ISYE 4803-REV: Advanced Manufacturing Systems

Instructor: Spyros Reveliotis

Midterm Exam II

November 11, 2019

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. What is the production rate of an unstable G/G/1 queue (i.e., a G/G/1 queue with $\lambda \geq \mu$) as $t \to \infty$?

Please, briefly explain your answer.

For an unstable queue, as $t \to \infty$,

WIP_q (i.e., the number of jobs waiting for processing) $\to \infty$

Hence, the station server will keep busy (i.e., it will reach a utilization of 100%.) and the production rate will be μ (i.e., one job every $1/\mu$ time units, on average).

2. An automated single-server manufacturing workstation that meets the operational assumptions of the G/G/1 queue, currently is found to be unstable. The shop-floor supervisor who has taken a two-days short course on factory physics and has heard about the negative impact of variability on the performance of any production system, tries to address this problem by replacing the station server with another one that has the same processing rate with the current one, but a smaller variability in its processing times. Do you think that the supervisor's approach can address effectively the experienced problem?

(a) YES (b) NO

Please, briefly explain your answer.

Instability means that $u = \lambda t_e = \lambda \frac{t_0}{A} \geq 1$

where

* u = the server utilization
* λ = job arrival rate
* t_0 = mean nominal proc. time (i.e. not accounting for experienced disruptions)
* A = server availability (after accounting for the potential disruptions)

In order to render the system stable, we need to modify some of its operational parameters so that the resulting value for u is less than 1.

But from the above is clear that the coefficient of variation of processing times, c_0, that characterizes the variability in these proc. times, is not one of the parameters that impacts u.

3. The total processing time of a batch of k parts where the part processing times are mutually independent, has a lower variability than the processing time of any single part.

(a) TRUE (b) FALSE

Please, briefly explain your answer.

We have that the batch proc. time T_b is expressed by

$$T_b = \sum_{i=1}^{k} T_i$$

where T_i is the proc. time of the i-th part in the batch. Then, we have

$$E[T_b] = k\,E[T_i]$$

while the independence of T_i also implies that

$$Var[T_b] = k\,Var[T_i].$$

Finally,

$$SCV[T_b] = \frac{Var[T_b]}{E^2[T_b]} = \frac{k\,Var[T_i]}{k^2 E^2[T_i]} = \frac{1}{k} SCV[T_i]$$

As discussed in class, the above result is a manifestation of a statistical phenomenon that is known as "pooling effect".

4. Discuss whether Kingman's approximation for CT_q of a stable G/G/1 queue gives a correct result for the case of a stable D/D/1 queue, i.e., a G/G/1 queue where all inter-arrival times are deterministically equal to t_a, all processing times are deterministically equal to t_p, and $t_a \geq t_p$.

For D/D/1 queue with $t_a \geq t_p$, every new arrival at the queue finds the server idle, and therefore $CT_q = \emptyset$.

Kingman's approximation for CT_q of a stable G/G/1 queue sets $CT_q = \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t$

In the case of a D/D/1 queue, $c_a = c_e = 0$, and therefore, the first factor in the above expression is equal to $\emptyset$.

When $t_a > t_p \Rightarrow 1 > \frac{1}{t_a} t_p = u$, the product $\frac{u}{1-u} t$ is finite, and therefore $CT_q = 0 \cdot \frac{u}{1-u} t = 0$, which is the correct result.

When $t_a = t_p \Rightarrow u = \frac{1}{t_a} t_p = 1$ and $\frac{u}{1-u} \cdot t = \infty$.

In this case, $CT_q = 0 \cdot \infty$, which is an indeterminate quantity; i.e., Kingman's approximation fails to provide a clear result for this second case.

$\Rightarrow$ (IN THE GRADING OF THIS QUESTION, I IGNORED THE ANALYSIS OF THE CASE $t_a = t_p$ DISCUSSED ABOVE.)

5. Consider two single-server workstations WS_1 and WS_2 with the same distribution for their nominal processing times. But these two workstations also experience preemptive nondestructive outages according to the model that was discussed in class, and it further holds that

$$m_{f1} = \frac{1}{2} m_{f2}$$

$$m_{r1} = \frac{1}{2} m_{r2}$$

$$\sigma_{r1} = \frac{1}{2} \sigma_{r2}$$

where m_{f_i} and m_{r_i}, $i = 1, 2$, are the corresponding MTTF and MTTR discussed in class, and σ_{r_i}, $i = 1, 2$, are the standard deviations for the distributions of the corresponding downtimes.

Show that these two workstations will either both of them be stable or both of them be unstable, for any job arrival process with rate λ. Also in the case of stable operation, discuss which workstation will present the highest congestion.

We have that

$$a_1 = \frac{m_{f_1}}{m_{f_1} + m_{r_1}} = \frac{\frac{1}{2} m_{f_2}}{\frac{1}{2} m_{f_2} + \frac{1}{2} m_{r_2}} = \frac{m_{f_2}}{m_{f_2} + m_{r_2}} = a_2 \Big\} \Rightarrow$$

Also, $t_{o_1} = t_{o_2}$

$$t_{e_1} = t_{o_1}/a_1 = t_{o_2}/a_2 = t_{e_2}$$

and eventually

$$u_1 = \lambda t_{e_1} = \lambda t_{e_2} = u_2$$

This answers the first part of this question. To answer the second part, we also notice that:

$$c_{r_1} = \frac{\sigma_{r_1}}{m_{r_1}} = \frac{\frac{1}{2}\sigma_{r_2}}{\frac{1}{2} m_{r_2}} = \frac{\sigma_{r_2}}{m_{r_2}} = c_{r_2} \quad \text{and} \quad c_{o_1} = c_{o_2}$$

