ISYE 4803-REV: Advanced Manufacturing Systems
Instructor: Spyros Reveliotis
Midterm Exam III
November 29, 2017

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. Discuss how the strategic concern of "responsiveness" is addressed by the typical specifications that are observed during the design of a production line in contemporary high-volume (also known as "repetitive") manufacturing.

This question intended to ask how "responsiveness" is addressed through the specifications that are considered during the design of a high-volume production line by means of the queueing theoretic tools that we have studied in this class.

The main specifications considered in the corresponding design process are: (i) the line maximal throughput, and (ii) the tolerated congestion, usually expressed in terms of an expected cycle time.

Clearly, both of these measures relate to the "responsiveness" of the line, since the first one determines how fast the line can produce, and the second one determines how fast any single job will go through this line.

2. Consider an arrival stream that results from the superposition of n Poisson arrival streams with rates λ_i, $i = 1, \ldots, n$ (i.e., by the merging of all these n streams into one arrival stream). The memoryless property of these n streams implies that the probability that the next arrival will occur through the i-th stream, is equal to $p_i = 1/n$, $\forall i$.

(a) YES (b) NO

Please, explain your answer.

When we were discussing the theory of Markovian queueing stations, I had pointed out that the aforementioned probability is equal to

$$\frac{\lambda_i}{\lambda_1+\lambda_2+\cdots+\lambda_n}$$

This is a result that you should have also seen in ISyE 3232, when you covered the properties of the exponential distribution and the Poisson process.

From a more intuitive standpoint, the explanation of the above result is as follows:

While the Poisson nature of each arrival stream, and the corresponding memoryless property, imply that we cannot predict anything about the time of the next arrival based on the knowledge of the time of the last arrival, we can still expect that the next part will come from a stream with higher arrival rate with a higher probability. In fact the above formula implies that these probabilities are <u>proportional</u> to the stream arrival rates.

3. Explain how the assumption of an "infinite buffering capacity" that is used in many (actually most of the) models that have been studied by queueing theory, is justified by the operational realities of the target applications.

This assumption is practically justified by the following two realities:

(i) Usually, the buffer sizes at the different workstations are set to levels that are considerably higher that the expected WIP level at that workstation (under stable operation), so that the probability that the actual WIP will reach the buffer capacity is rather small.

(ii) And in the (rare) cases where such an overflow might happen, the personnel who are in charge of this workstation will try to accommodate the excess WIP by "improvising" some extra buffering capacity (e.g., temporarily storing this material somewhere else, trying to increase the density of the available storage area, etc.)

Of course, all the above will work only if the line is operated within its effective processing capacity.

4. Briefly discuss the primary reason(s) that define a need for *serial* batching in a manufacturing operation.

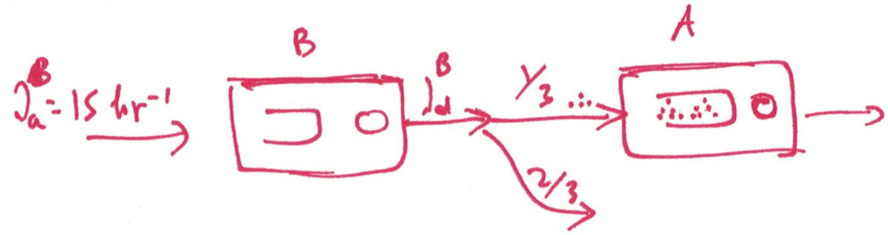
Serial batching is used primarily in an effort to control the capacity loss due to the set-ups that might be necessary in shared facilities like manufacturing cells.

Even when the system is stable without any batching serial batching might still occur in an effort to control the relevant set-up losses from a more economic standpoint (e.g., there might be considerable operational costs incurred by the execution of a setup). In such a case, the batch size is determined by an EOQ-type of analysis where the set up cost defines the "fixed" cost in the underlying operation.

Finally, serial batching at certain workstations might also result from a need to introduce "transfer" batches in an effort to control the overall material handling effort.

