Problem 7.4

$k = 4, \lambda(I) = 6$ jobs/hour $C_a^2(I) = 3, E[T_S(B)] = 3/5, C_S^2 = 4/5$

$$
\begin{aligned}
\lambda(B) &= \lambda(I)/k = 6/4 \text{Batches/hour} \\
C_a^2(B) &= 3/4 \\
u(B) &= u(I) = \lambda(B)E[T_S(B)] = 0.9 \\
CT_S &= \frac{(k-1)}{2}E[T_a(I)] + \frac{\left(C_a^2(B) + C_S^2(B)\right)}{2}\left(\frac{u(B)}{1-u(B)}\right)E[T_S(B)] + E[T_S(B)] \\
CT_S &= \frac{(4-1)}{2}\frac{3}{5} + \frac{(3/4+4/5)}{2}\left(\frac{0.9}{1-0.9}\right)\frac{3}{5} + \frac{3}{5} = 5.035 \text{ hours}
\end{aligned}
$$

Number of waiting batches

$$
\begin{aligned}
\text{WIP}_S &= \lambda(B)\frac{\left(C_a^2(B) + C_S^2(B)\right)}{2}\left(\frac{u(B)}{1-u(B)}\right)E[T_S(B)] \\
&= \frac{\left(C_a^2(B) + C_S^2(B)\right)}{2}\left(\frac{u(B)^2}{1-u(B)}\right) \\
&= \frac{(3/4+4/5)}{2}\left(\frac{0.9^2}{1-0.9}\right) = 6.2775 \text{ Batches}
\end{aligned}
$$

To find the optimal batch size that minimize the cycle time

$$
u(B) = \lambda(B)E[T_S(B)] = \frac{6}{k}\frac{3}{5} = \frac{3.6}{k} \Rightarrow k = \frac{3.6}{u(B)}
$$

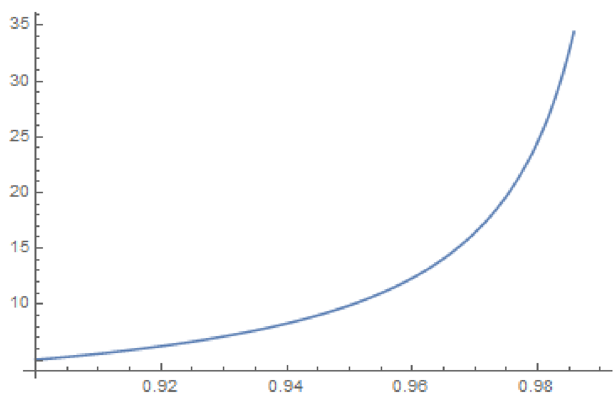
$$
CT_S = \left[\frac{k-1}{2ku} + \frac{C_a^2/k + C_s^2}{2}\frac{u}{1-u}\right]E[T_S(B)] + E[T_S(B)]
$$

Then, we replace k in the CT_S equation by $3.6/u$, then we got:

$$
CT_S = \left[\frac{3.6-u}{7.2u} + \frac{5u+4.8}{12}\frac{u}{1-u}\right]0.6 + 0.6
$$

But since the total capacity of the machine is 4, then we need to minimize CT_S with the constraint $k \leq 4 \Rightarrow 3.6/u \leq 4 \Rightarrow u \geq 0.9$.
The following figure shows the plot of CT_S versus u, from this picture you can notice that the value of u that minimizes CT_S is $u = 0.9$ and therefore, the optimal batch size will be $k^* = \frac{3.6}{0.9} = 4$.

Another way to obtain the last result above is by noticing that for stability $k > \lambda(I)E[T_S(B)] = 6\frac{3}{5} = 3.6$, and therefore, $k = 4$ is the only feasible solution.



35
30
25
20
15
10
0.92
0.94
0.96
0.98

Extra Credit Problem:

Chapter 7

Problem 1

data: $\lambda = 1/\text{hour}$, $C_a^2(I) = 1.5$, $E[T_s(I)] = 0.75$, $C_s^2(I) = 2$.

cycle time for individual move

$$CT_{IM} = \frac{\left(C_a^2(I) + C_s^2(I)\right)}{2} \frac{u(I)}{1-u(I)} E[T_s(I)] + E[T_s(I)]$$

$$CT_{IM} = \frac{(1.5+2)}{2} \frac{0.75}{0.25} 0.75 + 0.75 = 4.6875 \text{ hours}$$

cycle time for batch move as a function of batch size k

$$CT_{BM} = \frac{(k-1)}{2} \frac{1}{\lambda(I)} + \frac{\left(C_a^2(B) + C_s^2(B)\right)}{2} \frac{u(B)}{1-u(B)} E[T_s(B)] + \frac{(k-1)}{2} E[T_s(I)] + E[T_s(I)]$$

