ISYE 4803-REV: Advanced Manufacturing Systems

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Midterm Exam I

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Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. What are the basic mechanisms that can be used by a modern company in its efforts to cope with the uncertainly in the future demand for its product?

Traditionally, companies have tried to protect themselves against the uncertainty in their future demand by maintaining either a "material buffer" (i.e., safety stock) or a "capacity buffer" (i.e., excess capacity).

In the recent years, companies have also tried the available information technologies in order to control directly the uncertainty in their future demand. This can be attained by obtaining more pertinent and timely information for the behavior of the downstream parts of their supply chains, and ~~processing~~ integrating this information more effectively in their decision making processes.

Finally, some additional elements that introduce various notions of flexibility in the company operations – like modular product designs but also shop-floor structures, design for postponement, centralized inventories and more tightly interconnected supply chains, etc. – can also boost the company responsiveness to the experienced demand.

2. What necessitates the employment of the S-shaped layout in modern production facilities? Also, how can we cope with the shop floor congestion that might result from the employment of such a layout?

S-shaped layouts are used in an effort to fit a long production line in an available facility.

In the case that this production line is integrated by a conveyor belt, we can cope with the resulting congestion of the shop-floor by either trying to build "bridges" over the line or by elevating the conveyor belt itself (e.g., mounting it from the ceiling).

In the case of asynchronous production lines, that do not use a conveyor belt, we should try to decongest the shop-floor by keeping the line WIP as low as possible. This can be achieved through the proper management of the line workflow.

3. The problem of determining the smallest possible number of workstations for a synchronous transfer line that will support the execution of a set (indivisible) tasks, T, is a hard combinatorial optimization problem even in the absence of any precedence constraints among these tasks.

(a) TRUE (b) FALSE

Please, briefly explain your answer.

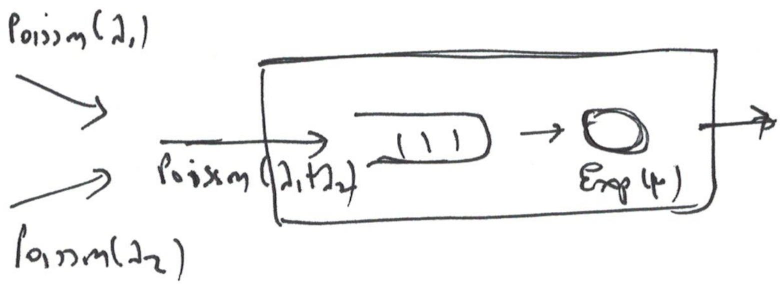
In class we said that even without the task precedence constraints, the resulting ALB problem corresponds to a "bin packing" problem where (i) the role of the bins is played by the line workstations, (ii) the bin capacity is the cycle time c that is suggested by the target throughput (i.e., $c = 1/TH$), and (iii) the items to be packed in these bins are the various tasks to be supported by the line. ~~Bin packing~~ Determining the smallest possible number of bins, of certain capacity, that can accommodate a given set of tasks T is a hard combinatorial problem due to the indivisibility of the tasks. Due to this indivisibility, the quantity $\left\lceil \frac{\sum_i t_i}{c} \right\rceil$ is only a <u>lower bound</u> for the necessary number of bins, i.e., in many cases, a solution utilizing only this number of bins might not be attainable. In fact, we demonstrated this issue with one of the small examples that were presented in class when this lower bound was introduced

4. Explain why in a highly utilized workstation that operates as a G/G/1 queue, the variability that appears in the inter-departure times of this workstation is determined primarily by the variability of the processing times of the station server.

In a highly utilized G/G/1 workstation, when job completes service it is very likely that another job will be waiting in the queue. As a result, the distribution of the job inter-departure times will be very similar to the distribution of the processing times themselves.

5. Consider a single-server workstation with an infinite buffer that processes two different types of jobs. The arrival processes for each job type are Poisson with corresponding rates λ_1 and λ_2, and these two Poisson processes are independent from each other. Furthermore, all arriving jobs enter the workstation buffer and they are served according to a FCFS policy. Finally, processing times at the station server are exponentially distributed with the same rate μ for both job types.

