**ISYE 4803-REV: Advanced Manufacturing Systems**
**Instructor: Spyros Reveliotis**
**Midterm Exam I**
**October 7, 2015**

**Name:** SOLUTIONS

Answer the following questions (**8 points each**):

**1.** Briefly discuss how the modern information technology helps companies control better their operational costs. Please, try to be specific about the various relationships / dependencies that are employed in your arguments.

Modern IT enables companies to acquire, process and disseminate more effectively information relating to various aspects of their operations. Therefore, they are able to support the responsiveness that is expected from them with less "buffering" in terms of finished and work-in-process inventories and/or extra processing capacity. Also, the visibility that is provided by modern IT enables the better coordination among the various parts of a company, and a more effective control of "waste" (this can materialize in various form in the operations of modern corporations). All the above have a positive impact in the companies' effort to control their operational costs.

**2.** What are some of the main advantages that have made the U-shaped layout popular in modern production and distribution environments? What is the key feature of the U-shaped layout that underlies and enables these advantages?

The U-shaped layout enables the fitting of long linear workflows in the rectangular area of a typical "shop floor", and furthermore, it increases the proximity of the various workstations that support the involved processing stages.
This last effect implies further important advantages like

* easier communication among the line workstations;
* better supervision of the entire line;
* the possibility of resource sharing across the line workstations;
* a more comprehensive view of the line operations by the employees that work on this line;
* etc.

**3.** Consider an Assembly Line Balancing (ALB) problem that is addressed through the relevant methodology that was presented in class. If the processing times of the tasks involved range from 5 secs to 30 secs, and the line will operate on an 8-hour daily shift, then,

i. we can infer that the maximum possible production rate for this assembly line is 960 product units per day.

ii. we can infer that the maximum possible production rate for this assembly line is 5760 product units per day.

iii. we can infer that the maximum possible production rate for this assembly line is 3360 product units per day.

iv. the provided information is not adequate for determining a maximum possible daily production rate.

Please, briefly explain your answer.

The key observations to answer this question are that

(i) $TH = 1/c$, where $c$ is the line cycle time, and

(ii) for the considered line, the minimum possible cycle time is 30 secs since some of the line workstations must support the tasks with the corresponding duration.

Hence

$$TH_{max} = \frac{1}{C_{min}} = \frac{1}{30\,sec} = \frac{60\frac{sec}{min} \times 60\frac{min}{hr} \times 8\frac{hr}{day}}{30\,sec} = 960\,\frac{parts}{day}$$

(where a "day" implies an 8-hr shift).

**4.** What is the meaning of PASTA in queueing theory? Explain how PASTA can be useful in the Mean Value Analysis (MVA) of queueing stations like the M/M/1 queue and the M/G/1 queue.

As an acronym, PASTA means that "Poisson Arrivals See Time Averages". The practical meaning of this expression is that when the arrival process at a queueing station is Poisson and this station operates at steady state, then, the observations of various statistics of the station operation (like the # of customers in it, the # of customers in the queue, the # of customers in service, etc) by a new arrival follow the same distribution with the distribution that characterizes these statistics as observed by an external observer that samples the system at some arbitrary point in time. But this last distribution is the corresponding "steady-state" distribution.

Since, both the M/M/1 and M/G/1 queue have Poisson arrivals, in class we used PASTA in order to argue that the expected # of customers that are observed at various parts in the system that are observed by a new arrival is equal to the corresponding expected values that are defined by the steady-state distributions (assuming, of course, that these queues operate in stable mode). Once this fact was established, then we were able to compute the expected delays for these new arrivals, and by Little's law, the expected customer concentrations in various parts of these systems.

**5.** Consider an M/M/1 queue with an arrival rate $\lambda = 20$ customers per hour and a processing rate $\mu = 30$ customers per hour. Explain that this queue will reach a steady-state operation, and determine the probability that a customer that arrives when the queue operates in steady-state will find the server busy.

Please, show clearly your calculations and explain clearly the rationale behind your response.

$\rho = \frac{\lambda}{\mu} = \frac{20}{30} < 1 \Rightarrow$ the queue is stable.