$$= \frac{(k-1)}{2} \frac{1}{\lambda(I)} + \frac{\left(\frac{C_a^2(I)}{k} + \frac{C_s^2(I)}{k}\right)}{2} \frac{u(I)}{1-u(I)} k\, E[T_s(I)] + \frac{(k-1)}{2} E[T_s(I)] + E[T_s(I)]$$

$$= \frac{(k-1)}{2} \frac{1}{\lambda(I)} + \frac{\left(C_a^2(I) + C_s^2(I)\right)}{2} \frac{u(I)}{1-u(I)} E[T_s(I)] + \frac{(k-1)}{2} E[T_s(I)] + E[T_s(I)]$$

$$= CT_{IM} + \frac{(k-1)}{2} \frac{1}{\lambda(I)} + \frac{(k-1)}{2} E[T_s(I)]$$

$k = 2$

$$CT_{BM} = \frac{1}{2}1 + \frac{1}{2}0.75 + \frac{(1.5+2)}{2} \frac{0.75}{0.25} 0.75 + 0.75 = 4.6875 + 0.875 = 5.5625 \text{ hours}$$

$k = 3$

$$CT_{BM} = \frac{2}{2}1 + \frac{2}{2}0.75 + \frac{(1.5+2)}{2} \frac{0.75}{0.25} 0.75 + 0.75 = 6.4375 \text{ hours}$$

$k = 4$

$$CT_{BM} = \frac{3}{2}1 + \frac{3}{2}0.75 + \frac{(1.5+2)}{2} \frac{0.75}{0.25} 0.75 + 0.75 = 7.3125 \text{ hours}$$

$k = 5$

$$CT_{BM} = \frac{4}{2}1 + \frac{4}{2}0.75 + \frac{(1.5+2)}{2} \frac{0.75}{0.25} 0.75 + 0.75 = 8.1875 \text{ hours}$$

Note: as batch size increases, so does the cycle time.

Problem 5

Due to the serial nature of the system: $\lambda_1 = \lambda_2 = \lambda_3 = 0.5$

Workstation 1:

$$u_1 = \frac{\lambda_1 E[T_1]}{c_1} = \frac{0.5(1.6)}{1} = 0.80$$

$$CT_s(1) = \left(\frac{2+0.75}{2}\right)\left(\frac{0.80}{0.20}\right)1.6 + 1.6 = 10.400$$

$$WIP_s(1) = \lambda_1 CT_s(1) = 0.5(10.400) = 5.200$$

$$C_d^2(1) = \left(1-u_1^2\right)C_a^2(1) + u_1^2 C_s^2(1) = (1-0.80^2)2 + 0.80^2(0.75) = 1.200$$

Workstation 2:

$$u_2 = \frac{\lambda_2 E[T_2]}{c_2} = \frac{0.5(1.5)}{1} = 0.75$$

$$CT_s(2) = \left(\frac{1.2+1.5}{2}\right)\left(\frac{0.75}{0.25}\right)1.5 + 1.5 = 7.575$$

$$WIP_s(2) = \lambda_2 CT_s(2) = 0.5(7.575) = 3.787$$

$$C_d^2(2) = \left(1-u_2^2\right)C_a^2(2) + u_2^2 C_s^2(2) = (1-0.75^2)1.2 + 0.75^2(1.5) = 1.369$$

Workstation 3:

$$u_3 = \frac{\lambda_3 E[T_3]}{c_3} = \frac{0.5(1.7)}{1} = 0.85$$

$$CT_s(3) = \left(\frac{1.369+2}{2}\right)\left(\frac{0.85}{0.15}\right)1.7 + 1.7 = 17.926$$

$$WIP_s(3) = \lambda_3 CT_s(3) = 0.5(17.926) = 8.963$$

$$C_d^2(3) = \left(1-u_3^2\right)C_a^2(3) + u_3^2 C_s^2(3) = (1-0.85^2)1.369 + 0.85^2(2) = 1.825$$

System Performance Measures

$$WIP_{sys} = WIP_s(1) + WIP_s(2) + WIP_s(3) = 17.951$$

$$Th_{sys} = \lambda_1 = 0.5$$

$$CT_{sys} = \frac{WIP_{sys}}{Th_{sys}} = \frac{17.951}{0.5} = 35.901$$

Problem 9

(a) $Th_{sys} = 0.51 \quad WIP_{sys} = 20.247 \quad CT_{sys} = 39.699$

$Th_{sys} = 0.53 \quad WIP_{sys} = 26.807 \quad CT_{sys} = 50.579$

$Th_{sys} = 0.55 \quad WIP_{sys} = 38.969 \quad CT_{sys} = 70.852$

(b) $C_s^2(3) = 1.31 \rightarrow Th_{sys} = 0.51 \quad WIP_{sys} = 18.297 \quad CT_{sys} = 35.876$

implies a 34.3% reduction

(c) $E[T_3] = 1.7 \rightarrow 1.343 \Rightarrow Th_{sys} = 0.55 \quad WIP_{sys} = 19.738 \quad CT_{sys} = 35.89$

(d) $E[T_3] = \dfrac{util_1}{th} = \dfrac{0.880}{0.55} = 1.6$ hrs, $SR = 1/E[T_3] = 1/1.6 = 0.625/hr$

See the next pages for a more detailed treatment of this problem.

