory of white dress shirts?

ii. Assuming that unmet demand is lost, what is the fill rate achieved by the current policy?

iii. Assuming that unmet demand is lost, compute an optimized implementation of the (Q, r) policy for the considered case, using the approximation

   - $Q^* = \sqrt{2AD/h}$
   - $G(r^*) = k'D/(k'D + hQ^*)$

   that was presented in class.

iv. What is the difference of the expected annual *ordering* costs that result from the current policy and the policy that you computed in item (3) above?

Hint: In your work, remember that for the considered model

$$S(Q,r) = 1 - \frac{1}{Q}[B(r) - B(r+Q)]$$

and that in the case that the *lead time demand*, D_l, is normally distributed with mean θ and st. dev. σ, the loss function $B(r)$ is given by

$$B(r) = \sigma L(\frac{r-\theta}{\sigma})$$

were $L()$ is the loss function of the standardized normal, tabulated at the end of this handout.

(i) $Q = 2 \cdot 120 = 240$

$r = 2 \cdot 120 - 1 = 239$

(Although, I took the answer $r = 240$ as correct, ...)

(ii) To answer this question, we need to characterize the distribution of the lead time demand D_L.
We know that the monthly demand

$D_m \sim N(120, 32^2)$

Assuming that weekly demands are independent from each other and that there are 4 weeks in a month, we get that the weekly demand

$$D_W \sim N\left(\frac{120}{4}, \left(\frac{32}{\sqrt{4}}\right)^2\right) = N(30, 16^2)$$

and

$$D_L \sim N\left(3 \cdot 30, (\sqrt{3} \cdot 16)^2\right) = N(90, 27.71^2)$$

Then, $S(Q,r) = 1 - \frac{1}{Q}\left[B(r) - B(r+Q)\right]$ and

$$B(r) = \sigma\, L\left(\frac{r-\theta}{\sigma}\right) = 27.71\, L\left(\frac{240-90}{27.71}\right) =$$

$$= 27.71\, L(5.4132) \simeq 0$$

$$B(r+Q) = \sigma\, L\left(\frac{r+Q-\theta}{\sigma}\right) = 27.71\, L\left(\frac{240+240-90}{27.71}\right)$$

$$\simeq 0.$$

and $S(Q,r) = 1.$

(iii) $Q^* \simeq \sqrt{\dfrac{2AD}{h}} = \sqrt{\dfrac{2\cdot 80\cdot 12\cdot 120}{0,2\cdot 6}} = 438,178 \simeq$

$\simeq 438$

$$g(r^*) = \frac{k'D}{k'D + hQ^*} = \frac{(15-6)\cdot 12\cdot 120}{(15-6)\cdot 12\cdot 120 + 1.2\cdot 438} = 0,961 \rightsquigarrow$$

$$\rightsquigarrow r^* = \theta + z_{0,961}\,\sigma = 90 + 1,77\cdot 27.71 \simeq 139.$$

(iv) As discussed in class, the correct expression for the expected annual ordering cost, in the case of lost sales, is:

$$\frac{A\cdot D\, S(Q,r)}{Q}$$

Hence, for the current policy:

$$AOC_1 = \frac{80\cdot 12\cdot 120\cdot 1}{240} = 480$$

To compute the expected annual ordering cost under (Q^*, r^*), we need to compute $S(Q^*, r^*)$

We have:

$$B(r^*) = \sigma\, L\left(\frac{r^*-\theta}{\sigma}\right) = 27.71\, L\left(\frac{139-90}{27.71}\right) =$$

$$= 27.71\, L(1,768) \simeq 27.71\cdot 0.158 = 4.38$$

$$B(r^*+Q^*) = \sigma\, L\left(\frac{r^*+Q^*-\theta}{\sigma}\right) = 27,71\, L\left(\frac{577-90}{27.71}\right) \simeq$$

$$\simeq \emptyset.$$

Hence,

$$S(Q^*, r^*) = 1 - \frac{1}{Q^*}\left[B(r^*) - B(r^*+Q^*)\right] =$$

$$= 1 - \frac{4.38}{438} = 0.99.$$

and $$AOC_2 = \frac{80 \cdot 12 \cdot 120 \cdot 0.99}{438} = 260$$

Finally

$$AOC_1 - AOC_2 = 480 - 260 = 220.$$

Problem 2 (20 points): Consider a three-station CONWIP line, where each station possesses a single server presenting exponentially distributed *effective* processing times with a mean value equal to 2 minutes.

i. What is the WIP level W at which we should operate this line, in order to achieve a target production rate $TH_d = TH(W) = 20$ parts per hour? What is the resulting cycle time, $CT(W)$?

ii. If we replace the servers of Stations 1 and 2 of the aforementioned line with faster ones, so that the corresponding mean effective processing times are reduced to 1 minute, but we maintain the applied operational policies, and therefore, the exponential nature of the relevant processing time distributions, what is the minimum WIP level, W', that can sustain the aforementioned throughput requirement, and what is the resulting cycle time $CT(W')$?

