ISYE 4803-C: Advanced Manufacturing Systems
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Midterm Exam II
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Name: SOLUTIONS

Answer the following questions **(8 points each)**:

1. In a stable flowline that can be modeled as a sequence of n $G/G/m$ queues according to the theory that was presented in class, the variability in the inter-arrival times of the workstation WS_{i+1}, as measured by the SCV of the corresponding distribution, is always higher than the variability in the inter-arrival times of workstation WS_i, for all $i = 1, 2, \ldots, n-1$.

(a) TRUE (b) FALSE

Explain your answer.

Remember that in the case of the G/G/1 queueing station, the SCV ~~[illegible]~~ of the inter-departure times, which essentially defines the variability in the inter-arrival times experienced at the downstream station, is given by

$$C_d^2 = (1-u^2)\, C_a^2 + u^2 C_e^2 \qquad (1)$$

where

- u = the station utilization
- C_a^2 = SCV of the inter-arrival times experienced by the station
- C_e^2 = SCV of the effective processing times at this station

From (1), it is easy to see that if u is pretty close to 1.0, and $C_e^2 \ll C_a^2$, then $C_d^2 < C_a^2$.

2. A manufacturing workstation that fits the assumptions of a $G/G/1$ queueing station currently is found to be unstable, i.e., unable to support the required production rate. Which (it may be more than one) of the following are reasonable options for addressing the faced problem, without compromising the posed throughput requirement?

i. Reduce the variability of the job processing times at the station.

ii. Add another machine at the station.

iii. Reduce the rate with which parts are fed to the station.

iv. Increase the availability of the station server.

Explain your answer.

We have that

$$u = \frac{r_a t_e}{k} = \frac{r_a \cdot t_s/a}{k} = \frac{r_a \cdot t_s}{k a} \quad (1)$$

where

- u = the station utilization
- r_a = the job arrival rate (that defines the required production rate)
- k = number of servers (currently 1)
- t_s = the mean "natural" proc. time (that does not account for the impact of the various disruptions)
- a = the server availability after accounting for the various disruptions.

From (1) we can see that we can effect a reduction of u either by increasing k or a. Hence, (ii) and (iv) are viable options

u could also be reduced by reducing r_a, but such reduction would compromise the target production rate. Hence, (iii) is not an option.

Finally (i) is not an option since (1) does not involve c_e.

3. For a manufacturing workstation that can be modeled as a $G/G/1$ queue, and experiences non-destructive preemptive outages of its server according to the model that was presented in class, explain how a reduction of the variability of the server downtime can lead to a reduction of the average WIP of the station.

This dependency is expressed by the following formulae:

$$C_e^2 = C_s^2 + (1+C_r^2)\,A(1-A)\frac{m_r}{t_s} \qquad (1)$$

$$CT = \frac{C_a^2 + C_e^2}{2}\,\frac{u}{1-u}\,t_e + t_e \qquad (2)$$

$$WIP = r_a \cdot CT \qquad (3)$$

The variability of the server downtime is expressed by C_r^2. Hence, a reduction of C_r^2 implies a reduction of C_e^2, through (1). But this last reduction implies a reduction of CT, through (2), which, in turn, implies a reduction of WIP, through (3).

4. What are the main reasons that lead to batch-based processing in contemporary manufacturing facilities?

a) In facilities where different part types are sharing a set of resources (as in cellular manufacturing) batching is used to control capacity losses due to the necessary setup time ~~every time~~ ~~that~~ for switching production among two part types. This is known as "serial" batching.

b) Capacity losses that lead to batching can also occur in the case of equipment with fairly long proc. times and large buffering capacity, like furnaces or fermentors. Since in this case all the batched units are processed in parallel, this case is known as "parallel" batching.

c) Batching can also result from the need to control material handling costs, especially in the case where the processed units are of a small size.

5. Consider a $G/G/1$ queueing station where arriving parts are processed in batches of k parts in a sequential manner, i.e., the k parts are processed on the station server one after the other. Assuming that the processing times of the parts in a batch are mutually independent, the total processing time of batch has

i. a smaller

ii. a larger

iii. the same

iv. an incomparable

amount of variability w.r.t. the variability in the part processing times.

In your response, consider that variability is measured by the SCV of the corresponding distributions, and provide a brief explanation of your answer.

As we demonstrated a number of times in class,

$$T_b = T_1 + T_2 + \dots + T_k$$

where r.v. T_b is the batch proc. time, and r.v.'s T_i, $i=1,\dots,k$ are the part proc. times.

Then $E[T_b] = k\,E[T_i]$, for any i, since T_i are assumed iid, and $V[T_b] = k\,V[T_i]$ (for the same reason)

So

$$SCV[T_b] = \frac{V[T_b]}{E[T_b]^2} = \frac{k\,V[T_i]}{k^2\,E[T_i]^2} = \frac{1}{k}\,SCV[T_i]$$

Since $k > 1$, the right answer is (i).

