# ISYE 4803-REV: Advanced Manufacturing Systems

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Midterm Exam II

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Name: SOLUTIONS

Answer the following questions (**8 points each**):

**1.** In a design specification for an asynchronous transfer line, would it make sense to try to specify, as distinct requirements, (i) the capacity of the line (i.e., the maximal throughput that can be delivered in a stable manner); (ii) the average delay of a part going through this line when the line is operated at capacity; and (iii) the average WIP experienced in the line when it is operated at capacity?

(i) YES (ii) NO

Please, explain your answer.

The emphasis in the above question is on "distinct".

While all three quantities listed in items (i)–(iii) are important metrics characterizing the steady-state performance of an ATL, they are also related by Little's Law: $WIP = TH \times CT$ (1)

Hence, only two of them can be specified as distinct requirements. Once, these two quantities are specified, the third one is determined by Eq. (1) above.

As we discussed in class, when we design an ATL, typically we specify the line capacity and one of the other two quantities, e.g., the tolerated CT. (or tolerated WIP). This second quantity can be perceived as a measure of the "congestion" that we can tolerate in the line.

**2.** It is possible to reduce the expected cycle time that a part experiences at the third workstation of an asynchronous transfer line by reducing the variability of the processing times at the first workstation?

(i) YES (ii) NO

Please, explain your answer.

We know from the corresponding theory that

(i) $CT_3 = \frac{c_{a_3}^2 + c_{p_3}^2}{2} \frac{u_3}{1-u_3} t_{p_3} + t_{p_3}$

(ii) $c_{a_3}^2 = c_{d_2}^2$ ; $c_{a_2}^2 = c_{d_1}^2$

(iii) for $i = 1, 2$ : $c_{d_i}^2 = (1 - u_i^2)\, c_{a_i}^2 + u_i^2\, c_{p_i}^2$

The above equations imply a dependence of $CT_3$ on $c_{a_i}$, especially in the case that $u_1$ and $u_2$ are not very high.

**3.** Consider a single-server station that experiences the type of preemptive outages that was discussed in class. Parts arrive at this station at a rate $r_a$ and it has been observed that the station operation is *unstable* at that arrival rate. Please, mark below *all* the possible courses of action that might help stabilizing this station, without compromising the target throughput of $r_a$.

i. Increase the server processing rate $r_p$ (this is the processing rate that does not account for the experienced failures).

ii. Decrease the part arrival rate $r_a$.

iii. Reduce the average downtime per failure.

iv. Reduce the frequency of the occurrence of failures.

v. Reduce the variability in the time between two consecutive failures.

vi. Reduce the variability of the server downtime.

We know that

$$u = r_a \cdot t_e = r_a \cdot \frac{t_p}{A} \quad (1) \quad \text{and} \quad A = \frac{m_f}{m_f + m_r} \quad (2)$$

Also $r_p = 1/t_p \iff t_p = 1/r_p$ (3)

To establish stability we need to reduce $u$

(i) (1) & (3) imply that $r_p \uparrow \Rightarrow u \searrow$

(ii) (1) also implies that $r_a \searrow \Rightarrow u \searrow$, but reducing $r_a$ essentially compromises throughput.

(iii) From (1) and (2), $m_r \searrow \Rightarrow A \nearrow \Rightarrow u \searrow$

(iv) This assumption essentially implies $m_f \uparrow$. Then write (2) as $A = \frac{1}{1 + m_r/m_f}$ and it becomes clear that $m_f \uparrow \Rightarrow A \uparrow$.

(v), (vi) From (1) and (2) it is clear that $u$ does not depend on the second moments of the involved quantities.

**4.** Please, discuss what defines the need for *"transfer"* batching in the context of manufacturing operations.

"Transfer" batching implies the transfer of parts between any two consecutive workstations in groups of more than one part.

This is typically done in an effort to control the associated material handling effort (and cost), especially in the case of lines that produce rather small items in large quantities.

**5.** During the discussion of parallel batching in class, I pointed out that the formula

$$u = \frac{r_a \cdot t}{k},$$

for the resulting utilization, implies that, for a fixed part arrival rate $r_a$ and a fixed mean processing time $t$, the server utilization will decrease as the batch size $k$ increases. Please, provide an intuitive explanation for this effect.

The most intuitive explanation of the utilization reduction that is implied by the above formula, is that, since processing time does not depend on the batch size, all parts that join the first part in the batch essentially get a "free ride"(!), i.e., the server processes these additional parts without having to do any extra work!

Another way to describe the above effect is as follows:
If parts are processed one at a time, the resulting utilization is $u(1) = r_a \cdot t$. For a batch of k parts, this amount of work is <u>ammortized</u> over these parts, and therefore, the effective server utilization per part is $u(k) = \frac{r_a \cdot t}{k}$

Since, however, the part arrival rate does not change, $u(k) = u < u(1)$, for $k \geq 2$.