Explain that the operation of this workstation can be modeled as an M/M/1 queue and use this fact in order to determine the stability condition for this workstation. Also, determine the probability that when the workstation server is busy, it is processing a type-1 job.



The Poisson nature of the arrival streams together with their independence imply that the combined stream is also Poisson with rate $\lambda = \lambda_1 + \lambda_2$.

Furthermore, since the jobs in the combined stream are processed FCFS and they are not differentiated in terms of their proc. time distribution, eventually the considered station operates as an M/M/1 queue with arrival rate λ and proc. rate μ.

Hence, its stability condition is $\lambda/\mu = \frac{\lambda_1+\lambda_2}{\mu} < 1$.

Finally, $\text{Prob}(\text{Type 1} \mid \text{busy}) = \frac{\text{Prob}(\text{Type 1} \wedge \text{busy})}{\text{Prob}(\text{busy})} =$

$$= \frac{u_1}{u_1+u_2} = \frac{\lambda_1/\mu}{\frac{\lambda_1}{\mu}+\frac{\lambda_2}{\mu}} = \frac{\lambda_1}{\lambda_1+\lambda_2}$$

Problem 1 (30 points): A local sports company will produce golf clubs on an assembly line, according to the eight operations that are listed in the following table:

Task	Req. Time (min)	Imm. Preds
1. Polish shaft	12	
2. Grind the shaft end	13	
3. Polish the club head	6	
4. Imprint number	4	3
5. Connect wood to shaft	6	1,2,4
6. Place and secure connecting pin	3	5
7. Place glue on other end of shaft	3	1
8. Set in grips and balance	12	6, 7

The above table also reports the required time for each task in minutes, and the immediate predecessors for each task.

Please, answer the following questions:

i. (5 pts) Draw the precedence diagram that represents the task precedence constraints that are specified by the above table.

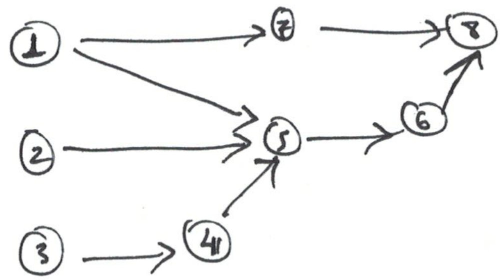
ii. (5 pts) What is the maximal throughput that can be supported by this line?

iii. (5 pts) Suppose that we want to set up this assembly line so that it produces 4 clubs per hour. What is the required *cycle time* for the operation of this assembly line that will deliver this production rate?

iv. (5 pts) What is a lower bound to the required number of workstations for this assembly line that is implied by the cycle time that you computed in item (iii) above?

v. (10 pts) Use the heuristic of the "ranked positional weights" in order to design an assembly line that will deliver the target throughput of 4 clubs per hour.

(i)



(ii) $\max TH = \dfrac{1}{\max_i\{t_i\}} = \dfrac{1}{13\text{ min}} \simeq 0.077 \text{ min}^{-1} = 4.615 \text{ hr}^{-1}$

(iii) $c = \dfrac{1}{TH} = \dfrac{1}{4 \text{ hr}^{-1}} = 0.25 \text{ hr} = 15 \text{ min}$

(iv) $N = \left\lceil \dfrac{\sum_i t_i}{c} \right\rceil = \left\lceil \dfrac{59}{15} \right\rceil = \lceil 3.9\overline{3} \rceil = 4.$

(v)

Task	Successors	PW	Rank
1	1, 5, 6, 7, 8	36	1
2	2, 5, 6, 8	34	2
3	3, 4, 5, 6, 8	31	3
4	4, 5, 6, 8	25	4
5	5, 6, 8	21	5
6	6, 8	15	6
7	7, 8	15	7
8	8	12	8

$\langle 1, 2, 3, 4, 5, 6, 7, 8 \rangle$

WS1	WS2	WS3	WS4	WS5
1, 7	2	3, 4	5, 6	8
~~15~~, 3, 0	~~15~~, 2	~~15~~, 9, 5	~~15~~, 9, 6	~~15~~, 3

Problem 2 (30 points): Consider a single-server queue operated similarly to an M/M/1 queue (i.e., with a Poisson arrival process and exponential service times) but with the following variation: Whenever a service is completed, a departure occurs only with probabilty a; with the remaining probability $1 - a$, the customer joins the end of the queue for another round of service. Furthermore, let the arrival rate be denoted by λ and the processing rate by μ.

