# ISYE 4803-REV: Advanced Manufacturing Systems
Instructor: Spyros Reveliotis
Midterm Exam II
November 7, 2018

Name: SOLUTIONS

Answer the following questions (**8 points each**):

**1.** Write down the formula for the expected cycle time in the queue, $CT_q$, for a stable M/G/1 queueing station. Also, discuss whether your formula is an exact estimate of this quantity or an approximation.

The requested formula is

$$CT_q = \frac{1+c_e^2}{2}\,\frac{u}{1-u}\,t$$

which can be perceived as the formula obtained by the corresponding formula for the G/G/1 queue by setting $Ca^2 = 1$, since, in this case, the inter-arrival times are exponentially distributed.

For the case of the M/G/1 queue, the above formula gives an exact estimate of $CT_q$, and it is part of the results that concern the MVA of the M/G/1 queueing station and are known as the Pollaçek-Kintchine formulae. We derived these results in class as an intermediate step towards Kingman's approximation for the G/G/1 queueing station.

**2.** Characterize the variability of the inter-departure times of an M/M/1 queueing station. Please, show clearly your entire argument and any computations involved in it.

As explained in class, the variability of a r.v. is characterized by the CV of the corresponding distribution.

Also, in class we saw that for G/G/1 queueing stations,

$$c_d^2 = (1-u^2)\,c_a^2 + u^2 c_p^2$$

For M/M/1 stations, $c_a^2 = c_p^2 = 1$ since the corresponding distributions are exponential.

Therefore, in this case,

$$c_d^2 = 1-u^2+u^2 = 1 \Rightarrow c_d = 1.$$

When I discussed this last result in class, I also told you that we can show the stronger result that the inter-departure times for a stable M/M/1 station are exponentially distributed with rate $r_d = r_a$ (i.e., the arrival rate).

**3.** Consider a G/G/1 queueing station experiencing non-destructive preemptive outages of the type that we discussed in class. The station is currently unable to deliver its target throughput $\overline{TH}$. Suggest **three** different (practical) approaches that can be pursued in an effort to establish a stable operation for this station without compromising the target throughput $\overline{TH}$.

Since the considered station is ustable, we have $u = \overline{TH}\, t_e = \overline{TH}\, t_P/A \geq 1$.
To stabilize this station, without compromising the target throughput $\overline{TH}$, we must reduce the factor $t_P/A$ in the above expression to the point that the new u-value drops below 1.
Two ways to effect such a reduction are

(i) either by reducing $t_P$ (or, equivalently, increasing the proc. rate $r_P$ of the station server),

(ii) or increasing the server availability A (This last effect can be attained either by increasing the mean time to failure, MTTF, or decreasing the mean time to repair, MTTR).

(iii) A third approach to reduce u is by adding one or more ~~another~~ servers, which will lead to a new server utilization $u' = u/k$, where k is the total number of servers in the new configuration.

**4.** In class we showed that the squared coefficient of variation, $SCV_b$, of the batch processing times for a batching scheme with (i) a batch size of $k$ parts and (ii) mutual independence among the processing times of the different parts in the batch, is equal to $SCV_p/k$, where $SCV_p$ is the squared coefficient of variation for the part processing times. Provide an intuitive explanation for this result.

We have $T_b = T_1 + T_2 + \dots + T_k$

where r.v. $T_b$ is the batch processing time, and the r.v.'s $T_i$, $i=1,\dots,k$, are the proc. times for each part in the batch.

Since $T_i$'s are iid, each of them will be above or below the corresponding mean, $E[T_i]$, independently of the fluctuation of the remaining r.v.'s with respect to their mean. But then, in the final sum $\sum_i T_i$, each fluctuation with opposite sign will tend to cancel each other, bringing the value of this sum closer to its mean $k E[T_1]$.

This cancelling effect occurs more generally in sums of iid r.v.'s, it is known as the "<u>pooling effect</u>", and it is recognized as an effective / useful mechanism for controlling the uncertainty in the r.v.'s that are defined by the aforementioned sums.

**5.** Consider a single-server queueing station processing two job types, $J_1$ and $J_2$. Each job type arrives according to a Poisson distribution with corresponding arrival rate $\lambda_i$, $i = 1, 2$. Also, processing times of type $J_i$ are distributed according to some general distribution $G_i$ with mean $t_i$ and CV $c_i$.

Finally, job type 1 has *preemptive priority* over job type 2; i.e., the station maintains a distinct queue for each job type, and the server allocation to the two job types observes the following two rules:

**Rule 1** When the server completes its current job, it will draw its next job from the type 1 queue if there are any jobs present in this queue.

**Rule 2** Whenever the station server works on a type 2 job and a type 1 job shows up, the server will send back its current job to the head of the queue for type 2 jobs, and it will shift its attention to the new arrival. Furthermore, the preempted job will resume its processing (only) when the system is clear again of type 1 jobs.

Provide the stability condition for this queueing station.

Also, assuming that the system is stable, provide a formula for the expected cycle time, $CT_1$, for type 1 jobs.

The utilization of the station server with respect to each of these two job types will be

$u_1 = \lambda_1 t_1$ and $u_2 = \lambda_2 t_2$.

