ISyE 6201: Manufacturing Systems
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Solutions to Homework 1

A.

Chapter 2, Problem 4.

(a)
$D = 60$ units/wk $\times$ 52 wk/yr $= 3120$ units/yr
$h = ic = 0.25$/yr $\times \$0.02 = \$0.005$/ yr
$A = \$12$
$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 12 \times 3120}{0.005}} = 3869.88 \approx 3870$
The time between orders is given by
$T^* = \frac{Q^*}{D} = \frac{3870}{3120} = 1.24$ yr $= 14.88$ mo

(b)
Set up cost is $A\frac{D}{Q} = \$12 \frac{3120 \text{ units/yr}}{3870 \text{ units}} = \$9.67/\text{yr}$
Holding cost is $\frac{Q}{2}h = \frac{3870 \text{ units}}{2} \times \$0.005/\text{yr} = \$9.675/\text{yr}$.
The costs are essentially the same. This is always true in the case of the EOQ model.

(c) The problem could be where to store all the items. If we were to order 1.24 years worth of styrofoam ice chests at a time it could take up a lot of room.

Chapter 2, Problem 5.
(a) Total (holding plus setup) cost would be
$TC = hQ/2 + DA/Q = (\$0.005/\text{yr})(3870\text{units})/2 + (6240\text{units/yr})(\$12)/(3870\text{units}) = \$29.02/\text{yr}$

(b) The optimum cost would be $\sqrt{2ADh} = \sqrt{2(12)(6240)(0.005)} = \$27.36/\text{yr}$.

(c) Using the wrong value for the demand (100% forecast error) in the EOQ formula results in an increase in cost of only 6%. EOQ is quite robust with respect to parameter values.

Chapter 2, Problem 6.
(a) The EOQ with 60 per week was computed to be 3,870 and the optimal reorder period was 1.24 years or 14.88 months. The closest power of two is 16 months or 1.33 years with a cost of
$TC(1.33) = TDh/2 + A/T =$ (1.33yr)(3120/yr)($0.005/yr)/2+$12/1.33yr=$19.37/yr
The power of two on the other side of 14.88 mo is 8 mo or 0.67 yr with a cost of
$TC(0.67) = TDh/2 + A/T =$(0.67yr)(3120/yr)($0.005/yr)/2+$12/0.67yr=$23.20/yr

(b) The minimum cost without the power of two restriction is
$\sqrt{2ADh} = \sqrt{2(12)(3120)(0.005)} = \$19.35/\text{yr}.$
so 16 months has a cost that is only 0.1% over the optimal while the 8 month solution is around 20% over optimal. The total cost in the EOQ model is relatively insensitive to order quantity used. Since the order period is directly proportional to the order quantity the cost is not very sensitive to the period used as well.

(c) The robustness of the EOQ to errors in parameter estimates and the effectiveness of a power-of-2 policy are both stemming from the fact that the total annual cost curve is relatively "flat" around its minimum point.

Chapter 2, Problem 7.
$D_b = 1000/\text{yr}, c_b = \$200, h_b = (0.2)(200) = \$40; D_s = 500/\text{ yr}, c_s = \$150, h_s = (0.2)(150) = \$30$

(a)

$$Q_b = \sqrt{\frac{2AD_b}{h_b}} = \sqrt{\frac{2 \times 50 \times 1000}{40}} = 50; \qquad T_b = \frac{Q_b}{D_b} = \frac{50}{1000} = 0.05 \text{ yrs} = 18.25 \text{ days}$$

$$Q_s = \sqrt{\frac{2AD_s}{h_s}} = \sqrt{\frac{2 \times 50 \times 500}{30}} = 40.82; \qquad T_s = \frac{Q_s}{D_s} = \frac{40.82}{1000} = 0.082 \text{ yrs} = 29.8 \text{ days}$$

Note that there will be virtually no chance to share trucks.