Please, answer the following questions:

i. (5 pts) Show that the dynamics of this queue can be modeled by a Continuous-Time Markov Chain (CTMC) with a state space structure that is similar to that of the CTMC that models the operation of the M/M/1 queue.

ii. (5 pts) What is the stability condition for this new queue?

iii. (5 pts) What is the average number of customers in this queue when it operates in steady state? (*Hint:* Use the result of item (i) above.)

iv. (5 pts) What is the expected cycle time for a customer that goes through this queue in steady state?

v. (5 pts) What is the expected waiting time of a customer from the time that she arrives until she enters the server for the first time, assuming that the queue operates in steady state?

vi. (5 pts) What is the expected number of times that the customer will receive service before leaving the queue?

(i) Arguing as we did when we developed the continuous-time Markov chain for the M/M/1 queue, first we see that at each state $i = 0, 1, 2, \ldots$ jobs arrive with rate λ, and therefore the chain transitions from any state i to state $i+1$ with this rate.
On the other hand, any job completion taking place at state $i = 1, 2, \ldots$ will result to an actual departure with prob. a while with the remaining probability $1-a$, the CTMC will maintain the same number of jobs. Hence, the rate for transitioning from a state $i = 1, 2, \ldots$ to state $i-1$ is μa.

From the previous remarks it follows that the CTMC modeling the dynamics of this queue is

$$\begin{tikzcd} 0 \arrow[r, bend left, "\lambda"] & 1 \arrow[l, bend left, "\mu a"] \arrow[r, bend left, "\lambda"] & 2 \arrow[l, bend left, "\mu a"] \arrow[r, bend left, "\lambda"] & 3 \arrow[l, bend left, "\mu a"] \arrow[r, bend left, "\lambda"] & 4 \arrow[l, bend left, "\mu a"] \arrow[r, bend left, "\lambda"] & \cdots \arrow[l, bend left, "\mu a"] \end{tikzcd}$$

This CTMC is similar to the CTMC of the $M/M/1$ queue when we set $\mu' = \mu a$.

(ii) From the last part of (i), it follows that the stability condition of this new queue is

$$\rho = \lambda/\mu' = \frac{\lambda}{\mu a} < 1.$$

(iii) Again, the developments of (i) imply that

$$WIP = \frac{\rho}{1-\rho} = \frac{\lambda/\mu a}{1-\lambda/\mu a} = \frac{\lambda}{\mu a - \lambda}$$

(iv) For a stable system, $TH = \lambda$. Then, from (iii) and Little's law we have that $CT = \frac{WIP}{TH} = \frac{1}{\lambda}\,\frac{\lambda}{\mu a - \lambda} = \frac{1}{\mu a - \lambda}$

(v) From PASTA, a new arrival will find WIP jobs in system, on average. Each of these jobs will require an average processing time of $1/\mu$. Furthermore, any new arrival or recycled job will join the end of the queue and therefore will not interfere with the effort of the considered job to access the server. Hence, the expected waiting time for the considered job till it accesses the server for the first time is

$$C\bar{T}_q^{(1)} = WIP_0\left(1/\mu\right) = \frac{\lambda}{\mu a - \lambda} \cdot \frac{1}{\mu}$$

(v) Every time a job receives service, it leaves with probability a; otherwise, it has to try again by rejoining the queue. Hence, we have a Bernoulli experiment with "success probability" equal to a. But then, the expected number of trials till success is $1/a$.

This last result explains also the stability condition obtained in part (ii): Because of the re-trials, the average amount of work for the server that is brought by a new (i.e., external) arrival is $\left(\frac{1}{\mu}\right) \cdot \left(\frac{1}{a}\right)$. Hence, we need that $\lambda\left(\frac{1}{\mu}\right)\left(\frac{1}{a}\right) < 1$.