ISYE 4803-REV: Advanced Manufacturing Systems
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Midterm Exam II
November 6, 2017

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. A recent IE graduate was approached by a company that asked her to design for them a single-server workstation that will deliver a throughput of 10 parts per hour while maintaining an average waiting time of 4 hours for the processed parts and an average WIP for the station buffer of 30 parts. In her enthusiasm as a novice engineer, she rushed to agree, but now she might be in trouble.

(i) TRUE (ii) FALSE

Please, explain your answer.

We know that for any stable workstation, TH, CT_q and WIP_q are related through Little's law: $WIP_q = TH \cdot CT_q$

But the reported specifications do not satisfy this equation.

More generally, this design problem is "over-specified". As we said in class, typically such a design specification will include (i) the maximum throughput (i.e., the proc. capacity) of this workstation, and (ii) either CT_q or WIP_q (or also CT or WIP) as a measure of the tolerated congestion ~~and~~ when the station is operated at capacity. Then, the workstation dynamics that are described by Little's law will determine the third number

2. In class we presented a model for extending the Mean Value Analysis for the basic G/G/m queueing station to stations that might experience non-destructive preemptive outages during the processing of the parts that are going through them. The development of this model was based on a set of assumptions regarding the experienced outages. Briefly discuss how these assumptions impact the practical applicability of the developed model.

As discussed in class, the most restrictive feature of this model in terms of its applicability is the assumption of exponentially distributed times between two consecutive failures.

Of course, we assume that we are dealing with preemptive, nondestructive outages; if this is not the case, then of course we need to develop a different model.

Some of you mentioned that even though the outages are nondestructive, they might still impact the processing of the part, incurring the need for some rework on it before its proper processing can be resumed. But any such "rework" time can be factored in the "downtime" associated with this outage (remember, that the statistical model for these downtimes is very general).

Some others mentioned the need to compute the necessary data/parameters for this model. But this is the case for any model.

3. Consider an M/G/1 workstation where the arriving jobs are of two different types, I and II, with respective probabilities p and 1-p. Jobs of type I (resp., II) have a mean processing time equal to t_1 (resp., t_2). The total arrival rate for this arrival process is λ. Write down the stability condition for this workstation.

In this case, the expected proc. time across both job types is $t = p t_1 + (1-p) t_2$. Hence, the stability condition becomes:

$$u = \lambda t = \lambda [p t_1 + (1-p) t_2] < 1.$$

* While I provide the requested analysis below, I want to add that some of you noticed that the configuration with two servers will be more robust to potential failures of these servers in the sense that any such failure will not stop completely the production of this workstation. This is a more qualitative argument, but it definitely merits consideration in the final decision.

* It is also interesting to notice that in both of the provided expressions, the factor $(1-u)$ in the denominator of CT_q now has become $(2-u)$!

4. Consider a stable M/G/1 station with arrival rate λ, mean processing time t_p and CV for the processing times equal to c_p. We want to double the processing capacity of this station, and for this, we consider two different options: (i) Option 1: Replace the current server with another one that has the same level of variability in its processing times, but the mean processing time for this new server is half the mean processing time of the original server. (ii) Option 2: Buy a second server of the same type with the current one.

Assuming that (i) both of the above options are technologically viable, and (ii) they have comparable deployment costs, discuss how we can use the Mean Value Analysis of stable G/G/m queueing stations in order to choose between them. Try to detail your response as much as possible.

This characterizes / explains quantitatively the drastic reduction of CT_q when adding capacity at a station.

This situation can be analyzed as follows:

(i) For option 1, the new parameters will be λ, $t_p/2$, c_p and 1 server. Hence, $u^{(i)} = \frac{\lambda t_p}{2} = \frac{u}{2}$ (as expected)

and
$$CT^{(i)} = \frac{1+c_p^2}{2}\,\frac{u/2}{1-u/2}\,\frac{t_p}{2} + \frac{t_p}{2} = \left[\frac{1+c_p^2}{2}\,\frac{u}{2-u} + 1\right]\frac{t_p}{2}$$

(ii) For option 2, the system parameters are λ, t_p, c_p but 2 servers.

Hence, $u^{(ii)} = \frac{\lambda t_p}{2} = \frac{u}{2}$ (again, consistent with our intention)

but
$$CT^{(ii)} = \frac{1+c_p^2}{2}\,\frac{(u/2)^{\sqrt{2\cdot 2+2}-1}}{2(1-u/2)}\,t_p + t_p =$$
$$= \frac{1+c_p^2}{2}\,\frac{u^{\sqrt{6}-1}}{2^{\sqrt{6}-1}(2-u)}\,t_p + t_p$$

Then, we can use $CT^{(i)}$ and $CT^{(ii)}$ to see which configuration results in less congestion. I don't think that we can establish a clear dominance of one over the other; i.e., the new configuration giving the smallest CT value, among these two, will be case-specific.

5. During the discussion of the example on the design of asynchronous transfer lines, we showed that if the processing times for the parts in a batch are independent and identically distributed, then the SCV for the batch processing time is equal to the SCV for the part processing times divided by the batch size B; i.e.,

$$SCV[\text{batch proc. time}] = SCV[\text{part proc. time}]\ /\ B.$$

Provide an **intuitive** justification for this result.

