ISYE 4803-REV: Advanced Manufacturing Systems
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Midterm Exam I
October 4, 2017

Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. What is the basic definition of *quality* for any product or service that is offered by modern companies?

The quality of some product or service is defined by the extent that it meets the customer requirements / expectations. From a more engineering standpoint, the latter are defined by a set of specifications that the considered product or service must adhere to.

2. From an operational standpoint, the U-shaped layout is a more efficient layout than the S(serpentine)-shaped layout.

(i) TRUE (ii) FALSE

Please, explain your answer. Also, discuss the reason(s) that might lead to the employment of the S-shaped layout for a certain flow line.

U-shaped layout is indeed much more efficient that the S-shaped layout since it establishes a better proximity for the line workstations, facilitating thus better communication and resource sharing among these workstations, better overall supervision of the entire line, easier cross-training of the line workers on the various tasks supported by the different workstations, etc.

On the contrary, S-shaped layout can result in a pretty congested shop-floor. But it might be necessary if we have to deploy a long line in a restricted area.

3. In a few words explain how the "pull" production system (according to the definition of this concept that was presented in class) provided a solution to the high levels of congestion of the "push"-based production flow lines that were deployed in the 70's and 80's. How modern "push"-based production lines have managed to overcome these congestion problems?

"Pull" systems established explicit WIP ceilings for various parts of the production line. When this ceiling was reached at a certain part, any further shipments from the previous part of the line were blocked, and in this way, information about some anomaly occurring at a certain part of the line was propagated to the previous parts. If this anomaly persisted for a certain amount of time, eventually even the first part of the line would reach its WIP ceiling, and the line would stop accepting any new parts in it.

These days, "push" systems have also become much more reactive to the aforementioned anomalies that might take place in them by using modern information technology to better monitor the line operations, and revise their production plans, if necessary.

4. What is the meaning of *cycle time* in the context of a synchronous transfer line (e.g., an assembly line integrated by a paced conveyor belt)? If we want to increase the production rate of an already deployed synchronous transfer line, its current cycle time should be

(i) increased. (ii) decreased.

Please, explain your answer.

In the terminology (and theory) of synchronous transfer lines, "cycle time" means the time between two consecutive advancements of the conveyor belt of the line. Equivalently, the cycle time c is the time that is spent by any part at one of the line workstations.

The above definition implies that

$$TH = 1/c$$

Hence, if we want to increase TH, we must decrease c.

5. Consider an M/M/1 queueing station with arrival rate λ and processing rate μ. Provide an *intuitive* explanation of the fact that such a workstation with $\lambda = \mu$ is unstable.

Since $\lambda = \mu$, the necessary utilization of the server workstation is 1; i.e., the server must be utilized 100% of its time. But the variability that is present in the arrival process and in the proc. times will inadvertently bring about some server idleness due to lack of available work (starvation), and therefore, in this regime, 100% utilization is not effectively attainable.

Problem 1 (30 points): An assembly process consists of ten atomic tasks. The processing times for these tasks and the precedence constraints among them are reported in the following table:

Task	Proc. Time (secs)	Imm. Predecessors
a	5	-
b	8	-
c	12	a
d	8	c, b
e	20	-
f	15	d, e

i. (5 pts) What is the highest possible throughput that can be attained by any synchronous transfer line that supports the execution of these tasks? Please, provide your response in parts per hour.

ii. (5 pts) Express the precedence constraints for the various tasks of this assembly line through a precedence graph.

iii. (15 pts) Use the Ranked Positional Weight heuristic presented in class in order to come up with an efficient design for this assembly line that will support a production rate of 80 parts per hour.

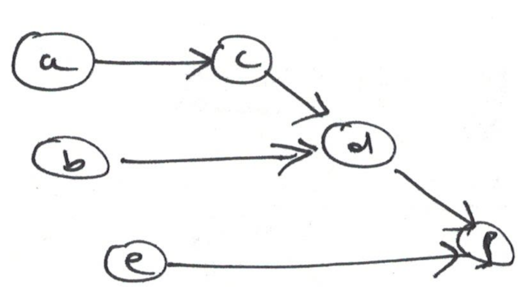
iv. (5 pts) Provide a lower bound for the number of workstations that are necessary for the line designed in part (iii) above. What is implied by this lower bound for the optimality of the design that you obtained in part (iii) above?