The probability that the server is busy is

$P_{busy} = 1 - P_{idle} = 1 - P_0 = 1 - (1-\rho) = \rho$

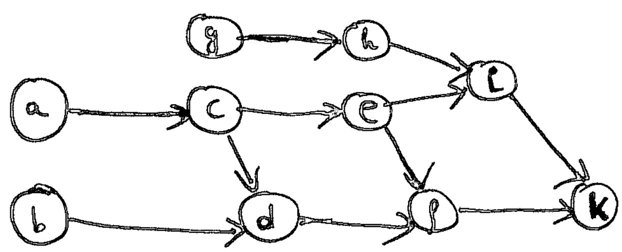
**Problem 1 (20 points):** An assembly process involves the following ten atomic tasks with the corresponding processing times and precedence constraints being reported in the following table:

| Task | Proc. Time (secs) | Imm. Predecessors |
|---|---|---|
| a | 10 | - |
| b | 8 | - |
| c | 12 | a |
| d | 7 | c, b |
| e | 5 | c |
| f | 15 | d, e |
| g | 11 | - |
| h | 20 | g |
| i | 10 | e, h |
| k | 5 | f, i |

i. (15 pts) Use the Ranked Positional Weight heuristic to design an assembly line for this process that possesses a production rate of 60 parts per hour. Please, show clearly all the computations of this algorithm.

ii. (5 pts) Provide a lower bound for the number of workstations that are necessary for the line designed in part #2 above. What are the implications of this bound for the line that you derived in part #2?

(i)




| Task | Successor Set | PW | Rank |
|---|---|---|---|
| a | a, c, d, e, f, i, k | 10+12+7+5+15+10+5 = 64 | 1 |
| b | b, d, f, k | 8+7+15+5 = 35 | 4 |
| c | c, d, e, f, i, k | 12+7+5+15+10+5 = 54 | 2 |
| d | d, f, k | 7+15+5 = 27 | 7 |
| e | e, f, i, k | 5+15+10+5 = 35 | 5 |
| f | f, k | 15+5 = 20 | 8 |
| g | g, h, i, k | 11+20+10+5 = 46 | 3 |
| h | h, i, k | 20+10+5 = 35 | 6 |
| i | i, k | 10+5 = 15 | 9 |
| k | k | 5 | 10 |

Ordered task list } a, c, g, b, e, h, d, f, i, k

$c = \frac{1}{TH} = \frac{1}{60}\, hr = 1\, min = 60\, sec.$

Hence,

| WS1 | WS2 |
|---|---|
| a, c, g, b, e, d | h, f, i, k |
| ~~60~~ | ~~60~~ |
| ~~50~~ | ~~40~~ |
| ~~38~~ | ~~25~~ |
| ~~27~~ | ~~15~~ |
| ~~19~~ | 10 |
| ~~14~~ | |
| 7 | |

(ii)

$$\underline{N} = \left\lceil \frac{\sum t_i}{c} \right\rceil = \left\lceil \frac{103}{60} \right\rceil = 2$$

Hence, by this calculation, our design in part (i) is optimal in that it utilizes the smallest possible number of workstations. We can also see that it is quite balanced in the way that it distributes the overall workload across the two workstations. Finally, the resulting utilizations for the two workstations are:

$u_1 = \frac{60-7}{60}\ 0.88\bar{3}$ and $u_2 = \frac{60-10}{60} = 0.8\bar{3}$

**Problem 2 (40 points):** Consider a gas station with 2 pumps. Filling up at any of the pumps of this gas-station takes an exponentially distributed time with mean 5 minutes (Although the exponential nature of the tank-filling times might sound a little strange, it becomes more realistic when considering all the different sizes of the tanks in the contemporary fleet of vehicles, and also the possibility that a car sitting at a pump might not be actually serviced, but the driver is shopping in the gas-station mini-mart, or s/he might be just sitting in the car checking email or talking on the phone...).

Cars drive by this gas station according to a Poisson process with rate of 100 cars per hour, and 10% of this stream of cars are in need of refueling. However, a passing car will just move on if there are no free pumps at the gas station at that time.