Problem 10

Workstation 1:

$$C_a^2(1) = 0.75, \quad E[T_s(1)] = 0.20, \quad C_s^2(1) = 2.0, \quad u_1 = 0.8$$

$$CT_s(1) = \frac{(0.75 + 2.0)}{2}\frac{0.8}{0.2}(0.2) + 0.2 = 1.300 \text{ hrs.}$$

$$C_d^2(1) = \left(1 - 0.8^2\right)0.75 + \left(0.8^2\right)2.0 = 1.550$$

Workstation 2:

$$E[T_s(2)] = 0.30, \quad C_s^2(2) = 0.7, \quad u_2 = 0.6$$

$$CT_s(2) = \frac{(1.550 + 0.7)}{2}\frac{\left(0.6^{\sqrt{6}-1}\right)}{2(0.4)}(0.3) + 0.3 = 0.5012 \text{ hrs.}$$

$$C_d^2(2) = 1 + \left(1 - 0.6^2\right)(1.550 - 1) + \frac{0.6^2(0.7 - 1)}{\sqrt{2}} = 1.276$$

Workstation 3:

$$E[T_s(3)] = 0.22, \quad C_s^2(3) = 1.0, \quad u_3 = 0.88$$

$$CT_s(3) = \frac{(1.276 + 1.0)}{2}\frac{0.88}{0.12}(0.22) + 0.22 = 2.056 \text{ hrs.}$$

System:

$$WIP_{sys} = 4(1.300 + 0.5012 + 2.0560) = 15.429 \text{ jobs}$$

$$\text{CT}_{\text{sys}} = \frac{WIP_{sys}}{Th_{sys}} = 15.429/4 = 3.857\ hrs.$$

$$Th_{sys} = 4 \text{ jobs / hour}$$

An expansive sol. of Problem 5.9

a) Working as in the solution of Problem 5.5, but using the new values for the arrival rate, we obtain the following table (this computation can be organized very conveniently in a small spreadsheet):

r_a	TH	u_1	CT_1	WIP_1	$c_{d_1}^2$	u_2	CT_2	WIP_2	$c_{d_2}^2$	u_3	CT_3	WIP_3	CT_{sys}	WIP_{sys}
0.5	0.5	0.8	10.4	5.2	1.2	0.75	7.575	3.787	1.369	0.85	17.926	8.963	35.901	17.951
0.51	0.51	0.816	11.356	5.79	1.168	0.765	8.013	4.087	1.362	0.867	20.33	10.368	39.70	20.247
0.53	0.53	0.848	13.874	7.353	1.101	0.795	9.065	4.805	1.353	0.901	27.64	14.65	50.58	26.807
0.55	0.55	0.88	17.733	9.753	1.032	0.825	10.452	5.749	1.350	0.935	42.667	23.467	70.852	38.969

b) From the above table, the required reduction is

$$\Delta CT_{sys} = 39.699 - 35.901 = 3.798$$

The "bottleneck" machine (according to the suggested definition) is the the 3rd one. The target cycle time for this machine is

$$CT' = 20.33 - 3.798 = 16.532$$

The above reduction will be effected by changing the scv of processing times at that machine. Let x denote the new value for the scv. Then, we need:

$$\frac{1.362 + x}{2} \; \frac{0.867}{1 - 0.867} \cdot 1.7 + 1.7 = 16.532 \iff x = 1.315$$

c) In this case, the requested CT reduction is:

$$\Delta CT_{sys} = 70.852 - 35.901 = 34.951$$

and the ~~target~~ CT_3 is

$$CT_3' = 42.667 - 34.951 = 7.716$$

Setting x = ~~new~~ mean proc.time for machine 3, we have:

$$\frac{1.302+2}{2} \; \frac{0.55 \cdot x}{1-0.55x} \, x + x = 7.716 \Leftrightarrow$$

$$\Leftrightarrow \; 0.92455 \cancel{x^2} = (7.716 - x)(1 - 0.55x) \Leftrightarrow$$

$$\Leftrightarrow \; 0.37455x^2 + 5.2438x - 7.716 = 0 \Rightarrow$$

$$\Rightarrow \; x = \frac{-5.2438 + \sqrt{5.2438^2 + 4 \cdot 0.37455 \cdot 7.716}}{2 \cdot 0.37455} = 1.343$$

d) The machine with the second highest utilization is machine 1, with $u_1 = 0.88$. Hence, we want:

$$0.55 \cdot x = 0.88 \Rightarrow \quad x = \frac{0.88}{0.55} = 1.6 \text{ hrs}$$ (which, of course, is the mean proc. time of machine!)