Therefore,

$$ce_1^2 = c_{o_1}^2 + (1 + c_{r_1}^2)\, a_1 (1 - a_1) \frac{m_{r_1}}{t_{o_1}} =$$

$$= c_{o_2}^2 + (1 + c_{r_2}^2)\, a_2 (1 - a_2) \frac{\frac{1}{2} m_2}{t_{o_2}} < ce_2^2$$

Finally, $CT_{q_1} = \frac{ca^2 + ce_1^2}{2} \frac{u_1}{1-u_1} t_{e_1} < \frac{ca^2 + ce_2^2}{2} \frac{u_2}{1-u_2} t_{e_2} = CT_{q_2}$

Problem 1 (30 points): Consider a single-server workstation that meets the operational assumptions of the G/G/1 queueing station discussed in class. The processing times at the workstation server are normally distributed with a mean of 3 min and st. deviation of 1 min. Furthermore, observation of this workstation indicates that it is in a stable operational mode, and the server utilization is 90%.

Please, answer the following questions:

i. (5 pts) What is the production rate of this workstation?

ii. (5 pts) If parts arrive for processing at this workstation according to a Poisson process, what is their cycle time at this workstation?

iii. (5 pts) Answer the question in part (ii) above for the case where parts are released for processing to this workstation from a neighboring storage area according to a deterministic pace that is specified by the production rate that is calculated in item (i).

iv. (10 pts) Determine the variability in the departure process of this workstation for each of the two operational regimes that are defined in items (ii) and (iii) above.

v. (5 pts) If the server of this workstation starts experiencing preemptive nondestructive outages with the time between two consecutive failures following an exponential distribution with rate 1 failure per hour, what is the maximal MTTR that will still ensure a stable operation for this workstation?

(i) $u = \lambda t_p \Rightarrow 3\lambda = 0.9 \Rightarrow \lambda = 0.9/3 = 0.3 \text{ min}^{-1} = 18 \text{ hr}^{-1}$

(ii) $CT = \frac{c_a^2 + c_p^2}{2} \frac{u}{1-u} t_p + t_p = \frac{1 + (1/3)^2}{2} \frac{0.9}{1-0.9} 3 + 3 = 18 \text{ min}$

(iii) $CT = \frac{0 + 1/9}{2} \frac{0.9}{1-0.9} 3 + 3 = 4.5 \text{ min}$

(iv) $c_d^2 = (1-u^2) c_a^2 + u^2 c_p^2 = \begin{cases} (1-0.9^2)\cdot 1 + 0.9^2\, 1/9 = 0.28 \\ (1-0.9^2)\cdot 0 + 0.9^2\, 1/9 = 0.09 \end{cases}$

(v) Let $A = \frac{MTTF}{MTTF + MTTR}$. We need $\frac{\lambda t_p}{A} < 1 \Rightarrow A > \lambda t_p = 0.9$

This further implies that $\frac{1}{1 + MTTR} > 0.9 \Rightarrow 1 - 0.9 > 0.9\, MTTR$

$\Rightarrow MTTR < 1/9 = 0.111 \text{ hr} = 6.67 \text{ min}$

Problem 2 (30 points): Consider a single-server workstation where parts arrive according to a Poisson process with rate $r_a = 10$ parts per hour, and, upon arrival, they are classified as type 1 and type 2 with corresponding probabilities equal to 0.4 and 0.6. Both part types are joining a single infinite-capacity queue, and they are processed according to a FCFS policy. Furthermore, the processing of both types of parts at the workstation server is fully automated, and therefore, the corresponding processing times are deterministically equal to 7 minutes for parts of type 1 and deterministically equal to 4 minutes for parts of type 2.

Please, answer the following questions.

i. (5 pts) Show that the operation of this workstation is stable.

ii. (10 pts) Define the distribution of the processing times that are experienced at the server of this workstation, treating jobs in both part types as a single stream of jobs, and compute the mean, the variance and the CV of this distribution.

iii. (5 pts) Compute the expected waiting time for the parts going through this workstation, and the average number of parts in the workstation buffer, when the workstation is operated in steady state.

iv. (5 pts) Compute the expected cycle time at this workstation for each part type.

v. (5 pts) Assuming that the server is busy, what is the probability that the part in processing is of type 1?

THIS PROBLEM IS THE SAME WITH PROBLEM 2 IN MIDTERM II FOR THE OFFERING OF THIS COURSE IN FALL 2018, EXCEPT FOR THE FOLLOWING MODIFICATIONS:

(I) THE CLASSIFICATION PROBABILITIES FOR THE ARRIVING PARTS NOW ARE 0.4 AND 0.6 INSTEAD OF 0.5 AND 0.5.

(II) QUESTION (V) IS NEW.

HENCE, WORKING AS SHOWN IN THE POSTED SOLUTION FOR THIS PROBLEM IN THE AFOREMENTIONED EXAM, YOU SHOULD BE ABLE TO GET THE FOLLOWING ANSWERS FOR THE FIRST FOUR QUESTIONS:

(i) $u = 0.867 < 1$

(ii) $E[T] = 5.2$ min

$Var(T) = 2.16 \text{ min}^2$

$SCV[T] = 0.08 \Rightarrow CV[T] = 0.283$

(iii) $CT_q = 18.30$ min

$WIP_q = 3.05$

(iv) $CT_1 = 25.30$ min

$CT_2 = 22.30$ min

(v) $Prob(\text{Type } 1 \mid \text{busy}) = \dfrac{Prob(\text{Type } 1 \wedge \text{busy})}{Prob(\text{busy})} =$

$$= \frac{u_1}{u} = \frac{10 \times 0.4 \times 7/60}{0.867} \simeq 0.54$$