ISyE 4803-REV: Advanced Manufacturing Systems Modeling and Analysis
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Homework # 3 Solutions

Problem 30

$$\begin{aligned}
\lambda &= 4 \\
C_a^2 &= 1 \ \ \textit{(Since Poisson arrivals, then exponential inter-arrival times)} \\
\mathbb{E}[T_s] &= = 0.2 \\
u &= \lambda\,\mathbb{E}[T_s] = 0.8
\end{aligned}$$

$$CT_s = \frac{(C_a^2 + C_s^2)}{2}\frac{u}{1-u}\,\mathbb{E}[T_s] + \mathbb{E}[T_s]$$

- First system ($C_s^2 = 1/2$)

$$\begin{aligned}
CT_s &= \frac{(1+1/2)}{2}\frac{0.8}{1-0.8}0.2 + 0.2 \\
&= 0.8
\end{aligned}$$

- Second system ($C_s^2 = 1$)

$$\begin{aligned}
CT_s &= \frac{(1+1)}{2}\frac{0.8}{1-0.8}0.2 + 0.2 \\
&= 1
\end{aligned}$$

- Third system ($C_s^2 = 2$)

$$\begin{aligned}
CT_s &= \frac{(1+2)}{2}\frac{0.8}{1-0.8}0.2 + 0.2 \\
&= 1.4
\end{aligned}$$

Problem 31

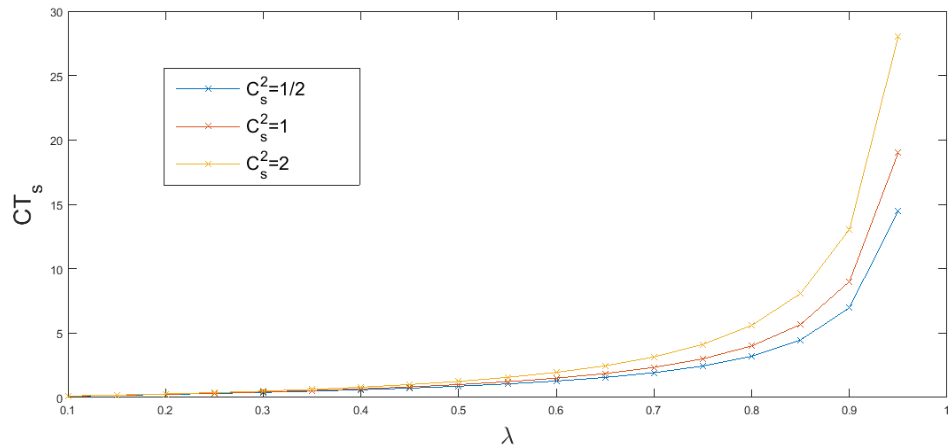

Figure 1: Problem 3.31 Solution

Problem 32

$$\begin{aligned}
\lambda &= 4 \\
C_a^2 &= 1 \quad \textit{(Since Poisson arrivals, then exponential inter-arrival times)} \\
\mathbb{E}[T_s] &= = 0.4 \\
c &= 2 \quad \textit{(2 machines)} \\
u &= \frac{\lambda\,\mathbb{E}[T_s]}{c} = 0.8
\end{aligned}$$

$$CT_s = \frac{(C_a^2 + C_s^2)}{2}\frac{u^{\sqrt{2c+2}-1}}{c(1-u)}\,\mathbb{E}[T_s] + \mathbb{E}[T_s]$$

- First system ($C_s^2 = 1/2$)

$$\begin{aligned}
CT_s &= \frac{(1+1/2)}{2}\frac{(0.8)^{\sqrt{6}-1}}{2(1-0.8)}0.4 + 0.4 \\
&= 0.9427
\end{aligned}$$

- Second system ($C_s^2 = 1$)

$$\begin{aligned}
CT_s &= \frac{(1+1)}{2}\frac{(0.8)^{\sqrt{6}-1}}{2(1-0.8)}0.4 + 0.4 \\
&= 1.1236
\end{aligned}$$

- Third system ($C_s^2 = 2$)

$$\begin{aligned} CT_s &= 1.4854 \frac{(1+2)}{2} \frac{(0.8)^{\sqrt{6}-1}}{2(1-0.8)} 0.4 + 0.4 \\ &= \end{aligned}$$

Chapter 5

Problem 1

$$C_d^2 = \left(1-u^2\right)C_a^2 + u^2 C_s^2, \quad \text{num. mach. } c=1$$

$$C_d^2 = (1-0.85^2)\times 1 + 0.85^2 \times 1.5 = 1.36125$$

Problem 2

$$C_d^2 = 1+\left(1-u^2\right)\left(C_a^2-1\right)\bullet u^2\left(1-C_s^2\right)/\sqrt{c}, \quad c>1$$

this formula simplifies to the corresponding formula in Property 5.2

$$C_d^2 = 1+(1-0.8^2)(2-1)+0.8^2\underbrace{(1/2-1)}_{}/\sqrt{2} = 1.1337$$

since service time is E_2

Problem 3

Due to the serial nature of the system: $\lambda_1=\lambda_2=\lambda_3=3$

and the fact that $u_i<1$, $\forall i$

Workstation 1:

$$u_1 = \frac{\lambda_1 E[T_1]}{c_1} = \frac{3(0.25)}{1} = 0.75$$

$$CT_s(1) = \left(\frac{1+4}{2}\right)\left(\frac{0.75}{0.25}\right)0.25 + 0.25 = 2.125$$

$$WIP_s(1) = \lambda_1 CT_s(1) = 3(2.125) = 6.375$$

$$C_d^2(1) = \left(1-u_1^2\right)C_a^2(1) + u_1^2 C_s^2(1) = (1-0.75^2)1 + 0.75^2(4) = 2.6875$$

Workstation 2:

$$u_2 = \frac{\lambda_2 E[T_2]}{c_2} = \frac{3(0.29)}{1} = 0.87$$

$$CT_s(2) = \left(\frac{2.6875+3}{2}\right)\left(\frac{0.87}{0.13}\right)0.29 + 0.29 = 5.809$$

$$WIP_s(2) = \lambda_2 CT_s(2) = 3(5.809) = 17.427$$

$$C_d^2(2) = \left(1-u_2^2\right)C_a^2(2) + u_2^2 C_s^2(2) = (1-0.87^2)2.6875 + 0.87^2(3) = 2.924$$

Workstation 3:

$$u_3 = \frac{\lambda_3 E[T_3]}{c_3} = \frac{3(0.30)}{1} = 0.90$$

$$CT_s(3) = \left(\frac{2.924+2}{2}\right)\left(\frac{0.90}{0.10}\right)0.30 + 0.30 = 6.9474$$

$$WIP_s(3) = \lambda_3 CT_s(3) = 3(6.9474) = 20.842$$

$$C_d^2(3) = \left(1-u_3^2\right)C_a^2(3) + u_3^2 C_s^2(3) = (1-0.90^2)2.924 + 0.90^2(2) = 2.1756$$

System Performance Measures

$$WIP_{sys} = WIP_s(1) + WIP_s(2) + WIP_s(3) = 44.645$$

$$Th_{sys} = \lambda_1 = 3$$

$$CT_{sys} = \frac{WIP_{sys}}{Th_{sys}} = \frac{44.645}{3} = 14.882 = CT_s(1) + CT_s(2) + CT_s(3)$$