ISyE 4803-REV: Advanced Manufacturing Systems Modeling and Analysis
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Homework #4

Due Date: Monday, 11/4/19

Reading Assignment

- The preamble and Sections 4.1 and 4.2 from Chapter 4 of your textbook. Section 4.2 corresponds to the material on preemptive non-destructive outages discussed in class.
- The slides on the G/G/m-based modeling of Asynchronous Transfer Lines and its applications that are posted at the course website (http://www2.isye.gatech.edu/~spyros/courses/IE4803/Fall-15/course_materials.html); this is item #2 in the "course materials" list in the above website.
- Your notes from the class lectures.

Problem set:

A. Solve problems 4.3, 4.4, and 4.7 at the end of Chapters 4 and 7 of your textbook.
Remark:
For Problem 4.7, you have to develop a Markov-chain model that also accounts for the machine failures. The rest of the analysis is similar to the M/M/1/c model that has been discussed earlier.

B. Also, solve the following problems:

1. Consider a G/G/1 station operated at 95% of its effective processing capacity. The station is fed with parts at a deterministically paced rate of one part per 10 minutes and the average waiting time experienced by a part before it enters the server is equal to 45 minutes. Use the above information in order to compute the part departure rate from this station and the coefficient of variation of the part inter-departure times.

2. Consider a production line consisting of two single-machine stations. The operational characteristics of these two stations are as follows:

Attribute	Station 1	Station 2
t_s	11min	11min
c_s	0.5	0.5
MTTF	7hrs	5hrs
MTTR	1.5hrs	0.5hrs
c_r	0.75	0.5

Also, at each station, the (operational) time between two consecutive failures is exponentially distributed.

Answer the following questions:
i. Which station is the effective bottleneck of the line? Consider that the bottleneck is defined as the station that has the highest effective utilization.
ii. Can the line sustain a production rate of 35 parts over an eight-hour shift?
iii. Suppose that parts are released to the line in a deterministic manner, with constant inter-release times, and the resulting mean cycle time at Station 1, CT_1, is equal to 2 hours and 7.815 minutes. What is the length of the inter-release

intervals?
iv. What is the average WIP waiting for processing at Station 1 under the assumptions of item (iii) above?
v. Provide estimates for the mean and the variance of the part inter-arrival times at Station 2, under the assumptions stated in item (iii) above.

3. Consider a variation of the single-server workstation model involving part post processing / "rework" that we discussed in class, where any necessary post-processing occurs at the same server that performed the first processing stage, immediately after this main processing stage (i.e., without the part leaving the server between the two processing stages). However, in this new situation, the processing time for the first processing stage has a general distribution with mean t_p = 4min and st. dev. σ_p = 2min, while the necessary rework might be either of type I, with probability p_I = 0.4, or of type II, with probability p_{II} = 0.6. The mean and the st. dev. for the processing times that are required for these two types of rework are, respectively:

$$t_{rI} = 1.5\text{min};\ \sigma_{rI} = 1\text{min};\ t_{rII} = 2.5\text{min};\ \sigma_{rII} = 3\text{min}$$

Finally, the probability that a processed part will need no rework at all, after its first processing stage, is q = 0.3, and parts arrive at this workstation according to a Poisson process with rate λ = 10 parts/hr.

Show that the operation of the above workstation is stable, and perform a "Mean Value Analysis (MVA)" for this workstation, i.e., compute its throughput, server utilization, expected cycle time, expected WIP, average waiting time, and average number of parts waiting in the station queue. Also, compute the coefficient of variation of the inter-departure time for this workstation.

C. Extra Credit (20%)

Consider a workstation that produces a final product by fastening together two major sub-assemblies. Jobs arriving at this workstation consist of kits containing one unit from each of the two sub-assemblies, and if both parts are in good order, the fastening operation can be performed at an average time of t=2min. However, each of the two parts in a kit can also be defective, with corresponding probabilities p1=0.3 and p2=0.2. A defective part must go through some additional rework that occurs locally on the server, while the kit is in processing, and requires an *exponentially* distributed time; the corresponding processing rates are $r1=0.2\text{min}^{-1}$ and $r2=0.1\text{min}^{-1}$. Part failures are independent from each other, and in the case that both parts in a kit are defective, the necessary reworks take place simultaneously. Use the above information in order to determine the effective processing capacity of this station. Express your result in product units per hour.

Hint: You need to compute the mean rework time for each of the three cases that (i) only part 1 is defective, (ii) only part 2 is defective, and (iii) both parts are defective. The first two cases are straightforward. For the third case, consider the subcases where (a) the rework of part 1 is finished first, or (b) second. What is the rate of getting a completion when both of the two rework processes are active? What is the completion rate of the remaining rework process when one of them has finished? How should these results be combined in order to get the result that you need for case (iii) above?