Homework 6

Problem 13 (Chapter 5)

Solving a simultaneous system of equations is not necessary for this problem since it can be solved in a straightforward manner:

$$\lambda_1 = 10$$

$$\lambda_2 = 0 + \frac{3}{4}\lambda_1 = 7.5$$

$$\lambda_3 = 0 + \frac{1}{4}\lambda_1 + \lambda_2 = 2.5 + 7.5 = 10$$

Problem 15

The system of equations relating these mean flow rates are:

$$\lambda_1 = 10 + \frac{1}{3}\lambda_2 + \frac{1}{5}\lambda_3$$

$$\lambda_2 = 2 + \frac{3}{4}\lambda_1$$

$$\lambda_3 = 0 + \frac{1}{4}\lambda_1 + \frac{2}{3}\lambda_2$$

and the from/to probability matrix is:

$$\mathbf{P} = \begin{bmatrix} 0 & 3/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/5 & 0 & 0 \end{bmatrix}$$

The system becomes $(\mathbf{I} - \mathbf{P}^{\mathrm{T}})\underline{\lambda} = \underline{\gamma}$

$$\lambda_1 - \frac{1}{3}\lambda_2 - \frac{1}{5}\lambda_3 = 10$$

$$-\frac{3}{4}\lambda_1 + \lambda_2 + 0\lambda_3 = 2$$

$$-\frac{1}{4}\lambda_1 - \frac{2}{3}\lambda_2 + \lambda_3 = 0$$

The solution $\underline{\lambda} = (\mathbf{I} - \mathbf{P}^{\mathrm{T}})^{-1}\underline{\gamma}$ via a compute code is:

```
external inflows by workstation (gam): { 10, 2, 0}

workstation from/to branching probabilities (P)
 0.0000  0.7500  0.2500
 0.3333  0.0000  0.6667
 0.2000  0.0000  0.0000

system of equations relating flow rates (lam)
M.lam = gam; where M = I-Transpose[P]
M
 1.0000 -0.3333 -0.2000
-0.7500  1.0000  0.0000
-0.2500 -0.6667  1.0000

lam: { 18.2222, 15.6667, 15.0000}
```

Problem 18 Part a

By the solution of Problem 5.13,

$\lambda_1 = 10$

$\lambda_2 = 0 + \frac{3}{4}\lambda_1 = 7.5$

$\lambda_3 = 0 + \frac{1}{4}\lambda_1 + \lambda_2 = 2.5 + 7.5 = 10$

$u_1 = 10(0.086) = 0.86,$

$u_2 = 7.5(0.110) = 0.825,$

$u_3 = 10(0.080) = 0.80.$

$\Rightarrow$ All stations are stable $\Rightarrow$ TH = 10 (i.e., the total inflow rate to this network).

Note: $C_d^2 = (1-u^2)C_a^2 + u^2 C_s^2$

$C_a^2(1) = 1.5,$

$C_d^2(1) = (1-0.86^2)1.5 + 0.86^2(1.352) = 1.391;$

$C_a^2(2) = (3/4)(\cancel{1.352}\ 1.391) + 1/4 = 1.293,$

$C_d^2(2) = (1-0.825^2)1.293 + 0.825^2(0.826) = 0.975;$

$C_a^2(3) = (7.5/10)(0.975) + (2.5/10)(\frac{1}{4}1.391 + \frac{3}{4}) = 1.006.$

Note that the number 1.352 should actually read 1.391.

Note: $CT_s(i) = \frac{\left(C_a^2(i) + C_s^2(i)\right)}{2}\left(\frac{u_i}{1-u_i}\right)E[T_s(i)] + E[T_s(i)]$

$$CT_s(1) = \left(\frac{1.5+1.352}{2}\right)\left(\frac{0.86}{0.14}\right)0.086 + 0.086 = 0.839,$$
$$CT_s(1) = \left(\frac{1.293+0.826}{2}\right)\left(\frac{0.825}{0.175}\right)0.110 + 0.110 = 0.660,$$
$$CT_s(1) = \left(\frac{1.006+1.562}{2}\right)\left(\frac{0.80}{0.20}\right)0.08 + 0.08 = 0.491.$$

Note: by Little's Law $WIP_s(i) = \lambda_i CT_s(i)$

$$WIP_s(1) = 10(0.8394) = 8.394,$$
$$WIP_s(2) = 7.5(0.660) = 4.946,$$
$$WIP_s(3) = 10(0.491) = 4.910,$$
$$WIP_{sys} = \sum_{i=1}^{3} WIP_s(i)$$
$$WIP_{sys} = 18.25 \text{ jobs}$$
$$\text{TH} = 10 \text{ jobs / hour}$$
$$\text{CT}_{\text{sys}} = \frac{WIP_{sys}}{TH} = 18.25/10 = 1.825 \text{ hours.}$$

