ISYE 4803-REV: Advanced Manufacturing Systems

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Final Exam

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Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. What is the primary motivation behind the notion of *cellular* manufacturing? What are the typical criteria for the definition of the corresponding cells?

Cellular manufacturing is used in an effort to retain the operational efficiencies and the conveniencies of the product layout, when the different "products" (or parts) do not have adequate demand volume to justify a dedicated flowline for each of them, but various groups of parts have considerable similarity in their processing requirements to facilitate the sharing of a flowline. In such a case, the corresponding groups of parts are characterized as "part families", and their supporting flowline is the "**cell**" supporting the production needs of the family. More specifically, these needs are supported in a switching (or batching) mode, with the line being periodically reconfigured for the production of certain parts of the family.

2. Consider a workstation where the processed jobs are batches of N parts, where N is *uniformly* distributed between 5 and 10. The processing time of each part is distributed according to some general distribution with mean equal to 5 minutes and st. deviation equal to 2 minutes. Compute the mean and the st. deviation for the batch processing time.

Hint: Notice that the batch process time is a random variable $T_b = \sum_{i=1}^{N} T_i$, where N is the random number of parts in the batch and T_i are the processing times of these parts.

As indicated in the hint, the batch process time is a compound random variable. Using the corresponding theory for this type of r.v. presented in class, we have:

$$E[T_b] = E[N] \cdot [T_i] = 7.5 \times 5 \text{ min} = 37.5 \text{ min} \quad (1)$$

$$Var[T_b] = Var[T_i] \cdot E[N] + E[T_i]^2 \, Var[N] =$$
$$= 2^2 \text{ min}^2 \cdot 7.5 + 5^2 \text{ min}^2 \, Var[N] \quad (2)$$

Next we compute $Var[N]$:

$$Var[N] = E[N^2] - E[N]^2 \quad (3)$$

$$E[N^2] = \frac{1}{6}\left[5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2\right] = 59.167 \quad (4)$$

$$Var[N] = 59.167 - 7.5^2 = 2.917 \quad (5)$$

From (2) and (5) we get

$$Var[T_b] = \cancel{59.167} \; (2^2 \times 7.5 + 5^2 \cdot 2.917) \text{ min}^2 =$$
$$= 102.925 \text{ min}^2 \Rightarrow$$

$$St.dev[T_b] = 10.145 \text{ min}$$

A quick derivation of the formula for $Var[T_b]$ is as follows:

$$Var[T_b] = E[T_b^2] - E[T_b]^2 \quad (*)$$

$$E[T_b^2] = E_N[E[T_b^2 | N]] = E_N[Var[T_b | N] + E[T_b | N]^2] =$$
$$= E_N[N \, Var[T_i] + N^2 \cdot E[T_i]^2] = E[N] \, Var[T_i] + E[N^2] \, E[T_i]^2$$

Hence $Var[T_b] = E[N] \, Var[T_i] + E[N^2] \, E[T_i]^2 - (E[N] \cdot E[T_i])^2 =$
$$= E[N] \, Var[T_i] + E[T_i]^2 (E[N^2] - E[N]^2) = E[N] \, Var[T_i] + E[T_i]^2 \, Var[N$$

3. Explain why the SPT rule minimizes the average WIP in static, single-machine scheduling.

An intuitive argument is that by pushing jobs out of the system as fast as possible, SPT minimizes the average WIP over the makespan (which is constant for any non-idling schedule). More formally, let $[i]$, $i=1,\ldots,n$ denote the job in the i-th position of any nonidling schedule. Then, the average WIP for this schedule can be expressed as follows:

$$WIP = \frac{1}{\sum_{i=1}^{n} t_i} \sum_{i=1}^{n} (n-i+1)\, t_{[i]} =$$

$$= \frac{1}{\sum_{i=1}^{n} t_i} \left\{ n\, t_{[1]} + (n-1)\, t_{[2]} + \cdots + 2\, t_{[n-1]} + 1 \cdot t_{[n]} \right\}$$

It is clear from the above expression that the (average) WIP is minimized by matching the largest coefficient (n) in the sum in the brackets with the smallest proc. time, the second largest coefficient $(n-1)$ with the second smallest proc. time, etc. But this is exactly the SPT rule.

4. Consider the re-entrant line with three single-server workstations WS_i, $i = 1, 2, 3$, where each station is revisited once; i.e., the line supports the production of a single part that is processed first at station WS_1, then at station WS_2, then at station WS_3, and subsequently the part comes back at station WS_1 for another run through the same three stations. Hence, the corresponding process plan has six processing stages in total, with processing times τ_j, $j = 1, \ldots, 6$. Finally, suppose that parts arrive at workstation WS_1 with processing rate λ.

The *necessary and sufficient* condition for the stability of this line is the satisfaction of the following three inequalities:

$$\lambda(\tau_1 + \tau_4) < 1$$
$$\lambda(\tau_2 + \tau_5) < 1$$
$$\lambda(\tau_3 + \tau_6) < 1$$

(a) TRUE (b) FALSE

Please, justify your answer.

