ISYE 4803-REV: Advanced Manufacturing Systems

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Midterm Exam I

October 5, 2016

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. In class we recognized *responsiveness* as one of the key strategic requirements for modern corporations. Please, (a) explain what is meant by this term, and (b) outline some of the primary concepts and practices that enable modern companies to support this requirement at the operational level.

Responsiveness essentially implies the ability of a company
(i) to meet the experienced demand in a timely manner, and also
(ii) to be able to follow effectively the various trends that arise in the market.

From an operational standpoint, responsiveness is facilitated by elements like
(a) the deployment of reliable and flexible production and distribution networks;
(b) the employment of "buffer", either in the form of stock or production capacity to cope with demand variability;
(c) the employment of modern IT tools in order to
(i) interface more effectively with the customers as well as the suppliers, and
(ii) establish better control of the internal operations;
(d) the employment of modularity and standardization in product design that facilitate
(i) "mass-customization" practices, and
(ii) the efficient evolution of the product portfolio;
(e) etc.

2. Identify a single feature of the U-shaped layout that has rendered it the most preferred layout in modern production and distribution environments.

The most important feature of the U-shaped layout is the proximity that is establishes among the line workstations, which further facilitates

(a) better supervision,
(b) better communication among the various workstations,
(c) easier resource sharing among the workstations, including operator/employee sharing,
(d) the development of a more comprehensive understanding of the line operations by the persons who work on it, and a better engagement of these persons to those operations
(e) etc.

3. In the ALB problem that we considered in class, there was an idle time of at least 5 secs at every station in the produced design. Noticing this result, somebody suggested that, in the final implementation of this design, it might be pertinent to reduce the line cycle time to 55 secs (instead of the original value of 60 secs). Do you agree with this idea?

(i) YES (ii) NO

Please, explain your answer.

- It is true that in the presented scenario every workstation will be able to complete its designated tasks within the new cycle time
- Furthermore, the resulting operation will be "stable" (i.e., practically feasible) even though some workstations will have a nominal utilization of 1.0 because, as we explained in class, the nominal task proc. times that are used during this design process are determined in a way that accounts for the inherent variability in these proc. times.
- The main objection is that such a reduction of the cycle time will increase the line throughput. Assuming that the throughput specified during the design process is the "<u>desired</u>" throughput, then any increase of this value might just imply <u>unnecessary production</u>, and the buildup of stock that eventually might have to be disposed in unprofitable manners, or even scrapped.

4. Consider an M/M/1 queue which serves its arriving stream of customers in a stable manner. In the current operational regime the average waiting time is 30min. In an effort to decongest the line, the server was replaced by a faster one, and now the average waiting time has been reduced to 15min. Based on this information, we can conclude that the average number of customers waiting in the queue (not accounting for those in service) in this new regime has been reduced to half the value of this quantity in the previous condition.

(i) TRUE (ii) FALSE

Please, explain your answer.

From the problem description, it is obvious that the arrival rate at this queue has not changed. Hence, from Little's law:

$$TH = \lambda = \frac{WIP_q^{(old)}}{CT_q^{(old)}} = \frac{WIP_q^{(new)}}{CT_q^{(new)}} \Rightarrow$$

$$\Rightarrow \frac{WIP_q^{(new)}}{WIP_q^{(old)}} = \frac{CT_q^{(new)}}{CT_q^{(old)}} = \frac{15 \text{ min}}{30 \text{ min}} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow WIP_q^{(new)} = \frac{1}{2} WIP_q^{(old)}$$

5. Consider a single-server workstation that operates in a stable regime, with a server utilization equal to ρ. The time between any two consecutive arrivals is *normally* distributed, a fact that reflects the distribution of the processing times of the heavily utilized upstream workstation.

Based on the above information, we can infer that the probability that a new arrival will find the station idle is equal to

i. 0

ii. $1-\rho$

iii. ρ

iv. we cannot tell

Please, explain your answer.

It is true that, under the typical assumptions of server nonfailure and nonidleness, the probability that this workstation is empty is equal to $1-\rho$.

But since the arrival process is not Poisson, PASTA does not hold anymore, and therefore the probability that a new arrival will find an empty system is not necessarily equal to the probability that the system is found empty at any arbitrary point in time.

