ISYE 4803-REV: Advanced Manufacturing Systems

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Final Exam

June 18, 2019

Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. Consider a single-server workstation that experiences preemptive, non-destructive outages of the type that we discussed in class. The original distribution for the processing times of this workstation has a mean of 2.5 minutes and a st. deviation of 1.1 minutes. Working through the corresponding method that was presented in class, we found that, when accounting for the server outages, the mean effective processing time should be 3.5 minutes, and the st. deviation for the effective processing times should be 1.5 minutes. Use the provided information in order to argue that our work was not correct.

As discussed in class, the experienced outages should increase, both, the mean and the variability of the eventually experienced proc. times (i.e., the effective proc. times).
In the considered case, the mean of the effective proc. times is indeed larger than the mean of the original distribution.
But when checking the corresponding CVs, that quantify the variability in these distributions, we find the respective values

$$CV_o = \frac{1.1}{2.5} = 0.44$$

and

$$CV_e = \frac{1.5}{3.5} \simeq 0.43 < CV_o$$

So, something has gone wrong here.

2. Consider a furnace that is operated according to a parallel batching scheme and where parts arrive *in pairs* according to a Poisson process with rate $r_a^p = 40$ pairs per hour. The batch processing time is equal to 45 minutes. What is the minimum batch size k for this furnace, **in terms of parts per batch**, that guarantees a stable operation for it?

Please explain your answer clearly.

According to the corresponding theory that was presented in class, we need to batch the arriving pairs in batches of size $k > r_a^p \cdot t = \frac{40}{60} \cdot 45 = 30$

Hence, when expressed in parts per batch, the minimum batch size is

$31 \times 2 = 62$ parts.

3. Consider an arrival stream that results from the superposition of n Poisson arrival streams with rates λ_i, $i = 1, \ldots, n$. The memoryless property of these n streams implies that the probability that the next arrival will occur through the i-th stream, is equal to $p_i = 1/n$, $\forall i$.

(a) YES (b) NO

Please, explain your answer.

We have repeated a number of times that when combining a number of independent arrival streams into a single stream, the probability that the next arrival will occur from the i-th stream is $P_i = \frac{\lambda_i}{\sum_j \lambda_j}$

So, the above statement is wrong.

4. Moore's algorithm is an appropriate method for static, single-machine, due-date-based scheduling, when

i. we are charged a fixed rate of x dollars per time unit of tardiness for the scheduled jobs with respect to their due dates.

ii. we are charged a fixed penalty of x dollars per tardy job.

iii. we are charged x dollars per time unit that the job completion deviates from the job due date, irrespective of whether the job is early or tardy.

iv. None of the above.

Please, provide a brief explanation for your answer.

As discussed in class, this algorithm assumes that every tardy job will incur the same penalty, and therefore, it tries to push to the end of the schedule the most lengthy jobs, in an effort to increase the chances of the remaining jobs to complete in time (i.e, before their due date).

5. Consider a car dealership that provides free rentals to its customers who bring their cars for service to it, at the cost of x dollars per rental per day. Currently, there are n cars that are waiting for service, and these cars will be served one by one at the local repair shop of the dealership. Furthermore, the service of each car i is expected to last d_i days. What is the sequence to serve these cars, if the company wants to minimize the cost that is accrued by the provided free rentals?

Obviously, the total cost is

$$x \sum_{i=1}^{n} C_i$$

where C_i is the completion time for car i.

Hence, the appropriate dispatching rule for this case is SPT.

Problem 1 (30 points): Consider a manufacturing cell with two single-server workstations, WS_1 and WS_2, where jobs arrive according to a Poisson process with rate $r_a = 10\ hr^{-1}$, they enter the cell at workstation WS_1, and subsequently they circulate among the cell workstations according to the following routing-probability matrix:

FROM \TO	WS_1	WS_2	OUT
WS_1	0.2	0.8	
WS_2	0.2		0.8

For this cell, please, do the following:

i. (10 pts) Provide a graphical representation of the cell workflow and compute the total arrival rate for each workstation.

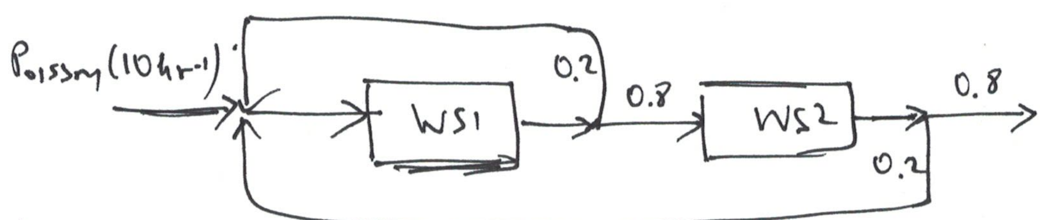
ii. (5 pts) Determine the mean processing time for the server of each workstation so that each server has a utilization level of 90%.

iii. (5 pts) What is the departure rate from this cell under the processing times that you computed in part (ii) above?

iv. (5 pts) Compute the expected number of visits at each of the two workstations for each job that goes through this cell.

v. (5pts) Suppose that the expected cycle times per visit for each of the two workstations are, respectively, $CT_1 = 5\ min$ and $CT_2 = 8\ min$. Compute the expected cycle time CT for going through the entire operation.