5. Consider a workstation, A, that processes its arriving parts in parallel batches of 15 parts per batch. Parts are directed to this workstation from an upstream workstation, B. Parts arrive at workstation B at a rate of 15 hr^{-1}, and when they leave this workstation they are directed to workstation A with probability of 1/3. Compute the *expected* **batch** *inter-arrival time* for workstation A.

The above situation can be depicted as follows:



Assuming a stable operation for this arrangement, we have:

$$\lambda_a^A = \tfrac{1}{3}\lambda_d^B = \tfrac{1}{3}\lambda_a^B = \frac{1}{3}\,15\ hr^{-1} = 5\ hr^{-1}$$

Hence, the mean part inter-arrival time at WS A is:

$$t_a^A = 1/\lambda_a^A = \frac{1}{5\ hr^{-1}} = \frac{1}{5}\ hr = 12\ min.$$

And since I need 15 parts for a batch, the mean batch inter-arrival time at this WS is:

$$t_{a_b}^A = 15 \times t_a^A = 15 \times 12\ min = 180\ min = 3\ hr.$$

Problem 1 (30 points): Consider a furnace that processes two different part types, A and B. The buffering capacity of the furnace is 200 parts, and each part type is organized into batches of some size k (the batch size is common for both part types). The batches of each part type are entering a common queue and they are processed on a first-come-first-serve basis. Batch processing times for the two part types are deterministic, and they are 10 minutes for part type A and 15 minutes for part type B. On the other hand, the arrival stream of each part type at this workstation is a Poisson process with rate of 10 hr^{-1} for part A and 5 hr^{-1} for part B.

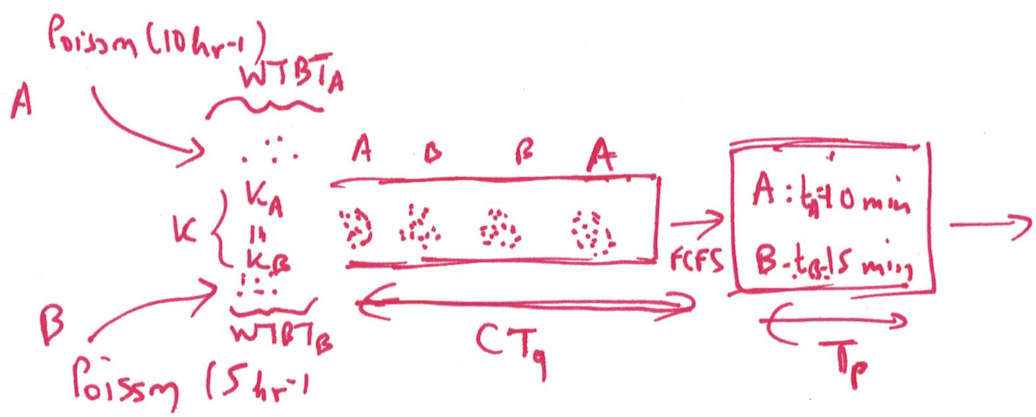
i. (10 pts) Compute the minimal value of the batch size k that will guarantee a stable operation for this workstation.

ii. (5 pts) Compute the server utilization when the employed batch size is equal to the full buffering capacity of the furnace. Also, break down this utilization in terms of each part type.

iii. (10 pts) Compute the expected cycle time for each part type at this workstation when the batch size is set as in part (ii) above. Also, provide the expected cycle time for any single part, irrespective of its type.

iv. (5 pts) What are the WIP levels for each part type at this workstation?