(i) The minimum possible cycle time is equal to the largest proc. time; i.e. $C_{min} = t_{max} = 20\,sec$

But then $TH_{max} = 1/C_{min} = \frac{1}{20\,sec} = \frac{3600\ sec/hr}{20\ sec}$

$= 180$ parts/hr.

(ii)



(iii) From the diagram of part (ii) we have the following successor sets and Positional weights for the various tasks:

Task i	Succ(i)	PW(i)
a	a, c, d, f	5+12+8+15 = 40
b	b, d, f	8+8+15 = 31
c	c, d, f	12+8+15 = 35
d	d, f	8+15 = 23
e	e, f	20+15 = 35
f	f	15

Hence, the task order that is suggested by the PW(i)'s is: $< a, c, e, b, d, f >$

Also the cycle time c that is implied by the target production rate is $c = 1/TH = \frac{1}{80\,hr^{-1}} = \frac{3600\,sec}{80} =$

$= 45 sec.$

Then, working with the above information as demonstrated in class, we get the following design for this line

WS1	WS2
$\boxed{a, c, e, b}$	$\boxed{d, f}$
~~45~~	~~45~~
~~40~~	~~37~~
~~28~~	22
~~8~~	
0	

This line is quite unbalanced in terms of the utilizations of the two workstations. One way to address this problem is to move task b to WS2.

(iv) A lower bound for the number of required workstations is given by $\left\lceil \frac{\sum_i t_i}{c} \right\rceil = \left\lceil \frac{68\text{ sec}}{45\text{ sec}} \right\rceil = 2.$

Based on this computation, we can infer that the design of part (iii) utilizes the smallest possible number of workstations, under the current parameterization of the problem, and therefore it is optimal in that sense.

On the other hand, the ratio $68/45 \simeq 1.51$ suggests considerable idleness for the line workstations

Problem 2 (30 points): Consider an M/M/2/3 queueing station. Processing times at the two servers of the station are exponentially distributed with rates $\mu_1 = \mu_2 = 10hr^{-1}$. Jobs arrive at this station from two different Poisson streams with corresponding rates $\lambda_1 = 10hr^{-1}$ and $\lambda_2 = 8hr^{-1}$, and they join the single queue of jobs waiting to be processed by the station servers.

Please, answer the following questions:

i. (5 pts) Argue that this workstation is stable, according to the definition of workstation stability that we have provided in class.

ii. (5 pts) What is the probability that the next arrival at the considered workstation will occur from stream 1?

iii. (5 pts) Model the operation of this workstation as a continuous-time Markov chain (CTMC). Please, show clearly the states of this CTMC, their semantics, and the transition rates among these states.

iv. (5 pts) Write down a system of equations that defines completely the steady-state distribution for the CTMC that you developed in item (iii) above. You don't have the solve these equations, but, please, show clearly their structure and the detailed values of all the involved parameters.

v. (5 pts) Using the distribution that you defined in item (iv) above, provide a formula that characterizes the line throughput.

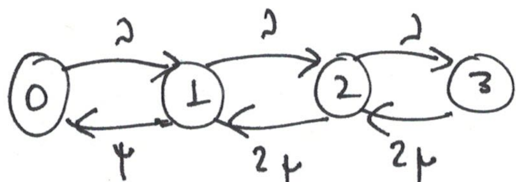
vi. (5 pts) Using the distribution that you defined in item (iv) above, provide a formula that characterizes the utilization of each of the station servers.

(i) This station can accommodate only upto 3 jobs, and any arrivals that find it full will just leave. So, its WIP will never explode.

(ii) Assuming that the two arrival streams are independent

Prob.(next arrival is type 1) $= \lambda_1 / \lambda_1 + \lambda_2 = \frac{10}{10+8} = 5/9.$

(iii) Since (i) the jobs that arrive from the two streams are eventually treated as one job type, and (ii) the proc. time distributions of the two servers are exactly the same, a simple CTMC model for this workstation, that is also adequate for answering parts (v) and (vi), is as follows:



where $\begin{cases} \lambda = \lambda_1 + \lambda_2 = 18\ hr^{-1} \\ \mu = \mu_1 = \mu_2 = 10\ hr^{-1} \end{cases}$

and the state is just the # of jobs in the station.

