ISYE 4803-REV: Advanced Manufacturing Systems
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Final Exam
December 11, 2019

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Explain why in a stable M/M/1 queue, the server utilization is equal to $\rho = \lambda/\mu$, where λ is the arrival rate and μ is the processing rate of the station.

$$\rho = \lambda/\mu = \lambda t_p$$

where t_p is the mean proc. time.

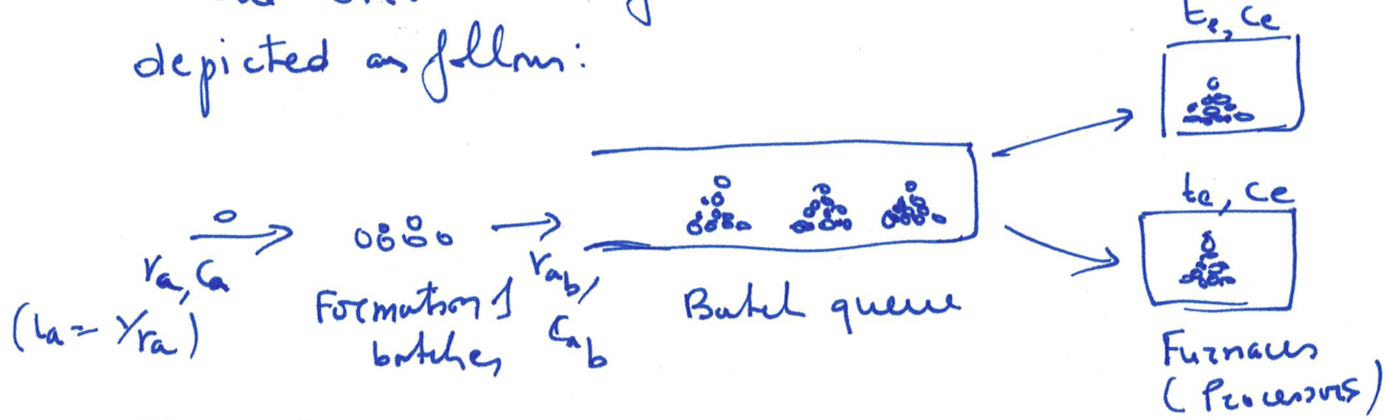
In class we explained that the last product is the average amount of work for the server that arrives per unit of time. Hence, the server must keep busy that percentage of its effective proc. time, which is the meaning of the server utilization.

2. Consider the model for parallel batching that we discussed in class, but suppose that the considered facility has two identical furnaces, instead of one. Hence, arriving parts are organized in batches of some size k, each completed batch enters a queue waiting for its processing, and when it gets at the head of the queue it is processed at one of the two furnaces. The rest of the assumptions and the notation is the same as in the model presented in class; i.e., B is the buffering capacity of each furnace in terms of number of parts; r_a is the part arrival rate in parts per hour; c_a is the CV for the part inter-arrival times; t is the mean processing time for the batches at any of the two furnaces; and c_e is the CV for the batch processing times.

Provide (i) the stability condition for this facility, and (ii) the formula for the expected cycle time CT for the parts that go through this facility, when the operation of the facility is stable.

Also, explain briefly the rationale behind the provided formulae.

The overall workflow at this workstation can be depicted as follows:



The situation is similar to the corresponding one that we analyzed in class, but now we have two identical processors instead of one. Hence:

Stability Condition: $u = \frac{r_{a_b} \cdot t_e}{2} < 1 \Rightarrow \frac{r_a}{k} t_e < 2 \Rightarrow k > \frac{r_a t_e}{2}$

Expected Cycle Time: $WTTB + CT_q + t_e =$

$$= \frac{(k-1)t_a}{2} + \frac{c_a^2/k + c_e^2}{2} \cdot \frac{u^{\sqrt{2\cdot 2+2}-1}}{2(1-u)} t_e + t_e$$