Hence, the above problem does not provide adequate information to compute the requested probability.

(In fact, even in the cases where it is feasible, the computation of this probability under non-Poisson arrivals is a pretty non-trivial task.)

Problem 1 (30 points): An assembly process involves the following ten atomic tasks with the corresponding processing times and precedence constraints being reported in the following table:

Task	Proc. Time (secs)	Imm. Predecessors
a	1	-
b	8	-
c	12	a
d	7	c, b
e	5	-
f	15	d, e

i. (5 pts) What is the highest possible throughput that can be attained by any synchronous transfer line that supports the execution of these tasks? Please, provide your response in items per hour.

ii. (10 pts) The RPW heuristic uses the positional weights (PW_i) of the different tasks in order to develop the ordered task list that is eventually used in the design process that specifies the line workstations. Explain (by providing the necessary mathematical argument) that in the aforementioned process, a valid task ordered list can also be obtained by replacing the positional weights, PW_i, by the cardinalities of the (task) successor sets S_i that we defined in class during the computation of the PW_i. In other words, instead of ranking the tasks by decreasing PW_i, now we shall rank them by decreasing $|S_i|$ (remember that for any given set A, $|A|$ denotes its cardinality, i.e., the number of the elements in this set – you need to justify that the suggested substitution will provide task ordered lists that still respect the precedence constraints among the tasks).

iii. (10) Use the heuristic that results from the substitution that is suggested in item (ii) above in order to design an assembly line for the considered process that will support a production rate of 100 parts per hour. Please, show clearly all the computations of this algorithm.

iv. (5 pts) Provide a lower bound for the number of workstations that are necessary for the line designed in part (iii) above. What can you infer from this computation regarding the optimality of the design that you obtained in part (iii) above?

(i) Since, for synchronous transfer lines,

$TH = 1/c$, the highest throughput is determined by the minimum possible cycle time. But then, the indivisibility of the supported tasks implies that $c_{min} = 15$ sec, which implies that

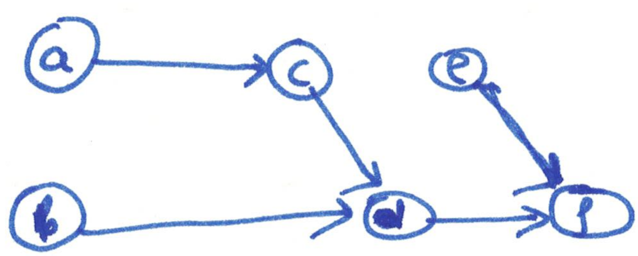
$$TH_{max} = \frac{1}{15 \text{ secs}} = \frac{3600 \text{ sec/hr}}{15 \text{ secs}} = 240 \text{ hr}^{-1}$$

(ii) If task i is a predecessor of task j, then all the successors of j are also successors of i. And since, by definition, the set S_i contains task i itself, we shall have $S_j \subset S_i$ (with the inclusion being strict), and therefore, $|S_j| < |S_i|$.

So, it is true that in any ordered list where tasks are listed in decreasing cardinality, a task i will appear before all of its successors.

Hence, we can rank tasks according to the cardinalities of the corresponding sets S_i instead of the PWi's.

(iii) In the considered case, the task precedence graph is as follows:



and

$S_f = \{f\}$; $S_e = \{e, f\}$; $S_d = \{d, f\}$; $S_c = \{c, d, f\}$;
$S_b = \{b, d, f\}$; $S_a = \{a, c, d, f\}$

Hence, $|S_f| = 1$; $|S_e| = |S_d| = 2$, $|S_c| = |S_b| = 3$,
$|S_a| = 4$, and a possible task ordering is

$\langle a, b, c, d, e, f \rangle$

Also, the desired throughput of 100 parts/hr implies a cycle time $c = \frac{3600}{100} = 36$ sec. Putting everything together we get the following design:

or

total time: 33 sec
idle " : 3 sec

total time: 15 sec
idle " : 21 sec

The better balanced line:

WS 1	WS 2
a, b, c, d	e, f

total time: 28 sec   total time: 20 sec
idle ,, : 8 sec   idle ,, : 16 sec

(iv) Applying the corresponding formula, we get:

$$N_{min} = \left\lceil \frac{\sum_i t_i}{c} \right\rceil = \left\lceil \frac{48}{36} \right\rceil = 2$$

Hence, the above designs utilize the minimum possible number of workstations under the current speed.