Problem 18 Part b (availability < 1)

This problem proceeds as Part a except that $E[S]$ and $C^2[S]$ are adjusted to account for breakdowns. These adjusted values are denoted herein as $E[S_e]$ and $C^2[S_e]$. The adjusts are in terms of the machine availability a, the mean and SCV of the repair time $E[R]$ and $C^2[R]$, all of this is given data (or computable data from given data) with:

$$E[S_e] = E[S]/a;$$
$$C^2[S_e] = C^2[S] + (1 + C^2[R])a(1-a)E[R]/E[S].$$

For this problem:

$$\underline{a} = (0.95, 0.93, 0.87),$$
$$E[R_1] = 0.2,\ C^2[R_1] = 1,$$
$$E[R_2] = 0.3,\ C^2[R_1] = 1,$$
$$E[R_3] = 0.4,\ C^2[R_1] = 1.$$

Thus, the adjusted service characteristics are:

$E[S_{e1}] = 0.086 / 0.95 = 0.0905,$
$E[S_{e2}] = 0.110 / 0.93 = 0.1183,$
$E[S_{e3}] = 0.080 / 0.87 = 0.0920,$
and
$C^2[S_{e1}] = 1.352 + (1+1)0.95(0.05)0.2 / 0.086 = 1.573,$
$C^2[S_{e2}] = 0.862 + (1+1)0.93(0.07)0.3 / 0.110 = 1.181,$
$C^2[S_{e3}] = 1.5625 + (1+1)0.87(0.13)0.4 / 0.080 = 2.693.$

This of course leads to increased utilizations for the machines which are:

$\underline{u} = (0.905, 0.887, 0.920).$

These can be handled by one machine in each workstation, so the number of machines is not changed. The rest of the solution proceeds as before but with the updated numbers that are provided above.
A computer solution of this problem follows → see the highlighted part for the new values

```
[* ========== start of user input data ============ *]

numprod = 1;
numstat = 3;
maxnumsteps = 3;

[*  inflow rates for each product by step  *]
Print["External Arrival Stream Mean Rates : G"];
G =   {{10,0,0}};

Print["External Arrival Streams Squared Coef. Var. by step: Cao"];
Cao = {{1.5,0,0,0}};

[* product routing probabilities: a matrix for each product *]
[* these are given as leaving probabilities (from - to) *]

P = {{0,3/4,1/4},{0,0,1},{0,0,0}};

 [* number of machines in stations *]
m = {1,1,1};
[* special workstations - batching size fixed *]
BatchWS = {1,1,1};

[*  prod serv time means and variances by step *]
ProdMeans = {{0.086, 0.110, 0.080}};

ProdSCV = {{1.3521,0.8264,1.5625}};

[* MACHINE AVAILABILITY DUE TO BREAKDOWNS AND REPAIRS *]
[* used only if avail < 1, and applies to each machine of a group *]
[* avail is the fraction of the time that the machine is operational *]
avail = {0.95, 0.93, 0.87};
[* repair time data not used unless avail < 1 *]
ERTime = {0.2,0.3,0.4};
C2RTime = {1,1,1};

[* ========== end of user input data ============ *]
```

```
 CaKorqK:  0  CdApprox:  1  CaApprox:  1
maxciter:  10  prtciter:  1  CaPrt:  0
-----------------------------------

External Arrival Stream Mean Rates : G
external inflow by product to Steps
  1  {  10,    0,    0}
External Arrival Streams Squared Coef. Var. by step: Cao
  1  {    1.500,    0,    0,    0}

Steps to WorkStations Conversion
  1   2   3
Routing Step Matrix by Product: QPRP
  0.000   0.750   0.250
  0.000   0.000   1.000
  0.000   0.000   0.000


Prod mean times
  0.086   0.110   0.080
Prod Serv Vars
  0.010   0.010   0.010
External Inflow by Product to WorkStations
  1  {  10.000,   0.000,   0.000}
External Arrival Squared Coef. Var. by WorkStation: Caows
uses asymptotic weighted Cao of steps into workstations
  1  {    1.500,   0.000,   0.000}
S - Serv Time Sq. Coef. Var. by Product
  1.352   0.826   1.563
QPRP   1
  0.000   0.750   0.250
  0.000   0.000   1.000
  0.000   0.000   0.000
G[i] by step: {  10,    0,    0}
Lam:    1  = {  10.000,    7.500,   10.000}
Lam by Product per Step
 10.000   7.500  10.000
Lam by Product and WorkStation
 10.000   7.500  10.000
Total wslam by WorkStation
 10.000   7.500  10.000
Branching Probabilities by Workstation: Qws
  0.000   0.750   0.250
  0.000   0.000   1.000
  0.000   0.000   0.000
Gam Inflow by Workstation and Product: Gws
 10.000   0.000   0.000
wsgam: {  10.000,   0.000,   0.000}
total gam:   10

mean proc times at WorkStations
mtws: {   0.086,   0.110,   0.080}
Sq. Coef Serv Times at WorkStations
Csws: {   1.352,   0.826,   1.562}

mean proc times at WorkStations adjusted for avail
```

```
mtws: {   0.091,   0.118,   0.092}
Sq. Coef Serv Times at WorkStations adjusted for avail
Csws: {   1.573,   1.181,   2.693}

station loads
util: {   0.905,   0.887,   0.920}
num machs at stations
m: {   1,    1,    1}

external arriv sq coef var to stations: asymptotic method
Caosws: {   1.500,   0.000,   0.000}

 citer:    0  Ca: {   1.000,   1.000,   1.000}
 citer:    1  Ca: {   1.500,   1.352,   1.136}
              Cd: {   1.470,   1.143,   1.000}
             Cda: {   1.117,   1.143,   1.000}

 citer:    2  Ca: {   1.500,   1.420,   1.198}
              Cd: {   1.560,   1.218,   1.000}
             Cda: {   1.140,   1.218,   1.000}

 citer:    3  Ca: {   1.500,   1.420,   1.209}
              Cd: {   1.560,   1.232,   1.000}
             Cda: {   1.140,   1.232,   1.000}

 citer:    4  Ca: {   1.500,   1.420,   1.209}
              Cd: {   1.560,   1.232,   1.000}
             Cda: {   1.140,   1.232,   1.000}

asymptotic method - Ca: {   1.500,   1.420,   1.209}
                    Cd: {   1.560,   1.232,   1.000}
                   Cda: {   1.140,   1.232,   1.000}

rho: {   0.905,   0.887,   0.920}

mtws: {   0.091,   0.118,   0.092}

 Results by WorkStation

WSi:    1  CTqi:    1.329  WIPqi:   13.291
           CTsi:    1.420  WIPsi:   14.197

WSi:    2  CTqi:    1.209  WIPqi:    9.066
           CTsi:    1.327  WIPsi:    9.953

WSi:    3  CTqi:    2.051  WIPqi:   20.507
           CTsi:    2.143  WIPsi:   21.426

total system inflow (tgam):   10.000

WIPsys:   45.576  CTsys:    4.558
```

