ISYE 6201: Manufacturing Systems
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Midterm Exam II
December 1, 2011

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. A manufacturing station that *fits* the assumptions of a $G/G/1$ queueing station currently is found to be unstable, i.e., unable to support the required production rate. Which of the following are reasonable options for addressing the faced problem, without compromising the posed throughput requirement?

i. Reduce the variablity of the job processing times at the station.

ii. Add another machine at the station.

iii. Reduce the rate with which parts are fed to the station.

iv. Increase the availability of the station server.

Explain your answer.

From a mathematical standpoint, the station instability means that

$$u = r_a t_e = r_a \frac{t_0}{A} \geq 1$$

where

r_a = job arrival rate

t_e = effective mean proc. time

u = the server utilization

To attain stability we need to attain a server utilization less than 1.

– (i) Cannot do that since server utilization depends only on the mean of the proc times and not their variance.

– By (ii) we will get $u = r_a t_e / 2$, which might get $u < 1$. So (ii) is a possible option for attaining stability.

– With (iii), we will get $u' = r_a' t$ and we might have $u' < 1$ since $r_a' < r_a$, but since r_a defines also the station throughput, we will have compromised the throughput.

– Finally, with (iv) $u' = r_a t_0 / A'$ and since $A' > A$ we might get $u' < 1$, without affecting r_a. Hence, this is another option.

2. Consider a synchronous transfer line supporting an assembly involving n tasks and with task processing times $\{t_1, t_2, \ldots, t_n\}$. Also, let TH denote the production throughput of this line. Then,

$$TH \leq \frac{1}{\max_i\{t_i\}}$$

(A) TRUE (B) FALSE

Explain your answer.

We know that for synchronous transfer lines, $TH = 1/C$ where C is the line cycle time.

But $C \geq \max_i \{t_i\}$, since, otherwise, the station that gets t_i will not be able to finish its task(s).

Since

$$C \geq \max_i \{t_i\} \Leftrightarrow \frac{1}{C} \leq \frac{1}{\max_i\{t_i\}}$$

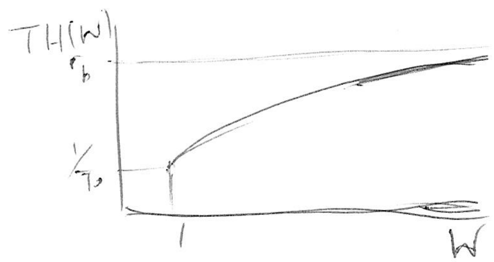
the above statement is true.

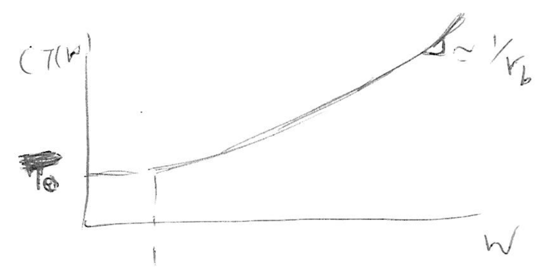
3. The most effective way for reducing the mean cycle time, CT, in a CONWIP line is by reducing the maximal WIP level W that is allowed in the line.

(A) TRUE (B) FALSE

Explain your answer.

In class we showed that for CONWIP lines the dependencies of the CT and TH on W are as follows:





Hence, reducing W will reduce CT but it will also reduce TH, which, most likely, will be an undesirable effect.

4. Provide *three* advantages of the U-shaped layout.

- Enable to fit long lines in typically available shop-floorst plants
- Increases the proximity of the stations, which further translates to
  * better communication among stations
  * ability to share labor across stations
  * better supervision
  * decreased distances among stations
  * etc.
- Enables the use of the same ~~outlets~~ for ingress and egress.

5. Consider a manufacturing station where jobs are classified into two types, A and B, and jobs of type A have a pre-emptive priority over jobs of type B; i.e., the server always picks jobs of type B only if there are no jobs of type A in the queue, and furthermore, if a job of type A arrives while a job of type B is in service, the currently served job is interrupted in order to provide service to the just arrived job of type A (and to any other type A jobs that arrive in the meantime); the service of the interrupted type B job is resumed (only) when the queue is again clear of type A jobs. Assuming that the arrival rates of jobs of type A and B are respectively r_A and r_B, and that the expected processing time is equal to t for both job types, provide

i. a necessary and sufficient condition for the stability of this station, i.e., a condition that will guarantee that there will be no "explosion" of the WIP of any type of job;

ii. a stability condition for the type A jobs.

Justify your answers.