The batch processing time is a r.v. T_b that can be written as $T_b = T_1 + T_2 + \dots + T_B$, where T_i, $i=1,\dots,B$ are the r.v.'s representing the proc. times for each part in the batch. If these r.v.'s are iid, then, in any single batch some of them will above the corresponding mean value and some others will be below, and therefore the ~~sum~~ $T_1 + T_2 + \dots + T_B$ will tend to keep closer to the mean $E[T_i] \cdot B$. Hence the variability of T_b will tend to be smaller than the variability of the individual r.v.'s T_i.

The above remark holds true for r.v. X that is the sum of a number of iid r.v.'s X_i, and it is known as a "pooling" effect that mitigates the relative uncertainty in X compared to the corresponding uncertainty in each component r.v. X_i. But this "pooling" effect attenuates as the component r.v.'s X_i become more positively correlated.

Problem 1 (30 points): Consider a G/G/1 queueing station with an arrival rate $\lambda = 10$ jobs per hour, CV for the inter-arrival times, $c_a = 0.7$, mean processing time $t_p = 4$ min, and st. deviation for the processing times $\sigma_p = 2$ min. However, the operator who runs the server of this workstation is also responsible for another operation of this shop-floor, and therefore, the server is not available 100%. The disruptive events that take away the operator from the considered workstation are randomly generated, according to a Poisson process. Furthermore, every time that the operator walks away from the workstation, she stays away for a random time period that has a mean of 30 minutes and a st. deviation of 10 minutes. If there is a part in processing when the operator walks away, the processing of this part is resumed upon the return of the operator.

1. (10 pts) What is the minimum availability of the operator at this workstation that will render the workstation stable?
2. (20 pts) Perform a MVA for this station, assuming that the operator is actually available at this workstation 80% of the time. In particular, compute the server utilization, the mean cycle time in the queue, the total expected cycle time in the system, and the corresponding WIP levels for the waiting queue and for the entire system. Also, please, notice that in this analysis, you need to consider the statistics of the *effective* processing times, T_e, that account for the operator unavailability; these statistics can be obtained from the provided information.

1. We need: $\frac{\lambda t_p}{A} < 1 \Rightarrow A > \lambda t_p = 10\,hr^{-1} \times \frac{4}{60}\,hr = \frac{2}{3}$

2. For $A = 0.8$:

$$u = \frac{\lambda t_p}{A} = \frac{10 \times 4/60}{0.8} = 0.8\bar{3} < 1$$

$$t_e = t_p/A = 4/0.8 = 5\text{min}.$$

$$c_e^2 = c_p^2 + (1+c_r^2)A(1-A)\frac{m_r}{t_p} = \left(\frac{2}{4}\right)^2 + \left[1+\left(\frac{10}{30}\right)^2\right]0.8 \times 0.2 \times \frac{30}{4} =$$

$$= 1.58\bar{3}$$

$$CT_q = \frac{c_a^2 + c_e^2}{2}\,\frac{u}{1-u}\,t_e = \frac{0.7^2 + 1.583}{2}\,\frac{0.833}{1-0.833}\,5 = 25.85\text{min}$$

$$CT = CT_q + t_e = 25.85 + 5 = 30.85 \text{ min}$$

$$WIP_q = \lambda \cdot CT_q = \frac{10}{60} \times 25.85 = 4.31$$

$$WIP = WIP_q + u = 4.31 + 0.833 = 5.143$$

Problem 2 (30 points): Consider an M/G/1 workstation where jobs arrive at a rate of 10 parts per hour, and where the processing of the arriving jobs takes place in two consecutive processing stages (without the jobs leaving the server between the two stages). The pairs of the mean and the CV of the processing time distribution of each stage are, respectively, (2min, 0.8) and (3min, 1.1).

1. (5 pts) Show that the operation of this workstation is stable.
2. (5 pts) Compute the SCV for the effective processing time of this workstation.
3. (5 pts) Compute the expected waiting time for the jobs going through this workstation.
4. (5 pts) Compute the average WIP at this workstation.
5. (5 pts) Compute the mean for the job inter-departure times.
6. (5 pts) Compute the SCV for the job inter-departure times.

1. $u = \lambda(t_1 + t_2) = 10\left(\frac{2}{60} + \frac{3}{60}\right) = \frac{5}{6} = 0.8\bar{3} < 1.$

2. $\left.\begin{aligned} \sigma_1 &= c_1 t_1 = 0.8 \times 2 = 1.6\text{min} \\ \sigma_2 &= c_2 t_2 = 1.1 \times 3 = 3.3\text{min} \end{aligned}\right\} \Rightarrow \sigma^2 = \sigma_1^2 + \sigma_2^2 = 1.6^2 + 3.3^2 = 13.45\text{min}^2$

$\Rightarrow c^2 = \frac{\sigma^2}{t^2} = \frac{13.45}{(2+3)^2} = 0.538.$

3. $CT_q = \frac{1+0.538}{2} \frac{5/6}{1-5/6} \cdot 5 = 19.225 \text{ min}$

4. $WIP = \lambda \cdot CT_q + u = 10 \times \frac{19.225}{60} + \frac{5}{6} = 4.0375$

5. $t_d = 1/\lambda_d = 1/\lambda_a = \frac{1}{10} \text{hr} = 6 \text{ min}$

6. $c_d^2 = (1-u^2) c_a^2 + u^2 c_p^2 = (1-0.833^2) \cdot 1 + 0.833^2 \cdot 0.538 \simeq 0.68$