Use the above information in order to answer the following questions:

i. (10 pts) Explain that the operation of the considered gas station can be modeled as a Continuous-Time Markov Chain (CTMC) and provide the corresponding state transition diagram (STD). Please, define clearly the meaning of the state of this CTMC and the transitions among these states.

ii. (5 pts) What is the arrival process of the cars that are in need for refueling? What is the corresponding arrival rate $\lambda$?

iii. (5 pts) Explain that the CTMC that you developed in item # 1 above corresponds to a stable queue, and compute the corresponding steady-state probability distribution $\pi$.

iv. (5 pts) Use your results for distribution $\pi$ in step (iii) above, in order to compute $\lambda_{eff}$, the rate that the cars needing refueling actually enter this gas-station.

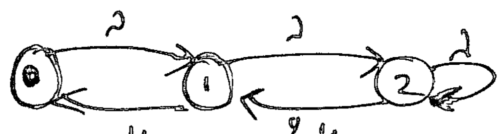
v. (5 pts) What is the expected cycle time $CT$ for a car that manages to enter the gas station?

vi. (5 pts) What is the expected number of cars at any point in time at this gas station?

vii. (5 pts) What is the expected number of cars at this gas station that is observed by an arrival that needs refueling (but not necessarily entering the gas station)?

(i) Since, both, the arrival rate and the processing rate are exponentially distributed, the operation of this gas station can be modeled by a Markovian queue. In particular, in Kendal's notation, this queue is an instance of the M/M/2/2 queue (the first '2' in this notation indicates the # of servers, and the second '2' the total buffering capacity of the station).

The State Transition Diagram (STD) of the CTMC that models the dynamics of this queue is as follows:




In the above STD, the number labelling each state indicates the # of customers in the station. $\lambda$ denotes the arrival rate of cars that need refueling and $\mu$ denotes the processing rate at each of the station pumps.

Obviously $\mu = \frac{1}{t_p} = \frac{1}{5}\ \text{min}^{-1} = 0.2\ \text{min}^{-1}$

(ii) To get $\lambda$, we observe from the problem data that the arrival process of cars that need refueling is also Poisson and its rate is $\lambda = 100 \times 0.1$ cars/hr $= 10$ cars/hr $=$

$$= \frac{10\ \text{cars/hr}}{60\ \text{min/hr}} = \frac{1}{6} \simeq 0.167\ \text{min}^{-1}$$

(iii) The Markovian queue developed in the previous two steps has finite buffering capacity, and as a result, the corresponding CTMC has a finite and fully connected STD. This implies the stability of the queue and the existence of a steady-state distribution $(\pi_0, \pi_1, \pi_2)$. This distribution can be obtained from the following equations:

$$\begin{cases} \pi_0 \lambda = \pi_1 \mu & \text{(flow balance at node 0)} \\ (\lambda+\mu)\pi_1 = \pi_0 \lambda + 2\mu \pi_2 & (\text{" " " } 1) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \pi_1 = \frac{\lambda}{\mu}\pi_0 \\ \pi_2 = \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 \pi_0 \end{cases}$$

Using the above results in $\sum_{i=0}^{2} \pi_i = 1$, we get:

$$\pi_0 \left[1 + \frac{5}{6} + \frac{1}{2}\left(\frac{5}{6}\right)^2\right] = 1 \Rightarrow \pi_0 = \frac{72}{157}$$

and $\pi_1 = \frac{5}{6} \times \frac{72}{157} = \frac{60}{157}$ ; $\pi_2 = \frac{1}{2}\left(\frac{5}{6}\right)^2 \frac{72}{157} = \frac{25}{157}$

(iv) $\lambda_{eff} = \lambda(1-\pi_2) = \frac{1}{6}\,\text{min}^{-1} \times \left(1 - \frac{25}{157}\right) = \frac{22}{157} = 0.14\,\text{min}^{-1}$

(v) The simplest way to answer this question is by noticing that the cars that will manage to enter this gas station will be served immediately. Therefore,

$CT = t_p = 5$ min

(vi) From (iv), (v) and Little's law: $WIP = \lambda_{eff} \times CT = 0.14\,\text{min}^{-1} \times 5\,\text{min} =$

$= 0.7$

(vii) In (iii) we argued that the car arrivals that will need refueling occur according to a Poisson process. But then PASTA (as explained in question #4 of this document) implies that the # of cars in the gas station that are observed by these arrivals follows the steady-state distribution of this statistic, and therefore, the corresponding average will be the same with that computed in item (vi) above, i.e., 0.7 cars.