(i) Recognizing that the total <u>workload</u> that arrives at the station per time unit is

$$r_a t + r_b t$$

and that all this workload can be supported by the single server of the station, it should be clear that the stability condition is

$$(r_a + r_b)t < 1$$

(ii) Since jobs of type B never get in the way of jobs of type A, the stability condition for the latter is:

$$r_a t < 1$$

Problem 1 (30 points): Consider a stable single-server manufacturing station with an 85% availability of its server. Jobs are released to this station at a constant pace, and the server utilization is measured at 90% of its capacity. The server *nominal* (or *"natural"*) processing time is (deterministically) equal to 2 minutes and it is also known that the times between failures are exponentially distributed, while downtimes are uniformly distributed between 10 and 30 minutes. Your task is to compute the following:

i. The throughput of this station in the operational regime that is described above.

ii. The expected cycle time CT for a job going through this station.

iii. The average WIP waiting in front of the station.

(i) $u = r_a \dfrac{t_0}{A} \Rightarrow r_a = \dfrac{uA}{t_0} = \dfrac{0.9 \times 0.85}{2 \text{ min}} =$

$= 0.3825 \text{ min}^{-1}$.

Since $u = 0.9 < 1$, the station is stable and

$TH = r_a = 0.3825 \text{ min}^{-1}$

(ii) We need to compute $CT = \dfrac{c_a^2 + c_e^2}{2} \dfrac{u}{1-u} t_e + t_e$

We have

$t_e = t_0/A = 2/0.85 = 2.353$ min

$c_a = 0$, since jobs are released with constant pace.

$c_e^2 = c_0^2 + (1 + c_r^2) A(1-A) \dfrac{m_r}{t_0}$

$c_0 = 0$, since nominal proc. times are constant

$m_r = \dfrac{30+10}{2} = 20$ min

$c_r^2 = \dfrac{\sigma_r^2}{m_r^2}$ and $\sigma_r^2 = E[T_r^2] - m_r^2$

$E[T_r^2] = \dfrac{1}{20} \displaystyle\int_{10}^{30} x^2 dx = \dfrac{1}{60}(30^3 - 10^3) = 433.33$ $\Big\} \Rightarrow$

$\Rightarrow \sigma_r^2 = 433.33 - 20^2 = 33.33$ and $c_r^2 = \dfrac{33.33}{20^2} = 0.0833$

Hence, $c_e^2 = 0 + (1 + 0.0833)\, 0.85\, (1 - 0.85)\, . \frac{20}{2} =$

$= 1.3812$

and

$$CT = \left(\frac{1.3812}{2} \; \frac{0.9}{0.1} + 1 \right) 9.353 = 16.98 \text{ min}$$

(iii) $WIP_q = TH \times CT - u = 0.3825 \times 16.98 -$

$- 0.9 = 5.595$

Problem 2 (30 points): We want to design a synchronous transfer line that will support an assembly process involving 5 tasks. The processing times and the precedence constraints for these tasks are as follows:

task	t_i (sec)	Imm. Pred
a	10	-
b	5	a
c	8	a
d	5	b, c
e	10	-

The required throughput is 100 parts per hour.

i. What is a lower bound to the minimun number of workstations required for this assembly line?

ii. Provide a design for this assembly line.

iii. Compute the utilizations of the different stations in the design that you developed in step (ii).

From the throughput requirement, we get that the line cycle time should be

$$C = \frac{3600 \text{ sec}}{100} = 36 \text{ sec}.$$

Then,

(i) the might lower bound is $\left\lceil \frac{\sum_i t_i}{C} \right\rceil = \left\lceil \frac{38}{36} \right\rceil = \lceil 1.055 \rceil = 2.$

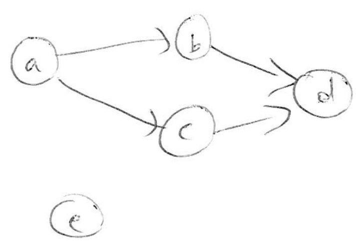
Notice that the fraction $38/36$ is just a little above 1, which implies that

(a) we should be able to get designs using 2 stations only,

and

(b) any such design will still suffer by low utilizations for at least one of its stations.

(ii) The precedence diagram for the considered tasks is



Notice that e is disconnected from all other tasks, and therefore it can "fit" anywhere in the line.

A possible design that tries to strike some balance for the station workloads is as follows:

	W1	W2
	a, e	b, d, c
Total Station workload:	10 + 10 = 20 sec	5 + 5 + 8 = 18 sec
Station utilization	$20/36 = 55.56\%$	$\frac{18}{36} = 50\%$

As expected, the station utilizations are quite low.