Problem 19 Part a

This problem must assume two machines per workstation. in order to have $u_i<1$

$$\lambda_1 = 10 + \frac{1}{3}\lambda_2 + \frac{1}{5}\lambda_3$$

$$\lambda_2 = 2 + \frac{3}{4}\lambda_1$$

$$\lambda_3 = 0 + \frac{1}{4}\lambda_1 + \frac{2}{3}\lambda_2$$

$$\underline{\lambda} = \{18.22\bar{2}, 15.66\bar{6}, 15\}$$

With two machines and the computed λ_i values, we have the following utilizations:

$u_1 = \lambda_1 \cdot E[T_s(1)]/2 = 18.222 \cdot 0.086/2 = 0.7835$

$u_2 = \lambda_2 \, E[T_s(2)]/2 = 15.666 \cdot 0.11/2 = 0.8617$

$u_3 = \lambda_3 \, E[T_s(3)]/2 = 15 \cdot 0.08/2 = 0.6$

Then,

$$C_a^2(1) = \frac{10(1.5)}{18.222} + \frac{15(1/5)}{18.222}\left[\frac{4}{5} + \frac{1}{5}\left\{1 + (1 - 0.6^2)(C_a^2(3) - 1) + \frac{0.6^2(1.563 - 1)}{\sqrt{2}}\right\}\right]$$
$$+ \frac{15.667(1/3)}{18.222}\left[\frac{2}{3} + \frac{1}{3}\left\{1 + (1 - 0.8617^2)(C_a^2(2) - 1) + \frac{0.8617^2(0.8264 - 1)}{\sqrt{2}}\right\}\right],$$

$$C_a^2(2) = \frac{2(1.5)}{15.667} + \frac{18.222(3/4)}{15.667}\left[\frac{1}{4} + \frac{3}{4}\left\{1 + (1 - 0.7835^2)(C_a^2(1) - 1) + \frac{0.7835^2(1.3521 - 1)}{\sqrt{2}}\right\}\right],$$

$$C_a^2(3) = \frac{0(1.5)}{15} + \frac{18.222(1/4)}{15}\left[\frac{3}{4} + \frac{1}{4}\left\{1 + (1 - 0.7835^2)(C_a^2(1) - 1) + \frac{0.7835^2(1.3521 - 1)}{\sqrt{2}}\right\}\right]$$
$$+ \frac{15.667(2/3)}{15}\left[\frac{1}{3} + \frac{2}{3}\left\{1 + (1 - 0.8617^2)(C_a^2(2) - 1) + \frac{0.8617^2(0.8264 - 1)}{\sqrt{2}}\right\}\right].$$