The above conditions are certainly necessary for the stability of the three workstations of this line (since each arriving job brings a total expected workload to each workstation WS_i, $i=1,2,3$, equal to the corresponding sum of τ_j's).

But in class we also gave an example that showed that for this type of re-entrant lines the above conditions might not be sufficient for the stability of the line, since the latter depends on the applied scheduling policy at each workstation. Characteristically we mentioned that in some re-entrant lines, we might have instability even if each workstation is run by a local FCFS rule and the above inequalities are also satisfied for each workstation. Some buffer-priority policies that guarantee the stability of the line as long as the above inequalities are satisfied, are the First-Buffer-First-Serve and Last-Buffer-First-Serve.

5. Consider a workstation with two identical servers and n jobs $J_1, \ldots, J_n$, waiting for processing at this workstation. The job processing times are t_i, $i = 1, \ldots, n$. Then, the makespan for the processing of these jobs is *minimized* by any *non-idling* schedule, i.e., any schedule that does not keep the workstation servers intentionally idle while there are jobs that are waiting for processing.

(a) YES (b) NO

Please, explain your answer.

Non-idleness guarantees a minimal makespan for static, single-workstation scheduling problems where the workstation has also a single server.
If you have more than one servers, a schedule that minimizes the makespan must also try to balance the workloads across the servers. This problem is NP-Hard, and therefore it does not admit a simple solution. But Longest Processing Time (LPT) is a typically used heuristic (for the reasons discussed in class).

Problem 1 (20 points): Parts arrive arrive at a certain single-server workstation according to a Poisson process with rate $\lambda = 3$ parts per hour. Each arriving part is classified as type 1 or 2, with corresponding probabilities 0.6 and 0.4. Parts of type 1 have a mean processing time $\tau_1 = 10$ minutes and parts of type 2 have a mean processing time $\tau_2 = 15$ minutes. Furthermore, during the processing of these parts, the workstation server experiences nondestructive outages with a mean repair time $m_r = 20$ minutes. These outages are generated while the server is in processing according to a Poisson process with a rate $\nu = 1$ outage per (working) hour.

i. (10 pts) Determine the stability of this workstation.

ii. (10 pts) Answer the question of part (i) above under the additional assumption that the arriving parts are processed according to the FCFS policy, and every time that the server must switch between part types, there is a set-up time $\tau_s = 5$ minutes.

(i) For this case,

$$\tau_e = \frac{\rho_1 \tau_1 + \rho_2 \tau_2}{A} \quad \text{where}$$

$$A = \frac{m_f}{m_f + m_r} = \frac{1\text{ hr}}{1\text{ hr} + \frac{1}{3}\text{ hr}} = \frac{3}{4} = 0.75$$

and

$$\tau_e = \frac{0.6 \times 10 + 0.4 \times 15}{0.75} = 16\text{ min}$$

Then

$$u = \lambda \tau_e = 3\text{ hr}^{-1} \times \frac{16}{60}\text{ hr} = 0.8 < 1 \Rightarrow \underline{\text{stable}}$$

(ii) Now we must also account for the presence of the set up times in the estimation of the mean effective proc. time. Let $\bar{\tau}_e$ denote this new quantity. We have:

$$\bar{\tau}_e = \tau_e + P_{switch}\, \tau_s$$

To estimate P_{switch} consider all the possible type pairs of consecutively arriving jobs and their probabilities

(1, 1) $0.6 \times 0.6 = 0.36$

switchings: (1, 2) $0.6 \times 0.4 = 0.24$; (2, 1) $0.4 \times 0.6 = 0.24$ $\rightarrow 0.48 = P_{switch}$

(2, 2) $0.4 \times 0.4 = 0.16$

Hence,

$$\bar{\tau}_e = 16 + 0.48 \times 5 = 18.4 \text{ min}$$

Finally

$$\bar{u} = \lambda \bar{\tau}_e = 3\, hr^{-1} \times \frac{18.4}{60}\, hr = 0.92 < 1 \Rightarrow \underline{\text{stable}}$$

Remark: The above computation assumes that there are no outages during the setups. If we assume otherwise, then we have:

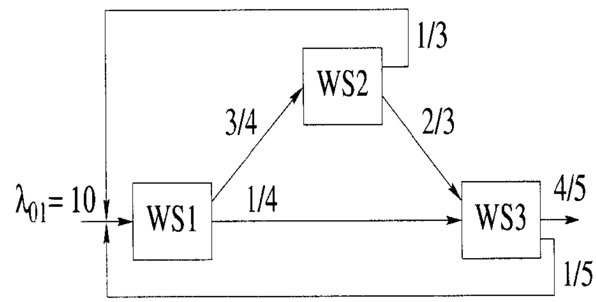
$$\bar{\bar{\tau}}_e = 16 + 0.48 \times \frac{5}{0.75} = 19.2 \text{ min}$$

and

$$\bar{\bar{u}} = \lambda \bar{\bar{\tau}}_e = 3\, hr^{-1} \times \frac{19.2}{60}\, hr = 0.96$$

(I took both approaches as correct during the grading).