The workflow at this workstation can be depicted as follows:



For any given batch size k,

(i) The total expected workload arriving at the queue of this workstation at each time unit is:

$\frac{\lambda_A}{k} t_A + \frac{\lambda_B}{k} t_B$, and since we have only one server, we must have:

$$\frac{\lambda_A}{k} t_A + \frac{\lambda_B}{k} t_B < 1 \Rightarrow k > \lambda_A t_A + \lambda_B t_B = 10\,hr^{-1} \times \frac{10}{60}\,hr + 5\,hr^{-1} \times \frac{15}{60}\,hr = 2.91 \Rightarrow \underline{k_{min} = 3}.$$

(ii) If $k = 200$, then:

$$\left.\begin{aligned} u_A &= \frac{10 \times 10/60}{200} = 0.008\overline{3} \\ u_B &= \frac{5 \times 15/60}{200} = 0.00625 \end{aligned}\right\} \Rightarrow u = u_A + u_B = 0.01455.$$

(iii) For each part type $x = A, B$, we shall have:

$$CT_x = WTBT_x + CT_q + t_x$$

$$WTBT_x = \frac{k-1}{2\lambda_x} \Rightarrow \left\{ \begin{aligned} WTBT_A &= \frac{199}{2 \times 10} = 9.95\ hr \\ WTBT_B &= \frac{199}{2 \times 5} = 19.9\ hr \end{aligned} \right.$$

Also, $t_x = \left\{ \begin{aligned} t_A &= 10/60 = 1/6\ hr \\ t_B &= 15/60 = 1/4\ hr \end{aligned} \right.$

To get CT_q, we work as follows: Since batches are processed on a FCFS basis, the expected waiting time of any batch is that of a $G/G/1$ queue where the single server mixes the corresponding processing times with the corresponding probabilities p_A and p_B of the type of ea

But $p_{x} = \dfrac{\lambda_x}{\lambda_A + \lambda_B} \Rightarrow \begin{cases} p_A = \dfrac{10}{15} = 2/3 \\ p_B = \dfrac{5}{15} = 1/3 \end{cases}$ and

$$E[T_p] = \frac{2}{3} \times 10 + \frac{1}{3} \times 15 = \frac{25}{3} \simeq 11.67 \text{ min}$$

$$Var[T_p] = \frac{2}{3}(10 - 11.67)^2 + \frac{1}{3}(15 - 11.67)^2 = 5.556$$

$$SCV[T_p] = \frac{Var[T_p]}{E[T_p]^2} = \frac{5.556}{11.67^2} \simeq 0.041$$

Also, since the two arrival streams are Poisson, the SCV for the batch inter-arrival times for each stream is equal to $1/k = \frac{1}{200}$, and therefore, the SCV for the inter-arrival times of the comingled stream is also $1/200$ (more formally $\dfrac{\lambda_A/k}{\lambda_A/k + \lambda_B/k}\dfrac{1}{k} + \dfrac{\lambda_B/k}{\lambda_A/k + \lambda_B/k}\dfrac{1}{k} = \dfrac{1}{k}$)

Hence, $CT_q = \dfrac{1/200 + 0.041}{2}\,\dfrac{0.01455}{1 - 0.01455}\,11.67 = 0.004 \text{ min} \simeq \emptyset$, and

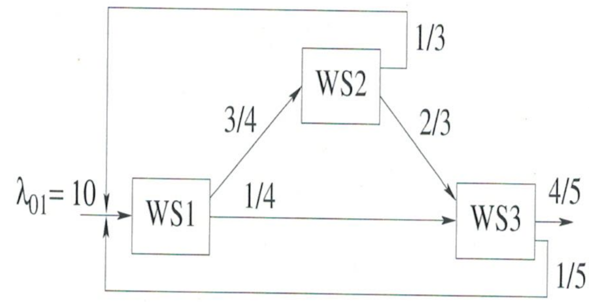
$CT_A = 9.95\,hr + \emptyset + 1/6\,hr \simeq 10.12\,hr$; $CT_B = 19.9\,hr + 1/4\,hr \simeq 20.15\,hr$

Finally $CT = p_A\,CT_A + p_B\,CT_B = \frac{2}{3} \times 10.12 + \frac{1}{3} \times 20.15 \simeq 13.46\,hr.$

(iv) $WIP_A = \lambda_A \cdot CT_A = 10\,hr^{-1} \times 10.12\,hr = 101.2$

$WIP_B = \lambda_B \cdot CT_B = 5\,hr^{-1} \times 20.15\,hr = 100.75$

Problem 2 (30 points): Consider a manufacturing cell of three workstations, where the arriving jobs circulate among these workstations as indicated in the following figure.