$COST_b = \sqrt{2AD_bh_b} = \sqrt{2(50)(1000)(40)} = \$2,000$
$COST_s = \sqrt{2AD_sh_s} = \sqrt{2(50)(500)(30)} = \$1,224.74$
TOTAL COST = \$3,224.74

Of this, the fixed trucking part is:

$$FIXED_b = 40\left(\frac{D_b}{Q_b}\right) = 40\left(\frac{1000}{50}\right) = \$800; \qquad FIXED_s = 40\left(\frac{D_s}{Q_s}\right) = 40\left(\frac{500}{40.82}\right) = \$489.96$$

TOTAL = \$1289.96

Note that trucking is 1,289.96/3,224.74 = 40% of annual cost.

(b) Round $T_b = 14$ days $= 2^1$ weeks, $T_s = 28$ days $= 2^2$ weeks, so

$$Q_b = \frac{T_bD_b}{365} = \frac{14(1000)}{365} = 38.35 \approx 38; \qquad Q_s = \frac{T_sD_s}{365} = \frac{28(500)}{365} = 38.35 \approx 38$$

Cost, not including fixed trucking (i.e., using A=\$10) is:

$$COST_b = \frac{h_bQ_b}{2} + \frac{AD_b}{Q_b} = \frac{40(38)}{2} + \frac{10(1000)}{38} = \$1023.15;$$

$$COST_s = \frac{h_s Q_s}{2} + \frac{AD_s}{Q_s} = \frac{30(38)}{2} + \frac{10(500)}{38} = \$701.58;$$

TOTAL = \$1,724.73

Note that sheet stock orders always overlap with bar stock orders and so never incur a trucking cost. Total trucking cost is cost of bar stock orders:
$TruckingCost_b = 40\frac{D_b}{Q_b} = 40\frac{1000}{38} = \$1,052.63$

Hence, total cost is \$1052.63 + 1724.73 = \$2777.36, which is 13.9% less than original cost of \$3224.74.

(c) Suppose we leave $Q_b = 38$, but decrease $T_s = 14$ days, so $Q_s = T_s D_s/365 = 14(500)/365 = 19.17 \approx 19$ units. The $COST_b$ is the same, as is fixed trucking cost, but
$COST_s = \frac{h_s Q_s}{2} + \frac{AD_s}{Q_s} = \frac{30(19)}{2} + \frac{10(500)}{19} = \$548.16$
which is even lower. The reason is that since we are getting fixed trucking for free for the sheet stock, the true fixed cost is lower than the \$50 we originally used. Hence, it makes sense to order more frequently to reduce the holding costs.

(d) Note that had we known sheet stock would require no additional trucking cost, then we should have used $A = \$10$ for it, which would have led to

$$Q_s = \sqrt{\frac{2AD_s}{h_s}} = \sqrt{\frac{2(10)(500)}{30}} = 18.26$$

which would round to 14 days on a power-of-two schedule using weeks as a base.

Under a power-of-two schedule, we know that an item with a larger order interval T will always overlap with a part with a smaller T after rounding. So, a heuristical approach to this type of problem would be:
i) compute EOQ's and hence T values for all parts
ii) round smallest T to nearest power-of-two (call it T_0^*)
iii) recompute Q_i^* and T_i^* using the reduced value of A (i.e., without trucking cost) for all other parts
iv) round the T_i^* of the remaining parts to nearest power-of-two (but not below T_0^*)

Chapter 2, Problem 9.
(a)

Last Period of Production	Planning Horizon 1	2	3	4	5	6
1	2000	3200	4200	4800	8000	
2		4000	4500	4900	7300	
3			5200	5400	7000	
4				6200	7000	
5					6800	7800
6						8800
Z_t^*	2000	3200	4200	4800	6800	7800
j_t^*	1	1	1	1	5	5

Optimal solution is to produce 2900 units in month 1 (for months 1-4) and 1800 units in month 5 (for months 5 and 6).

(b) Monthly planning periods are only appropriate if we make "monthly runs". If we set up and run a product many times in a month, then a month is too long and maybe we should use weeks or something shorter. Factors affecting the choice of planning period include rate of production (slower enables longer periods), setup cost/time (higher/longer enables longer periods), and maybe number of products in the system (where more means fewer runs per year).

(c) While a fixed setup cost will limit the number of setups, and hence put a premium to the available capacity, it is not sensitive to varying load. The thing is that when there are relatively few products being produced, capacity is not stretched and so setups are not so costly. But when a lot of products are being produced and capacity is tight, then they are very costly. This effect can be potentially captured by using time-varying setup costs.