**Problem 1 (40 points):** Consider a single-server station where the parts to be processed arrive in *kits* of 10 parts per kit. Kits arrive according to a Poisson process with a rate of 4 kits per hour. Each kit is processed *serially* by the station server, one part at a time, with the *part* processing time being equal to 1 minute per part and the coefficient of variation for these processing times being 0.5. Parts in a kit leave the server only when the entire kit has been processed. Furthermore, the server experiences preemptive outages with normally distributed downtimes with mean 5 minutes and st. deviation 2 minutes, while the time between two consecutive outages is exponentially distributed with a mean of 30 minutes.

Use the above information in order to perform the following:

i. (15 pts) Show that the station operation under the considered regime is stable.

ii. (15 pts) Compute the average cycle time of a part going through this station.

iii. (10 pts) Compute the average number of *parts* waiting for processing in this station.

(i) The kit arrival rate is $r_{a_b} = 4\,hr^{-1} =$

$= \frac{4}{60}\,min^{-1} = \frac{1}{15}\,min^{-1} \simeq 0.667\,min^{-1}$

The kit nominal proc. time is $t_b = 10 \times 1\,min = 10\,min$

The server availability is

$$A = \frac{m_f}{m_f + m_r} = \frac{30}{30+5} = \frac{30}{35} = \frac{6}{7} \simeq 0.857$$

Hence, $u = r_{a_b} \cdot t_{e_b} = r_{a_b}\, t_b / A = \frac{1}{15} \times 10 \times \frac{7}{6} =$

$= \frac{14}{18} = \frac{7}{9} \simeq 0.778 < 1 \Rightarrow$ stable.

(ii) The SCV for the kit proc. times is

$$c_b^2 = \frac{c_p^2}{K} = \frac{0.5^2}{10} = 0.025$$

and the SCV for the kit effective proc. times (accounting for failures) is :

$$c_e^2 = c_0^2 + (1 + c_r^2)\, A(1-A)\, \frac{m_r}{t_0} =$$

$$= 0.025 + \left[1 + \left(\frac{2}{5}\right)^2\right] 0.857\,(1 - 0.857)\,\frac{5}{10} \simeq 0.096$$

Hence,

$$CT_p = CT_b = \frac{c_{a_b}^2 + c_e^2}{2}\,\frac{u}{1-u}\,t_e + t_e =$$

$$= \frac{1 + 0.096}{2}\,\frac{0.778}{1 - 0.778}\,\frac{10}{0.857} + \frac{10}{0.857} = 34.078 \text{ min}$$

(iii) $\# \text{ kits} = CT \times r_{a_b} = 34.078 \text{ min} \times \frac{1}{15 \text{ min}} \simeq 2.272 \Rightarrow$

$\Rightarrow \# \text{parts} = \# \text{kits} \times 10 = 22.72$

To compute the corresponding numbers for the queue (excluding the server), we can work as follows:

$\#$ of kits in queue $=$ ($\#$ of kits) $- u = 2.272 - 0.778 =$

$= 1.494$

Hence, $\#$ parts in queue $=$ ($\#$ of kits in queue) $\times 10 = 14.94$

**Problem 2 (20 points):** Parts are released for processing at an oven with a deterministic release pace and organized in batches of $k$ parts per batch. The processing time of a batch is also deterministic, and equal to 15 minutes.

i. (10 pts) For a desired throughput rate of 100 parts per hour, what is the *minimum* batch size, $k_{min}$, that will provide this throughput?

ii. (10 pts) For the value of $k$ that you computed in part (i) above, define a batch-release policy that minimizes the part cycle time at this oven.

The key feature for this problem is that both, the arrival process and the server processing times are deterministic; so, we can aim for 100% utilization of the server and zero batch queueing times (as in the case of the "thought" experiment that we discussed at some point in class).

Then, for (i) we have:

$$K_{min} = r_a \cdot t = \frac{100}{60\,min} \times 15\,min = 25$$

Also, according to the above remarks, the key observation for answering (ii) is that we need to synchronize new batch arrivals with the completions of the batch in processing. Hence, we need to have one new batch released every 15 min. Since parts are released in a deterministic pace, and I need to have 25 parts released in 15 min, the part release rate is $r_a = \frac{25}{15\,min} \simeq 1.67\,min^{-1}$ and the part inter-release time is

$$t_a = \frac{1}{r_a} = \frac{15}{25} = \frac{3}{5}\,min = 0.6\,min \left(= \frac{60\,min}{100}\right)$$

The resulting expected cycle time is

$$CT = WTBT + \cancel{CT_q}^{\,0} + t =$$

$$= \frac{k-1}{2r_a} + t = \frac{25-1}{2(25/15)} + 15 =$$

$$= \frac{12 \times \cancel{15}^{\,3}}{\cancel{25}_{\,5}} + 15 = 22.2 \text{ min}$$