(i)



Also

$$\left.\begin{array}{l}\lambda_1 = 10 + 0.2\lambda_1 + 0.2\lambda_2 \\ \lambda_2 = 0.8\lambda_1\end{array}\right) \Rightarrow \left(\begin{array}{l}(1 - 0.2 - 0.2\times 0.8)\lambda_1 = 10 \\ \lambda_2 = 0.8\lambda_1\end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{l}\lambda_1 = 15.625\ hr^{-1} \\ \lambda_2 = 12.5\ hr^{-1}\end{array}\right.$$

(ii) We need to have

$u_1 = \lambda_1 t_1 = 0.9 \Rightarrow t_1 = \frac{0.9}{15.625} = 0.0576 \text{ hr} = 3.456 \text{ min}$

$u_2 = \lambda_2 t_2 = 0.9 \Rightarrow t_2 = \frac{0.9}{12.5} = 0.072 \text{ hr} = 4.32 \text{ min}$

(iii) Since the operation of this network is stable (all workstation have $u<1$), we shall have

$TH = r_a = 10 \text{ hr}^{-1}$.

(iv) $V_1 = \lambda_1 / r_a = \frac{15.625}{10} = 1.5625$

$V_2 = \lambda_2 / r_a = \frac{12.5}{10} = 1.25$

(v) $\left.\begin{aligned} WIP_1 &= \lambda_1 CT_1 = 15.625 \times 5/60 \cong 1.302 \\ WIP_2 &= \lambda_2 CT_2 = 12.5 \times 8/60 \cong 1.667 \end{aligned}\right\} \Rightarrow$

$\Rightarrow$ Total $WIP = WIP_1 + WIP_2 = 2.969$

and from Little's law:

$$CT = \frac{WIP}{TH} = \frac{2.969}{10} = 0.2969 \text{ hr} = 17.814 \text{ min.}$$

Alternatively, we can get CT from the following formula:

$CT = V_1 CT_1 + V_2 CT_2 = 1.5625 \times 5 + 1.25 \times 8 = 17.8125 \text{ min}$

The difference of the two results ~~is because of~~ is because of the rounding that we applied in the calculation of WIP_1 and WIP_2.

Problem 2 (30 points): We need to schedule five jobs $J_1, \ldots, J_5$ on **two** identical machines. The corresponding processing times, t_i, for these jobs are 3, 5, 7, 4 and 3 time units. Furthemore, each job J_i has a due date d_i associcated with it, and the corresponding values are 4, 9, 9, 6 and 3 time units from the current time.

i. (15 pts) Use the relevant algorithm that was presented in class, to show that there is a partition of the 5 jobs into two subsets with a total workload of $\frac{1}{2} \cdot \sum_{i=1}^{5} t_i$ per subset.

ii. (15 pts) Use your results from part (i) above, in order to provide a **minimum-makespan** schedule that also seeks to **minimize the maximum tardiness** across all the five jobs.

Also, please, present the developed schedule as a Gantt chart.

(i) First notice that with the provided t_i's, the minimum possible makespan, under an even distribution of the ~~total~~ workload to the two machines, is

$$\frac{1}{2}\sum_{i=1}^{5} t_i = \frac{1}{2}(3+5+7+4+3) = 11$$

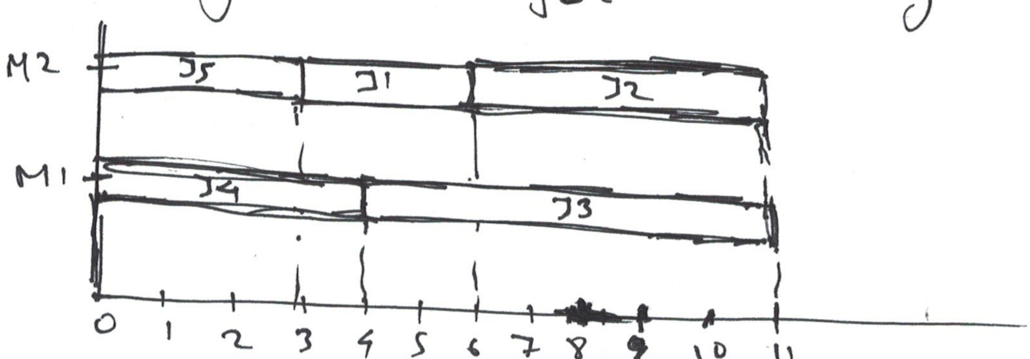
Then, the considered algorithm will execute according to the following table:

i \ k	0	1	2	3	4	5	6	7	8	9	10	11
1	1			1								
2	1			1		1			1			
3	1			1		1		1	1		1	
4	1			1	1	1		1	1	1	1	1
5	1			1	1	1	1	1	1	1	1	1

From the "1" in the last column of the previous table we can conclude that there exists a perfect partitioning of the workload for this problem, and backtracing the indicated dependencies in this table, we can see that the two job sets of this partition are $\{4,3\}$ and $\{5,2,1\}$

(The order in the listing of the above sets follows the sequence in which these two sets will be revealed when tracing the aforementioned dependencies, starting, respectively, from the "1" in rows 4 and 5 in column 11.)

(ii) Since we also want to minimize the max tardiness across all these jobs, we need to apply the EDD rule for scheduling the set of jobs that is assigned to each machine. The Gantt chart for the resulting schedule is as follows:



Juxtapposing the job completion times in the above chart with the quoted due dates, we can see that the vector containing the job tardiness for these jobs is $[2, 2, 2, 0, 0]$, and the max tardiness is 2.