(iv) Letting p_i, $i = 0, 1, 2, 3$, denote the "steady-state" probability for state i, we can get these p_i's by solving the following system of equations:

$$\begin{cases} \lambda p_0 = \mu p_1 \\ (\lambda + \mu) p_1 = \lambda p_0 + 2\mu p_2 \\ (\lambda + 2\mu) p_2 = \lambda p_1 + 2\mu p_3 \\ p_0 + p_1 + p_2 + p_3 = 1 \end{cases}$$

Notice that the <u>irreducibility</u> of the considered CTMC implies that the "flow balance" equation for one of the four nodes will be redundant (i.e., it does not convey any additional information w.r.t. the information conveyed collectively by the other three "flow balance" equations, and therefore, it can be dropped from the overall formulation; here I chose to drop the equation corresponding to node 3).

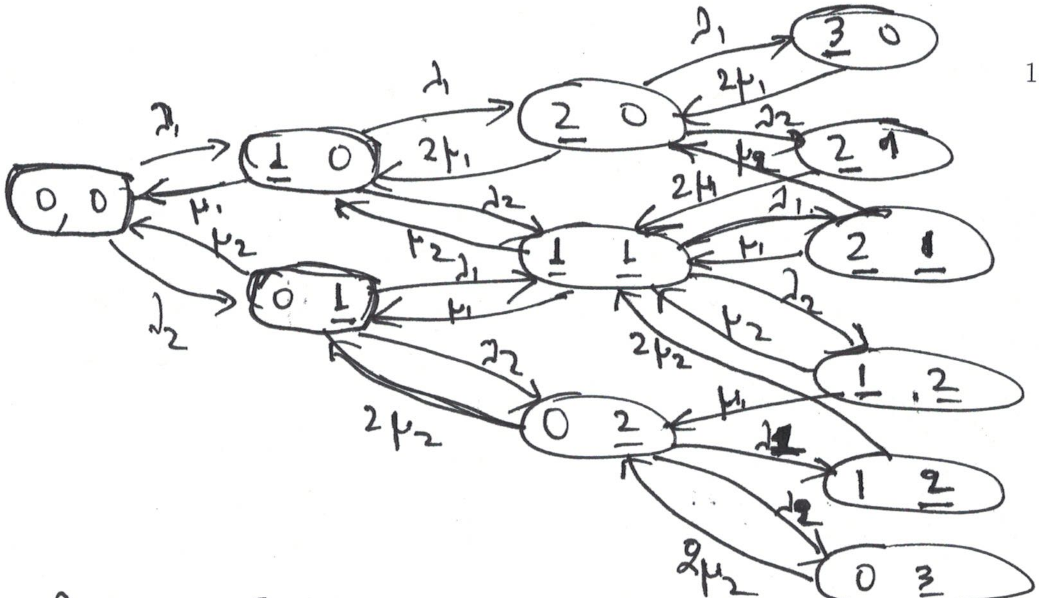
(v) $TH = \lambda_{effective} = \lambda[1 - P(\text{station is full})] = \lambda(1-P_3)$.

(vi) $u = \dfrac{\lambda_{eff}}{2\mu} = \dfrac{\lambda(1-P_3)}{2\mu}$

Also, $u = 0 \cdot P_0 + \frac{1}{2} \cdot P_1 + 1 \cdot P_2 + 1 \cdot P_3$

Using the equation of part (iv), try to verify that the above two expressions for u are equivalent.

Finally, if we try to develop a more refined CTMC model that keeps track of the composition of the queue w.r.t. the two arriving streams and the server allocation to these two job types (as some of you tried to do), then a compact way to encode this information is through the following state definition: $s = (\underset{(-)}{n_1}, \underset{(-)}{n_2})$ where, for $i=1,2$, n_i = # of jobs from stream i in the system, and the presence of the underscore ('−') indicates whether there is at least one server allocated to this stream. Then, the previous CTMC structure can be expanded as follows:



Let $P_x = P[\underset{(-)}{n_1}(x), \underset{(-)}{n_2}(x)]$ denote the "steady-state" prob. for state x. The distribution $[P_x]$ can be computed by writing down the corresponding balance equations for all states x but one, plus the normalizing equation $\sum_x P_x = 1$. Also,

$$TH = \lambda_{eff} = \lambda \Big(1 - \sum_{\substack{x: \\ n_1(x)+n_2(x)=3}} P_x\Big)$$

and $$u = \frac{\lambda_{eff}}{2\mu}$$