Chapter 17, Problem 1.
If capacity is not an issue and the cost of a purchase order is independent of volume, then the assumptions of EOQ (modified to allow for a delay delivery) may be approximately satisfied for procurement. In production systems, where the cost of replenishments depends on capacity, the EOQ assumption of a fixed setup cost is not even close to valid.

B.

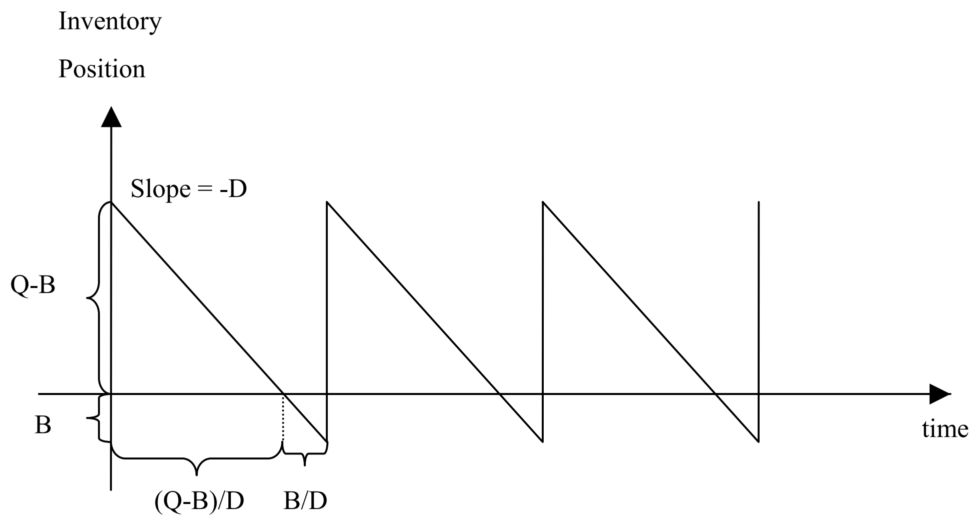
a. The evolution of the Inventory Position for this model is plotted in the next page. As in the basic EOQ model, the **setup** and **production** costs per time unit are AD/Q and DC, respectively. However, the **holding** cost is different because the holding cost is incurred only when there is inventory on hand, i.e. when the IP is positive. The average inventory level = $(Q-B)/2$ units. The fraction of time when the IP is positive = $(Q-B)/Q$. So the holding cost per time unit is = $h(Q-B)^2/2Q$. Similarly, the **shortage** cost is incurred only when the IP is negative. The average backorder level = $B/2$ units, and the fraction of time when the IP is negative = B/Q. So the shortage cost per time unit = $bB^2/2Q$. The total cost per time unit is

$$Y(Q,B) = A\frac{D}{Q} + h\frac{(Q-B)^2}{2Q} + b\frac{B^2}{2Q} + DC$$

Solve for the optimal pair (Q^*, B^*):

$$\begin{cases} 0 &= \dfrac{\partial}{\partial Q}Y(Q,B) = \dfrac{h}{2} - \dfrac{1}{Q^2}\left(B^2\dfrac{h+b}{2} + AD\right) \\ 0 &= \dfrac{\partial}{\partial B}Y(Q,B) = -\dfrac{h}{2Q}2(Q-B) + \dfrac{b}{2Q}2B = B\dfrac{h+b}{Q} - h \end{cases} \Rightarrow \begin{cases} Q^* &= \sqrt{2AD\left(\dfrac{1}{h}+\dfrac{1}{b}\right)} \\ B^* &= \dfrac{h}{h+b}Q^* \end{cases}$$

b. When the backorder cost b is high, it is not economical to have any backorder. Indeed, as b tends to infinity, B^* tends to 0. That makes backorders practically not allowed, which is in line with the basic EOQ model assumptions. At the same time, the $1/b$ term in Q^* vanishes, leaving Q^* tending to $\sqrt{2AD/h}$, which is the EOQ formula in the basic model.


Inventory
Position
Slope = -D
Q-B
B
time
(Q-B)/D
B/D