Hint: Remember that the performance evaluation of a CONWIP-based production line is enabled by the result provided in the following equation:

$$CT_j(W) = \frac{t_e^2(j)}{2} \cdot [c_e^2(j) - 1] \cdot TH(W-1) + [WIP_j(W-1) + 1] \cdot t_e(j)$$

Also, consider the possibility of shortcuts that might abbreviate your calculations.

i) This case corresponds to the "practical worst case" of your textbook, and therefore we can use the formula:

$$TH(W) = \frac{W}{W_0 + W - 1} r_b$$

We have $r_b = \frac{1}{t_e} = \frac{1}{2/60} = 30$ parts/hr

$W_0 = T_0 r_b = (3 t_e) r_b = 3 \cdot \frac{2}{60} \cdot 30 = 3$

Hence,

$$20 = \frac{W}{3 + W - 1} \cdot 30 \iff \underline{W = 4}$$

But then, $CT_j(W) = \frac{W-1}{n} t_e + t_e = \frac{4-1}{3} \cdot 2 + 2 = 4$ min

and $CT(W) = n \cdot CT_j(W) = 3 \cdot 4 = \underline{12 \text{ min}}$

(ii) Once we unbalance the line, we loose the capability to characterize the performance of the line as a function of W in closed-form. However, we can still use the evaluation capabilities offered by the "Mean Value Analysis" for general CONWIP lines. More specifically, we shall start with W = 1, and we shall keep increasing W by one unit, until $TH(W) \geq 20$ parts/hr.

<u>For W = 1</u>

$$CT(1) = 1 + 1 + 2 = 4 \text{ min} = \frac{1}{15} \text{ hrs}$$

$$TH(1) = \frac{1}{CT(1)} = \frac{1}{1/15} = 15 \text{ parts/hr} < 20.$$

$$WIP_{1,2}(1) = 15 \cdot \frac{1}{60} = \frac{1}{4}$$

$$WIP_3(1) = 15 \cdot \frac{2}{60} = \frac{1}{2}$$

<u>For W = 2</u>

$$CT_{1,2}(2) = \cancel{\tfrac{1}{2}[1-1]}^{\,0}\, TH(W-1) + [W_{1,2}(1)+1] \cdot t_{1,2} =$$

$$= \left[\frac{1}{4} + 1\right] \cdot 1 = \frac{5}{4} \text{ min}$$

$$CT_3(2) = \cancel{\tfrac{1}{2}[1-1]}^{\,0}\, TH(W-1) + [W_3(1)+1]\, t_3 =$$

$$= \left[\frac{1}{2} + 1\right] \cdot 2 = 3 \text{ min}$$

$$CT(2) = 2 \cdot CT_{1,2}(2) + CT_3(2) = 2 \cdot \frac{5}{4} + 3 = 5.5 \text{ min}$$

$$TH(2) = \frac{2}{CT(2)} = \frac{2}{5.5} = 21.82 \text{ parts/hr} > 20!$$

Problem 3 (20 points): The table below reports the quarterly demand that has been experienced by a certain company for one of its products over the last four years. Use an appropriate forecasting method to estimate the expected quarterly demands for the next year, taking also into consideration that in the last two years the company went through an extensive expansion of its operations in a new market.

	Y1	Y2	Y3	Y4
Q1	30	30	65	68
Q2	40	38	85	81
Q3	20	17	42	31
Q4	10	5	18	20
Total	100	90	210	200

Explain clearly the selection of your model and the underlying computations.

Taking the ratios $\frac{Q_{ij}}{Total_j}$ $i=1,\dots,4;\ j=1,\dots,4$

we get the following table

	Y_1	Y_2	Y_3	Y_4	$\overline{Q_{ij}/Total_j}$
Q_1	0.3	0.33	0.31	0.34	0.32
Q_2	0.4	0.42	0.4	0.4	0.405
Q_3	0.2	0.19	0.2	0.16	0.1875
Q_4	0.1	0.06	0.09	0.1	0.0875

The above data suggests a seasonal behavior, where ~~the percen~~ each quarterly demand constitutes the percentage of the corresponding total annual demand defined by the fifth column of the above table (each element of this column averages the other four elements in the corresponding row, providing thus a more robust estimate of the relevant percentage)

Given the information derived in the previous page, in order to estimate the quarterly demands for Y5, all we need to do is to estimate the total demand for that year, and then apportion it to the different quarters according to the estimated percentages.

The provided data on the total demand experienced over the last four years, combined with the provided input about the company's expansion, suggest that the current trends in this quantity are more pertinently characterized by the last two observations.

Hence, given the scarcity of the relevant data, and in lack of any further observation, the most reasonable approach for estimating the total demand in Y5 is to average the last two observations. This gives $Q_5 = (210 + 200)/2 = 205$

and

$$Q_{15} = 205 \cdot 0.32 = 65.6 \simeq 66$$

$$Q_{25} = 205 \cdot 0.405 = 83.025 \simeq 83$$

$$Q_{35} = 205 \cdot 0.1875 = 38.4375 \simeq 39$$

$$Q_{45} = 205 \cdot 0.0875 = 17.9375 \simeq 18$$