# ISYE 4803-REV: Advanced Manufacturing Systems

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Midterm Exam II

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**Name:** SOLUTIONS

Answer the following questions (**8 points each**):

**1.** Discuss the meaning of the statement that from the standpoint of the workflow that is materialized at a production or service station, the accumulated WIP and the experienced delays are dual manifestations of the same problem. What is the essence of this problem and how it can be controlled?

From Little's law we know that for a stable system:

$$TH = \frac{WIP}{CT} \qquad (1)$$

i.e., for a fixed throughput level, the WIP varies proportionately with the CT. Both of them are manifestations of the congestion that takes place at the underlying station.

From the basic formula $CT = \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t_e + t_e$

we see that CT can be reduced

a) by reducing the variability in the arrival process and the (effective) processing times;

b) by reducing $u$, either by adding another server or by decreasing the mean proc. time at the current server(s).

Of course, from (1), the reduction of CT will also reduce the WIP accordingly.

**2.** Consider an asynchronous transfer line operating in a stable mode, but containing a workstation with a single server that experiences long preemptive but non-destructive outages during the processing of the parts going through it. The disruptive effect of these outages on the performance of the line will be more severe if the considered workstation is located closer to

i. the beginning of the line.

ii. the middle of the line.

iii. the end of the line.

Please, briefly explain your answer.

Since the line is stable, the outages experienced at the considered station will impact the rest of the line through the fundamental dependency

$c_a^2(i+1) = c_d^2(i)$ and $c_d^2(i) = (1-u_i^2)\, c_a^2(i) + u_i^2\, c_e^2(i)$

Hence, putting the station at the beginning of the line will increase the number of the other workstations that can be potentially affected by the high value of $c_e^2(i)$ at this workstation (due to the experienced) outages.

On the other hand, putting this workstation at the end of the line will localize the experienced disturbance at that workstation (remember that the basic "push" model does not involve any blockages that might backpropagate the disturbance experienced at this station to the earlier stations).

**3.** What is the meaning and the role of *serial* batching in modern manufacturing facilities? What is the primary cost involved with this concept?

"Serial" batching characterizes the fact that parts in a batch are processed sequentially at any given server of the considered system, and not simultaneously as in the case of "parallel" batching.

Serial batching schemes are used in a effort to control

(i) potential setup costs, as in the case of cellular manufacturing, and also

(ii) material handling costs concerning the transferring of parts from one workstation to the next.

**4.** Consider a single-server workstation where the part processing time $T_s$ has a mean of $t$ time units and a st. deviation of $\sigma$ time units. Furthermore, part processing times are independent from each other. Show that the necessary time $T_b$ to process a batch of $k$ parts at this workstation is less variable than the part processing time $T_s$.

*Hint:* Remember what is the primary statistic that expresses the variability in some random variable $X$, and try to compare this statistic for the two random variables $T_s$ and $T_b$ mentioned above.

As shown in class (a number of times), if the ~~batch~~ processing times of the parts in a batch of size $k$ are independent from each other, the

$$E[T_b] = k(T_s) \quad \wedge \quad Var[T_b] = k\,Var[T_s]$$

Hence,

$$SCV[T_b] = \frac{Var[T_b]}{E^2[T_b]} = \frac{k\,Var[T_s]}{k^2 E^2[T_s]} = $$

$$= \frac{1}{k} SCV[T_s]$$

And since $k>1$, $\quad SCV[T_b] < SCV[T_s] \Leftrightarrow$

$$\Leftrightarrow \quad CV[T_b] < CV[T_s].$$

**5.** During a busy day, visitors of the local amusement park arrive at the park roller coaster according to a Poisson process with rate 5 persons per minute. The train of this roller coaster has a capacity of 30 persons, and the ride lasts 3 minutes. The company policy is that the roller coaster will start a new ride as soon as there are at least 10 persons on the train (of course, if there are more persons waiting, additional persons will get on the train until the sitting capacity is reached). Based on this information, we can infer that the current operational policy of this facility will meet the experienced demand in a stable manner.

(a) YES (b) NO

Please, explain your answer.