$\underline{C}_a^2(iter\ 0) = \{1.0, 1.0, 1.0\}$

$\underline{C}_a^2(iter\ 1) = \{1.274, 1.232, 0.997\}$

$\underline{C}_a^2(iter\ 2) = \{1.276, 1.233, 1.005\}$

$\underline{C}_a^2(iter\ 3) = \{1.276, 1.234, 1.005\}$

$\underline{C}_a^2(iter\ 4) = \{1.276, 1.234, 1.005\}$

$\Rightarrow$ $\underline{C}_a^2 = \{1.276, 1.234, 1.005\}$

Workstation Performance Measures

$$CT_s(1) = \left(\frac{1.276 + 1.352}{2}\right)\frac{0.784^{\sqrt{2(2)+2}-1}}{2(1-0.784)}0.086 + 0.086 = 0.269$$

$$TH_s(1) = 18.222$$

$$WIP_s(1) = 0.269 * 18.222 = 4.902$$

$$CT_s(2) = \left(\frac{1.234 + 0.826}{2}\right)\frac{0.862^{\sqrt{2(2)+2}-1}}{2(1-0.862)}0.110 + 0.110 = 0.440$$

$$TH_s(2) = 15.667$$

$$WIP_s(2) = 0.440 * 15.667 = 6.893$$

$$CT_s(3) = \left(\frac{1.005 + 1.563}{2}\right)\frac{0.6^{\sqrt{2(2)+2}-1}}{2(1-0.6)}0.110 + 0.110 = 0.141$$

$$TH_s(3) = 15$$

$$WIP_s(3) = 0.141 * 15 = 2.118$$

Factory Performance Measures

$$TH_{sys} = 10 + 2 + 0 = 12 \text{ jobs / hour}$$

$$WIP_{sys} = WIP_s(1) + WIP_s(2) + WIP_s(3) = 4.908 + 6.894 + 2.118 = 13.920 \text{ jobs}$$

$$CT_{sys} = \frac{13.920}{12} = 1.160 \text{ hours}$$

Problem 19 Part b As in Problem 18(b), we need to adjust the data of the first table to reflect the effective proc. times due to the experienced outages.

This problem must assume two machines per workstation.