For this cell, please, do the following:

i. (10 pts) Compute the average arrival rate for each workstation.

ii. (5 pts) Assuming that each workstation has only one server, determine the mean processing time for each of these servers so that each workstation has a utilization level of 90%.

iii. (5 pts) What is the departure rate from this cell under the processing times that you computed in part (ii) above?

iv. (5 pts) Write down the equation that expresses the SCV of the inter-arrival times at workstation $WS3$ in terms of the SCV of the inter-departure times at the two workstations that feed workstation $WS3$. Provide numerical values for all the coefficients of this equation.

v. (5 pts) Compute the expected number of visits at each of the three workstations for each job that goes through this cell.

Hint: For the last question above, try to use the information that is provided in your response to the first question.

(i).
$$\begin{cases} \lambda_1 = 10 + \frac{1}{3}\lambda_2 + \frac{1}{5}\lambda_3 \\ \lambda_2 = \frac{3}{4}\lambda_1 \\ \lambda_3 = \frac{1}{4}\lambda_1 + \frac{2}{3}\lambda_2 \end{cases} \Rightarrow \ldots \begin{cases} \lambda_1 = 50/3 \simeq 16.67 \\ \lambda_2 = \lambda_3 = \frac{25}{2} = 12.5 \end{cases}$$

(ii) $u_1 = \frac{50}{3} t_1 = 0.9 \Rightarrow t_1 = \frac{0.9 \times 3}{50} = 0.054$

$u_2 = \frac{25}{2} t_2 = 0.9 \Rightarrow t_2 = \frac{2 \times 0.9}{25} = 0.072 = t_3$

(iii) Since each workstation has a utilization of 0.9, the entire facility is stable, and therefore $\lambda_d = \lambda_a = 10$

(iv) We have:

$$C_a^2(3) = \frac{2/3\,\lambda_2}{2/3\,\lambda_2 + 1/4\,\lambda_1} C_d^2(2,3) + \frac{1/4\,\lambda_1}{2/3\,\lambda_2 + 1/4\,\lambda_1} C_d^2(1,3)$$

$$\frac{1/4\,\lambda_1}{2/3\,\lambda_2 + 1/4\,\lambda_1} = \frac{\frac{1}{\cancel{4}\,2}\frac{\cancel{50}\,25}{3}}{\frac{2}{3}\frac{25}{2} + \frac{1}{4}\frac{\cancel{50}\,25}{\cancel{3}\,2}} = \frac{1}{3}$$

$C_d^2(1,3) = 1/4\, C_d^2(1) + 3/4$; $C_d^2(2,3) = \frac{2}{3} C_d^2(2) + 1/3$

Putting everything together, we get:

$$C_a^2(3) = \frac{2}{3}\left[\frac{2}{3} C_d^2(2) + 1/3\right] + 1/3\left[1/4\, C_d^2(1) + 3/4\right] =$$

$$= \frac{4}{9} C_d^2(2) + 2/9 + 1/12\, C_d^2(1) + 3/12 =$$

$$= 1/12\, C_d^2(1) + 4/9\, C_d^2(2) + 17/36$$

(v) Let V_i, $i=1,2,3$, denote the corresponding values for the three workstations. Then,

$$V_1 = \lambda_1/\lambda_a = \frac{50/3}{10} = 5/3.$$

$$V_2 = \frac{\lambda_2}{\lambda_a} = \frac{25/2}{10} = \frac{25}{20} = \frac{5}{4} = V_3$$

To understand the above computation, just notice that, at the end, it is the external arrivals that generate all the internal traffic among the system workstations.