Problem 2 (20 points): Consider a manufacturing cell of three workstations, where the arriving jobs circulate among these workstations as indicated in the following figure.



For this cell, please, do the following:

i. (5 pts) Compute the average arrival rate for each workstation.

ii. (5 pts) Assuming that each workstation has only one server, determine the mean processing time for each of these servers so that each workstation has a utilization level of 90%.

iii. (5 pts) What is the departure rate from this cell under the processing times that you computed in part (ii) above?

iv. (5 pts) Compute the expected number of visits at each of the three workstations for each job that goes through this cell.

(i) From the provided drawing we have:

$$\left.\begin{aligned} \lambda_1 &= 10 + \tfrac{1}{3}\lambda_2 + \tfrac{1}{5}\lambda_3 \\ \lambda_2 &= \tfrac{3}{4}\lambda_1 \\ \lambda_3 &= \tfrac{1}{4}\lambda_1 + \tfrac{2}{3}\lambda_2 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \lambda_1 &= 50/3 = 16.67 \\ \lambda_2 &= 50/4 = 12.5 \\ \lambda_3 &= 50/4 = 12.5 \end{aligned}\right.$$

(ii)

$$\left.\begin{aligned} u_1 &= \lambda_1 \tau_1 = 0.9 \Rightarrow \tau_1 = 0.9/16.67 = 0.054 \\ u_2 &= \lambda_2 \tau_2 = 0.9 \Rightarrow \tau_2 = 0.9/12.5 = 0.072 \\ u_3 &= \lambda_3 \tau_3 = 0.9 \Rightarrow \tau_3 = 0.9/12.5 = 0.072 \end{aligned}\right\}$$

The units of these time are the inverse of the units of λ_{01}.

(iii) Since all workstations are stable, conservation of mass requires that the total inflow rate is equal to the total outflow rate; hence, the departure rate from this cell is equal to 10.

(iv) As shown in class, the corresponding expected visits at each workstation are:

$$f_1 = \lambda_1 / \lambda_{01} = \frac{16.67}{10} = 1.667$$

$$f_2 = \lambda_2 / \lambda_{01} = \frac{12.5}{10} = 1.25$$

$$f_3 = \lambda_3 / \lambda_{01} = \frac{12.5}{10} = 1.25$$

Problem 3 (20 points): Mike's Auto Body Shop has five cars waiting to be repaired. The shop is quite small, so only one car can be repaired at a time. The number of days required to repair each car and the promised date (w.r.t. the current day) for each are given in the following table.

Cars	Repair Time (days)	Promised Date
1	3	5
2	2	6
3	1	9
4	4	11
5	5	8

Propose appropriate schedules for the repair of these five cars under the following two scenaria:

i. (10 pts) Mike has agreed to provide a rental car to each customer whose car is not repaired on time.

ii. (10 pts) Mike will give a voucher of $100 to each customer whose car is not repaired on time, to be used at her next visit at the repair shop.

I am answering parts (i) and (ii) in reverse order, since the second part is more straightforward. Hence, for part (ii) it is obvious that we want to minimize the number of tardy jobs, since each tardy job costs $100 (in future revenue). For this objective we need to apply Moore's algorithm:

EDD	1	2	5	3	4
t_i	3	2	5	1	4
C_i	3	5	10	11	15
D_i	5	6	8	9	11
T_i	0	0	2	2	4

first tardy job

$\longrightarrow$

	1	2	3	4	5
	3	2	1	4	5
	3	5	6	10	15
	5	6	9	11	8
	0	0	0	0	7

Hence, this is the optimal schedule!

For part (i), we need to minimize the total number of tardy days across all jobs, since each tardy day for a job implies the cost of a car rental for that particular day. Since the total # of job is fixed, this criterion is equivalent to minimizing the average tardiness. We know, however, that this problem is NP-hard, and therefore it is not easy (no simple dispatching rule) to identify the optimal sequence.

In the considered case, it can be checked that the schedule developed for part (ii) in the previous page results in 7 days of tardiness (all for job # 5). And this seems to be an optimal schedule, although I did not check exhaustively. Another schedule with the same # of tardy days is: < 1, 2, 3, 5, 4 >

The SPT and the EDD schedules result in 8 ~~tardy days~~ days of tardiness, while the Least Slack schedule is < 1, 5, 2, 4, 3 > and gives 13 days of tardiness!

Remark: For part (i), it is also interesting to notice that if Mike paid a rental for his clients for the entire period that they had their cars at the body shop (not just for the tardy days), then, the corresponding schedule should minimize the total (or average) lateness, since finishing a car before its due date would imply a gain for Mike that should be accounted to the total cost of the schedule (assuming that he had already arranged for car rentals specified by the designated due dates). Then, the optimal schedule is that of SPT.