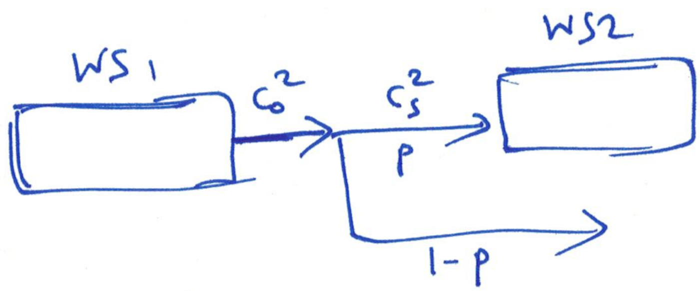
and $u = \frac{r_a t_e}{2k}$ / Notice that for CT_q we must use the corresponding expression for G/G/2 queue.

3. Consider a sub-stream of departures from a certain workstation WS_1 that is directed to another workstation WS_2. Departures from the overall stream are directed to the considered sub-stream with probability p. Also, let c_o^2 denote the SCV for the inter-departure times of the original departure stream, and c_s^2 denote the SCV of the inter-departure times for the departures in the direction of the considered sub-stream. Then, it will always hold that $c_s^2 > c_o^2$.

(a) TRUE (b) FALSE

Please, explain your answer.

The described situation is depicted as follows:



In class we showed that:

$$c_s^2 = p \cdot c_o^2 + 1 - p \qquad (1)$$

So

$$c_s^2 > c_o^2 \iff p \cdot c_o^2 + 1 - p > c_o^2 \iff$$

$$\iff (1-p) > (1-p)\, c_o^2 \Rightarrow c_o^2 < 1 \Rightarrow c_o < 1.$$

Since we can have $c_o \geq 1$, the answer is "FALSE".

To get a feeling of the above result, notice that Eq. 1 implies that $c_s^2 = p \cdot c_o^2 + (1-p) \cdot 1$, i.e., c_s^2 is the weighted average of c_o^2 and 1. So, $c_s^2 > c_o^2$ only if c_o^2 is averaged with something bigger than itself, i.e., only if $1 > c_o^2$.

4. In class we showed that Moore's algorithm is an appropriate method for minimizing the number of tardy jobs in the context of static, single-machine, due-date-based scheduling. Briefly explain how this algorithm works, and the basic logic behind the computation of this algorithm.

Please, see the coverage of Moore's algorithm in the excerpt on operational scheduling that is posted at the electronic reserves for this course.

5. In class we defined the notion of "lateness" for any given job in the context of static, single-machine, due-date-based scheduling as the completion time of this job minus its due date. For this definition of lateness, the problem of minimizing the *average lateness* across all jobs in the context of static, single-machine, due-date-based scheduling

i. is solved optimally by the EDD dispatching rule.

ii. is solved optimally by the SPT dispatching rule.

iii. is solved optimally by the LS dispatching rule.

iv. cannot be solved optimally by any simple dispatching rule.

Please, provide a brief explanation for your answer.

Let $L_i = C_i - D_i$

where L_i = lateness of job i; C_i = the job completion time and D_i = the job due date.

Then,

$$\text{Average lateness } L = \frac{1}{n}\sum_{i=1}^{n} L_i =$$

$$= \frac{1}{n}\sum_{i=1}^{n}(C_i - D_i) = \frac{1}{n}\sum_{i=1}^{n} C_i - \frac{1}{n}\sum_{i=1}^{n} D_i$$

Since $\frac{1}{n}\sum_{i=1}^{n} D_i$ is just a constant value, independent from the adopted schedule, in order to minimize L we need to minimize $\frac{1}{n}\sum_{i=1}^{n} C_i$. In class we showed that this quantity is minimized by the SPT rule (c.f. the discussion on minimizing average flowtime)