Problem 2 (30 points): Consider an M/M/1/c queue operating in steady-state. For this queue, we have that:
the average inter-arrival time t_a = the average processing time t_p = 5min.

Use the above information in order to answer the following questions:

i. (5 pts) What is the minimum possible value for c so that the percentage of the lost customers is no more than 15%?

ii. (25 pts) For the value that you computed in item (i) above, compute also

- the server utilization;
- the average number of customers in the entire system;
- the average number of customers waiting in the queue;
- the expected cycle time for the customers that enter the queue;
- the expected cycle time for an arriving customer, irrespective of whether s/he entered the queue or not.

(i) For this particular M/M/1/c queue,

$$\rho = \lambda/\mu = \frac{1/t_a}{1/t_p} = \frac{t_p}{t_a} = 1.$$

Hence for any finite c, the dynamics of this queue will be described by a Markov chain

with all the states 0, 1, 2, —, c being equiprobable in steady-state; i.e., the steady-state distribution will be $P_i = \frac{1}{c+1}$, $\forall i \in \{0, 1, .., c\}$

We also have that

$\lambda_{eff} = \lambda(1-P_c) \Rightarrow$ percentage of lost customers $= P_c =$ $= \frac{1}{c+1}$. We want $\frac{1}{c+1} \leq 0.15 \Rightarrow c+1 \geq \frac{1}{0.15} \cong 6.67 \Rightarrow c \geq 5.67 \Rightarrow$ $\Rightarrow c_{min} = 6.$

(ii) Since in the resulting queue the server is utilized at every state except for the empty state, we have

$$\text{server utilization} \equiv u = \sum_{i=1}^{6} P_i = \sum_{i=1}^{6} \frac{1}{c+1}\Big|_{c=7} = \frac{6}{7} \simeq 0.857.$$

It is important to notice that in this context, u is <u>not</u> given by $\rho = \lambda/\mu = \lambda t_p$. If we wanted to obtain u in a way that reflects the employment of this last formula for the computation of the utilization in some other settings like the M/M/1, M/G/1, G/G/1, and even the M/M/k queue, we need to use

$u = \lambda_{eff} \cdot t_p$ (since it is this last product that characterizes the workload that is actually admitted in the queue at each time unit).

$$= \lambda(1-P_c)\cdot t_p = (\lambda \cdot t_p)\left[\sum_{i=0}^{c} \frac{1}{c+1}\right]_{c=7} = 1 \cdot \frac{6}{7} = \frac{6}{7}$$

For the average # of customers in the system, we have:

$$WIP = 0\cdot\frac{1}{7} + 1\cdot\frac{1}{7} + 2\cdot\frac{1}{7} + \dots + 6\cdot\frac{1}{7} = \frac{1}{7}(1+2+\dots+6) =$$

$$= \frac{1}{\cancel{7}}\,\frac{6\cdot\cancel{7}}{2} = 3.$$

Also,

$$WIP_q = WIP - u \simeq 3 - 0.857 \simeq 2.143$$

$$TH = \lambda_{eff} = \lambda(1-p_c) = \frac{1}{5} \cdot \frac{6}{7} = \frac{6}{35} \simeq 0.171 \text{ min}^{-1}$$

$$CT = \frac{WIP}{TH} = \frac{3}{6/35} = \frac{35}{2} = 17.5 \text{min}.$$

Finally, for the last quantity, i.e, the expected cycle time of an arriving customer irrespective of whether she joins the queue or not, we have:

$$\overline{CT} = \frac{6}{7} CT + \frac{1}{7} \cdot 0 = \frac{6}{7} \cdot \frac{35}{2} = 15 \text{ min}.$$

This last result can also be obtained through a Little's law type of computation, by recognizing that diverted arrivals do not create any further WIP. Therefore:

$$\overline{CT} = \frac{WIP}{\lambda} = \frac{3}{1/5 \text{ min}^{-1}} = 15 \text{ min}$$