From the results on parallel batching presented in class, we know that for stability we need

$$k > V_a t = 5\ \text{min}^{-1} \times 3\ \text{min} = 15 \quad (1)$$

From the problem description, we know that the company utilizes a <u>minimum</u> batch size of 10 persons. If the batch size was fixed to 10 persons, then the resulting operational scheme would be unstable, as implied by (1). But if the queue is building up, then the running batches will be higher than 10. In particular, any time the queue gets above 15 persons, the corresponding batches will be of size 16 or higher, i.e, adequately large to keep the system stable. This "<u>self-regulating</u>" mechanism that is naturally built in the considered operation will keep it stable.

**Problem 1 (40 points):** Consider a single-server workstation where parts arrive according to a Poisson process with rate 10 parts per hour. The part processing times are distributed according to some general distribution with a mean of 5 minutes and a st. deviation of 1 minute. Furthermore, during the processing of these parts, the server experiences non-destructive outages, and from empirical observation of these outages, they have been found to have an average duration of 20 minutes and a st. deviation of 10 minutes. The occurrence of these outages follows a Poisson process (on the operational time of the server) with rate of one outage every 3 hours.

Please, answer the following questions, showing clearly your calculations.

i. (10 pts) Compute the mean and the SCV for the effective processing times of this station.

ii. (10 pts) Show that this station is stable.

iii. (10 pts) Compute the expected cycle time and the average WIP at this station.

iv. (10 pts) Compute the departure rate and SCV for the inter-departure times at this station.

Applying the relevant formulae from the theory of G/G/1 queues with non-preemptive outages, we get:

(i) $a = \frac{m_f}{m_f + m_r} = \frac{3\text{ hr}}{3\text{ hr} + \frac{1}{3}\text{ hr}} = \frac{9}{10} = 0.9$

$t_e = t_s / a = \frac{5\text{ min}}{0.9} = 5.56\text{ min}$

$c_e^2 = c_s^2 + (1 + c_r^2)\, a (1 - a) \frac{m_r}{t_s} =$

$= \left(\frac{1}{5}\right)^2 + \left(1 + \left(\frac{1}{2}\right)^2\right) \times 0.9 \times (1 - 0.9) \times \frac{20\text{ min}}{5\text{ min}} = 0.49$

(ii) $u = r_a t_e = 10\text{ hr}^{-1} \times \frac{5.56\text{ min}}{60\text{ min/hr}} = 0.9267 < 1 \Rightarrow$

$\Rightarrow$ stable.

(iii) $CT = \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t_e + t_e =$

$$= \frac{1+0.49}{2} \frac{0.9267}{1-0.9267} 5.56 + 5.56 = 57.928 \text{ min}$$

$$WIP = r_a\, CT = 10\ hr^{-1} \times \frac{57.928\ min}{60\ min/hr} = 9.655$$

(iv) from stability, $r_d = r_a = 10\ hr^{-1}$

$$c_d^2 = (1-u^2) c_a^2 + u^2 c_e^2 = (1-0.9267^2) \times 1 + 0.9267^2 \times 0.49 = 0.562$$

**Problem 2 (20 points):** Consider a medical facility where patients go through a first testing process that requires an average time of 20 minutes. The standard deviation of this testing time is 5 minutes. If the test result is negative, then, the patient is free to go. If, on the other hand, the test result is positive, then, the patient has to undergo a second testing stage with a mean processing time of 30 minutes and a st. deviation of 15 minutes. Past observations have shown that in the considered population, tests are negative 80% of the time.

Please, answer the following questions, showing clearly your calculations.

i. (10 pts) What is the effective mean processing time for any patient that goes through this facility?

ii. (10 pts) If patients arrive at a rate of one patient every 10 minutes, how many testing stations are necessary in order that the operation of this facility is stable? (each station can perform both of the aforementioned tests).

(i) $t_e = 0.8 \times 20 + 0.2 \times 50 = 26$ min

(ii) Let $n$ denote the number of stations.
We need:

$$\frac{\text{rate}}{n} < 1 \Rightarrow 1/10\text{min} \times 26\text{min} < n \Rightarrow$$

$$\Rightarrow n > 2.6 \Rightarrow n \geq 3.$$