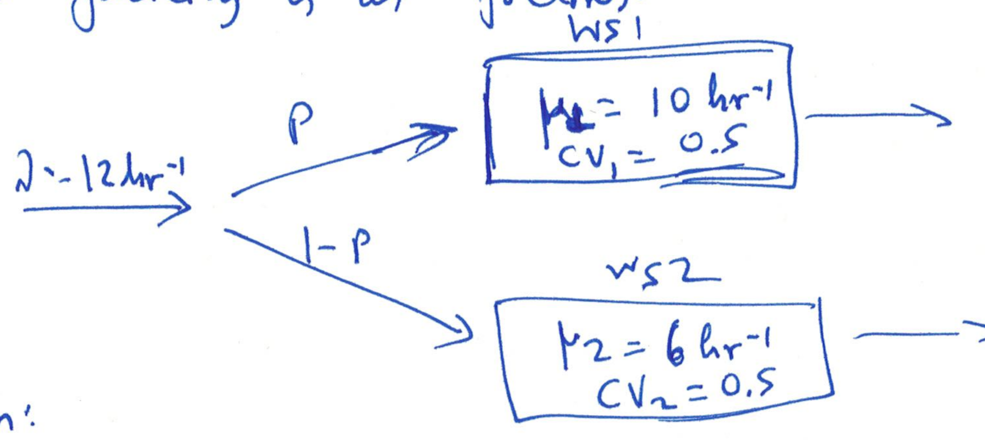
Problem 1 (20 points): Consider a facility consisting of two single-server workstations, WS_1 and WS_2, with infinite buffers, and respective processing rates $\mu_1 = 10\ hr^{-1}$ and $\mu_2 = 6\ hr^{-1}$. Jobs arrive at this facility according to a Poisson process with rate $\lambda = 12\ hr^{-1}$, and they are directed to WS_1 with probability p, and to WS_2 with the remaining probability $1 - p$.

i. (5 pts) Determine the range for p so that both workstations are stable.

ii. (5 pts) Determine the value for p that balances the workload of the two workstations (i.e., both workstations are utilized at the same level).

iii. (10 pts) Assuming that p is equal to the value that you computed in item (ii) above, and that the CV of the processing times is equal to 0.5 for both workstations, determine the expected cycle time, CT, for a job that is processed through this facility.

A schematic representation of the operation of this facility is as follows:



Then:

(i) $u_1 = \lambda/\mu_1 = \dfrac{12p}{10}$; $u_2 = \lambda/\mu_2 = \dfrac{12(1-p)}{6}$

We need

$$u_1 < 1 \wedge u_2 < 1 \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{12p}{10} < 1 \\ \dfrac{12(1-p)}{6} < 1 \end{cases} \Rightarrow \begin{cases} p < 5/6 = 0.8\overline{3} \\ p > 1/2 = 0.5 \end{cases}$$

(ii) $u_1 = u_2 \Rightarrow \dfrac{12p}{10} = \dfrac{12(1-p)}{6} \Rightarrow p = 5/8 = 0.625$

which belongs in the range specified in part (i).

Also, the resulting utilization for the two stations for this value of p is

$$u_1 = u_2 = \frac{12}{10} \cdot \frac{5}{8} = 0.75$$

(iii) We know that the two subprocesses that result from the Bernoulli splitting of the original arrival process, are also Poisson with rates $\lambda_1 = \lambda p$ and $\lambda_2 = \lambda(1-p)$.

Hence,

$$CT_1 = \frac{1+0.5^2}{2} \frac{0.75}{1-0.75} \cdot \frac{1}{10} + \frac{1}{10} = 0.2875 \text{ hrs}$$

$$CT_2 = \frac{1+0.5^2}{2} \frac{0.75}{1-0.75} \frac{1}{6} + \frac{1}{6} = 0.4792 \text{ hrs}$$

and finally

$$CT = p \cdot CT_1 + (1-p) CT_2 =$$

$$= 0.625 \times 0.2875 + (1-0.625) \times 0.4792 =$$

$$= 0.3594 \text{ hrs} \simeq 21.56 \text{ min}$$

Problem 2 (20 points): Consider a manufacturing cell with two single-server workstations, WS_1 and WS_2, where jobs arrive according to a Poisson process with rate $r_a = 10\ hr^{-1}$. The processing of these jobs starts at workstation WS_1 where they execute a first processing stage, and subsequently they move to workstation WS_2 for the execution of a second processing stage. But the execution of this second processing stage by a part at workstation WS_2 is successful only with probability 0.8. Successfully processed parts leave the cell, and this completes the processing of the corresponding job. In the opposite case, the current part is scrapped, and a new part is initiated at workstation WS_1, to replace the original one.