Obviously, we need $u = u_1 + u_2 = \lambda_1 t_1 + \lambda_2 t_2 < 1$.
This is the required stability condition for this station.
To compute $CT_1$, it suffices to notice that the preemptive priority of job type 1 over job type 2 that is described in this problem, implies that the jobs of type 1 experience this station as if it was totally dedicated to them. Hence, assuming that $u_1 = \lambda_1 t_1 < 1$, we shall have:

$$CT_1 \approx \frac{1 + c_1^2}{2} \frac{u_1}{1 - u_1} t_1 + t_1$$

In the above expression, we recognize that the arrival process of job type 1 is Poisson.

**Problem 1 (30 points):** Consider a single-server station where the parts to be processed arrive in *kits* of 10 parts per kit. Kits arrive according to a Poisson process with a rate of 4 kits per hour. Each kit is processed *serially* by the station server, one part at a time, with the *part* processing time being equal to 1 minute per part and the coefficient of variation for these processing times being 0.5. Parts in a kit leave the server only when the entire kit has been processed. Furthermore, the server experiences preemptive outages with normally distributed downtimes with mean 5 minutes and st. deviation 2 minutes, while the time between two consecutive outages is exponentially distributed with a mean of 30 minutes.

Use the above information in order to perform the following:

i. (10 pts) Show that the station operation under the considered regime is stable.

ii. (10 pts) Compute the average cycle time of a part going through this station.

iii. (10 pts) Compute the average number of *parts* waiting for processing at this station.

Please, see Problem 1 in the 2016 Midterm II Exam that is posted at the course website.

**Problem 2 (30 points):** Consider a queueing station where parts arrive according to a Poisson process with rate $r_a = 10$ parts per hour, and, upon arrival, they are classified as type 1 and type 2 with corresponding probabilities equal to 0.5. Both part types are joining a single infinite-capacity queue, and they are processed according to a FCFS policy. Processing times for parts of type 1 are deterministically equal to 7 minutes and processing times for parts of type 2 are deterministically equal to 4 minutes.

Answer the following questions.

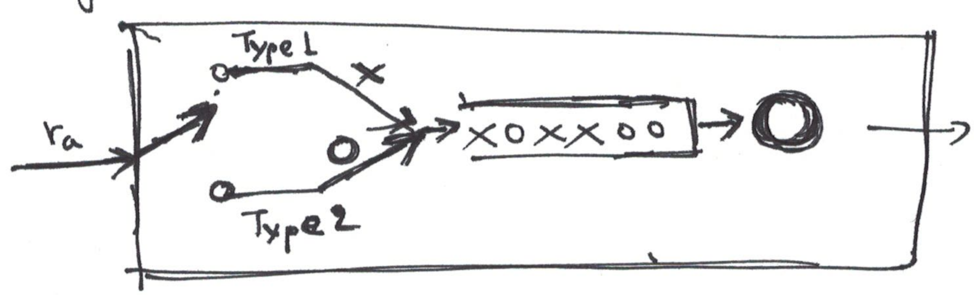
i. (5 pts) Show that the operation of this queueing station is stable.

ii. (10 pts) Define the distribution of the processing times that are experienced at the server of this workstation, treating jobs in both both part types as a single stream of jobs, and compute the mean, the variance and the CV of this distribution.

iii. (10 pts) Compute the expected waiting time in the queue for the parts going through this workstation, and the average number of parts in this queue, when the station is operated in steady state.

iv. (5 pts) Compute the expected cycle time at this station for each part type.

A schematic describing the operation of this queue is as follows




Each new arrival eventually reaches the queue as type 1 (X) or type 2 (O) job, with prob. 1/2.
Furthermore, since jobs are processed from this queue on a FCFS basis, each time the server picks a job for processing, it can be type 1 or type 2 with corresponding prob. 0.5.
Hence, for the stability and the MVA of this station,

We can treat these two job types as a single job type with the following distribution of proc. times:

$$T \sim \begin{cases} 7 & \text{w.p. } 0.5 \\ 4 & \text{w.p. } 0.5 \end{cases}$$

Then $E[T] = 7 \times 0.5 + 4 \times 0.5 = 5.5$ min

Also,

$$E[T^2] = 7^2 \times 0.5 + 4^2 \times 0.5 = 32.5 \text{ min}^2$$

$$Var[T] = E[T^2] - E^2[T] = 32.5 - 5.5^2 = 2.25 \text{ min}^2$$

$$SCV[T] = Var[T] / E^2[T] = 2.25 / 5.5^2 = 0.07438$$

$$CV[T] = \sqrt{SCV[T]} = \sqrt{0.07438} = 0.273.$$

Furthermore,

$$u = r_a \times E[T] = \frac{10}{60} \times 5.5 = \frac{55}{60} = 0.917 < 1 \Rightarrow \underline{\text{stable}}.$$

$$CT_q = \frac{1 + SCV[T]}{2} \frac{u}{1-u} E[T] = \frac{1+0.07438}{2} \frac{0.917}{1-0.917} \cdot 5.5 =$$

$$= 32.64 \text{ min}$$

$$WIP_q = r_a \cdot CT_q = \frac{10}{60} \times 32.64 = 5.44$$

$$CT_1 = CT_q + t_1 = 32.64 + 7 = 39.64 \text{ min}.$$

$$CT_2 = CT_q + t_2 = 32.64 + 4 = 36.64 \text{ min}.$$