Computer Model Data and Solution (non-perfect machines)

```
[* ========= start of user input data ============ *]
numprod = 1;
numstat = 3;
maxnumsteps = 3;
[* product titles *]
ProdTitles = {"HW5_17b"};
Print["Products: ",ProdTitles];
[* time units for all model data *]
TimeUnit = "hours";
UnitConv = 24;
TimeConv = "days";
Print["------------------- model analysis in: ",TimeUnit];

[*  inflow rates for each product by step  *]
Print["External Arrival Stream Mean Rates : G"];
G =  1.0 {{10,2,0}};
```

```
Print["external inflow by product to Steps"];
For[i=1,i<=numprod,i++,
  Print[i," ",G[[i]] ];
];
Print["External Arrival Streams Squared Coef. Var. by step: Cao"];
Cao = {{1.5,1.5,0.0}};
For[i=1,i<=numprod,i++,
  Print[i," ",Cao[[i]] ];
];
[* steps to work station conversion *]
StoWS = Table[{},{i,numprod}];
StoWS[[1]] = {1,2,3};
Print[" "];
Print["Steps to WorkStations Conversion"];
MatrixPrint[StoWS];
[* workstation processing titles *]
WSTitles = {"one","two","three"};
Print["Workstations:",WSTitles];
[* product routing probabilities: a matrix for each product *]
[* these are given as leaving probabilities (from - to) *]
QPRP = Table[{},{i,numprod}];
QPRP[[1]] = {{0,3/4,1/4},{1/3,0,2/3},{1/5,0,0}};
Print["Routing Step Matrix by Product: QPRP"];
MatrixPrint[1.0QPRP ];
Print[" "];
[* number of machines in stations *]
m = {2, 2, 2};
[* special workstations - batching size fixed *]
BatchWS = {1,1,1};
[*  prod serv time means and variances by step *]
ProdMeans = {{0.086, 0.110, 0.080}};
ProdSCV = {{1.3521,0.8264,1.5625}};
ProdVar = (ProdSCV)*ProdMeans^2;
[* MACHINE AVAILABILITY DUE TO BREAKDOWNS AND REPAIRS *]
[* used only if avail < 1, and applies to each machine of a group *]
[* avail is the fraction of the time that the machine is operational *]
avail = {0.95, 0.93, 0.87};
[* repair time data not used unless avail < 1 *]
ERTime = {0.2,0.3,0.4};
C2RTime = {1,1,1};
[* ========== end of user input data ============ *]
```

```
==================================================
  CaKorqK:  0  CdApprox:  1  CaApprox:  1
 maxciter:  10  prtciter:  1  CaPrt:  0
 ---------------------------------
```

```
Products: { HW5_19b}
------------------- model analysis in:  hours
External Arrival Stream Mean Rates : G
external inflow by product to Steps
  1  {  10.000,   2.000,   0.000}
External Arrival Streams Squared Coef. Var. by step: Cao
  1  {   1.500,   1.500,   0.000}

Steps to WorkStations Conversion
  1   2   3
Workstations:{ one, two, three}
Routing Step Matrix by Product: QPRP
  0.000   0.750   0.250
  0.333   0.000   0.667
  0.200   0.000   0.000

---- machine availability data ----
avail : {   0.950,   0.930,   0.870}
E[R]  : {   0.200,   0.300,   0.400}
C^2[R]: {   1,   1,   1}

Prod mean times
  0.086   0.110   0.080
Prod Service SCVs
  1.352   0.826   1.563
Prod Serv Vars
  0.010   0.010   0.010
External Inflow by Product to WorkStations
  1  {  10.000,   2.000,   0.000}
External Arrival Squared Coef. Var. by WorkStation: Caows
uses asymptotic weighted Cao of steps into workstations
  1  {   1.500,   1.500,   0.000}
S - Serv Time Sq. Coef. Var. by Product
  1.352   0.826   1.563
QPRP   1
  0.000   0.750   0.250
  0.333   0.000   0.667
  0.200   0.000   0.000
G[i] by step: {  10.000,   2.000,   0.000}
Lam:    1  = {  18.222,  15.667,  15.000}
Lam by Product per Step
 18.222  15.667  15.000
Lam by Product and WorkStation
 18.222  15.667  15.000
Total wslam by WorkStation
 18.222  15.667  15.000
Branching Probabilities by Workstation: Qws
  0.000   0.750   0.250
  0.333   0.000   0.667
  0.200   0.000   0.000
Gam Inflow by Workstation and Product: Gws
 10.000   2.000   0.000
wsgam: {  10.000,   2.000,   0.000}
total gam:   12

mean proc times at WorkStations
```

```
mtws: {   0.086,   0.110,   0.080}
Sq. Coef Serv Times at WorkStations
Csws: {   1.352,   0.826,   1.563}


mean proc times at WorkStations adjusted for avail
mtws: {   0.091,   0.118,   0.092}
Sq. Coef Serv Times at WorkStations adjusted for avail
Csws: {   1.573,   1.181,   2.694}

station loads
util: {   0.825,   0.927,   0.690}
num machs at stations
m: {   2,   2,   2}

external arriv sq coef var to stations: asymptotic method
Caosws: {   1.500,   1.500,   0.000}

 citer:    0  Ca: {   1.000,   1.000,   1.000}
 citer:    1  Ca: {   1.304,   1.244,   1.072}
              Cd: {   1.276,   1.110,   1.570}
             Cda: {   1.069,   1.073,   1.114}

 citer:    2  Ca: {   1.308,   1.308,   1.095}
              Cd: {   1.373,   1.145,   1.607}
             Cda: {   1.093,   1.096,   1.121}

 citer:    3  Ca: {   1.309,   1.309,   1.100}
              Cd: {   1.374,   1.154,   1.620}
             Cda: {   1.094,   1.102,   1.124}

 citer:    4  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:    5  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:    6  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:    7  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:    8  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:    9  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
             Cda: {   1.094,   1.103,   1.124}

 citer:   10  Ca: {   1.310,   1.309,   1.100}
              Cd: {   1.375,   1.154,   1.622}
```

```
             Cda: {   1.094,   1.103,   1.124}

asymptotic method
WS Ca : {   1.310,   1.309,   1.100}
WS Cd : {   1.375,   1.154,   1.622}
WS Cda: {   1.094,   1.103,   1.124}

WS Utils: {   0.825,   0.927,   0.690}

WS E[Ts]: {   0.091,   0.118,   0.092}

 Results by WorkStation

WSi:    1   one  CTqi:    0.282  WIPqi:    5.132
                     CTsi:    0.372  WIPsi:    6.782

WSi:    2   two  CTqi:    0.897  WIPqi:   14.057
                     CTsi:    1.016  WIPsi:   15.911

WSi:    3   three  CTqi:    0.164  WIPqi:    2.460
                     CTsi:    0.256  WIPsi:    3.839


total system inflow (tgam):   12.000

WIPsys:   26.531  CTsys:    2.211   hours
                  CTsys:    0.092   days
```