For this cell, please, do the following:

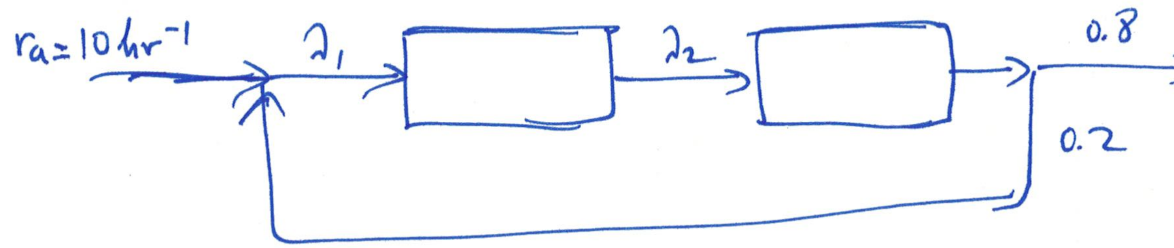
i. (5 pts) Represent the workflow taking place in this cell as a queueing network, and compute the total part arrival rate for each workstation.

ii. (5 pts) Determine the mean processing time for the server of each workstation so that each server has a utilization level of 90%.

iii. (5 pts) What is the departure rate of completed jobs from this cell under the processing times that you computed in part (ii) above? Please, explain clearly your answer.

iv. (5 pts) How many parts must be processed, on average, in order to get a job completed?

(i) Working as in the corresponding example presented in class, we have:



$$\begin{cases} \lambda_1 = 10 + 0.2\lambda_2 \\ \lambda_2 = \lambda_1 \end{cases} \Rightarrow \lambda_1 = \lambda_2 = 12.5\ hr^{-1}$$

(ii) $\lambda_1 t_1 = 0.9 \Rightarrow t_1 = 0.9/\lambda_1 = 0.9/12.5 = 0.072$ hrs

Similarly, $t_2 = 0.072$ hrs.

(iii) Since all jobs are completed, even if some initial attempts for each of these jobs are scrapped, and the system is stable, it must be that the corresponding rate is equal to $r_a = 10\ hr^{-1}$.

Alternatively, we can see that good jobs leave the cell with a rate of $0.8 \times \lambda_2 = 0.8 \times 12.5 = 10\ hr^{-1}$.

(iv) Each newly initiated part for some given job is a Bernoulli trial with success probability 0.8.

Hence, the expected number of trials till success is equal to $\frac{1}{0.8} = 1.25$.

Problem 3 (20 points): A consultant has fallen behind with the due dates of her five current projects, P_i, $i = 1, \ldots, 5$. Furthermore, given the current status of each of these projects, she estimates that the necessary amount of work for the completion of each of them is, respectively, 5, 2, 3, 7 and 5 workdays. According to the corresponding contracts that have been signed for these projects, the consultant is charged the rates of \$800, \$500, \$300, \$400 and \$600, for every extra workday past the project due date. Determine a schedule that she should follow in her work on these five projects in order to minimize the total penalties that she will have to pay for all these projects.

Let the current time point be time $\emptyset$, and C_i denote the completion of project P_i with respect to this time point. Also, let t_i denote the remaining processing time (in workdays) for project P_i, and W_i denote the daily penalty for project P_i (in \$100). Then, from the project description, we need a nonidling schedule that will minimize the total penalties accrued from now till the completion of all five projects; i.e the proper objective is $\min \sum_{i=1}^{5} W_i C_i$.

As discussed in the last lecture, the optimal rule for this objective is weighted SPT, i.e., we must schedule the five projects in increasing order of t_i / W_i. To determine such a schedule we can tabulate our work as follows:

P_i	1	2	3	4	5
t_i	5	2	3	7	5
w_i	8	5	3	4	6
t_i/w_i	0.625	0.4	1	1.75	0.833

So, the optimal schedule is:

$$\langle 2, 1, 5, 3, 4 \rangle$$