In class, we also remarked that this result is another manifestation of the "pooling" effect.

Problem 1 (30 points): Consider a stable single-server manufacturing station which constitutes the first station of a longer production line. Jobs are released to this station at a constant pace, and the server utilization is measured at 90% of its effective capacity. The server *nominal* (or *"natural"*) processing time (i.e., the processing time that does not include the experienced outages) is deterministically equal to 2 minutes, and the estimated availability is 85%. It is also known that the times between failures are exponentially distributed, while downtimes are normally distributed with mean 15 minutes and st. deviation 7.5 minutes. Your task is to compute the following:

i. (10 pts) The throughput of this station in the operational regime that is described above.

ii. (10 pts) The expected cycle time CT for a job going through this station.

iii. (10 pts) The variability that is induced by this station to the rest of the production line, as measured by the SCV of the inter-departure times.

We have:

- utilization $u = 0.9$
- mean natural proc. time $t_s = 2$ min
- $c_s = 0$ (since proc. times are deterministically equal to t_s, i.e., constant)
- Availability $A = 0.85$

Then,

(i) $u = r_a \frac{t_s}{A} \Rightarrow TH = r_a = \frac{uA}{t_s} = \frac{0.9 \cdot 0.85}{2 \text{ min}} = 0.3825 \text{ min}^{-1} = 22.95 \text{ hr}^{-1}$

(ii) $CT = \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t_e + t_e$ (1)

But

$c_a^2 = 0$ (2), since parts are released to this line at a constant pace and the considered station is the first station of the line.

$c_e^2 = c_s^2 + (1 + c_r^2) A (1-A) \frac{m_r}{t_s}$ (3)

$m_r = 15$ min (4)

$$C_r^2 = \frac{\sigma_r^2}{m_r^2} = \frac{7.5^2}{15^2} = 0.25 \quad (5)$$

from (3), (4), (5) and $C_s = 0$,

$$c_e^2 = (1+0.25) \cdot 0.85 \, (1-0.85) \frac{15}{2} = 1.195 \quad (6)$$

Also,

$$t_e = \frac{t_s}{A} = \frac{2 \text{ min}}{0.85} = 2.35 \text{ min} \quad (7)$$

From (1), (2), (6), (7) and $u = 0.9$,

$$CT = \frac{1.195}{2} \, \frac{0.9}{1-0.9} \, 2.35 + 2.35 = 14.99 \text{ min}$$

(iii) $C_d^2 = (1-u^2) C_a^2 + u^2 C_e^2 = (1-0.9^2) \cdot 0 + 0.9^2 \cdot 1.195 \simeq 0.968$

Problem 2 (30 points): Parts arrive for processing at a furnace according to a Poisson process with rate $\lambda = 30$ parts per hour. The processing time of these parts at the furnace is deterministically equal to 30 minutes. Answer the following questions:

i. (10 pts) What is the minimum batch size k that ensures a stable operation for this furnace?

ii. (10 pts) What is the part cycle time at this furnace when it is operated with the batch size that you determined in part (i)?

iii. (10 pts) What is the batch size that leads to the minimum cycle time at this furnace? What is this minimum cycle time?

From the discussion on this problem that was provided in class and the accompanying slides, we have the following:

(i) minimum batch size k is defined by the stability condition:
$k > r_a t = 30\ hr^{-1} \cdot \frac{1}{2}\ hr = 15$. Since the inequality is strict (for the corresponding utilization u to be strictly less than 1.0) and k must be an integer, we pick $k = 16$.

(ii) Then, $u = \frac{r_a t}{k} = \frac{30 \cdot 1/2}{16} = \frac{15}{16} = 0.9375$
and from the relevant formula discussed in class, we obtain:

$$CT = \frac{k-1}{2r_a} + \frac{c_a^2/k + c_e^2}{2} \frac{u}{1-u} t + t =$$

$$= \frac{15}{2 \cdot 30\,hr^{-1}} + \frac{1/16 + 0}{2} \frac{0.9375}{1-0.9375} \cdot 0.5\,hr + 0.5\,hr = 0.984\ hrs =$$

$$= 59.04\ min$$

In the above formula we have set
- $c_a^2 = 1$ since part arrivals is a Poisson process
- $c_e^2 = 0$ since proc. times are constant

(iii) Following the relevant analysis presented in class, and in the corresponding slide, in a first approximation we could pick

$$k^* \simeq r_a t\,(1 + c_e) = 30 \cdot 0.5\,(1+0) = 15$$

But such a selection is problematic since it leads to

$$u = \frac{r_a t}{k} = \frac{30 \cdot 0.5}{15} = 1,$$ and therefore to instability!

This is not that surprising, though, since the above formula results from taking the term $b = c_a^2/k$, that appears in the calculation of CT for any given k, as negligible with respect to c_e^2 (which is the other term in the fraction $\frac{c_a^2/k + c_e^2}{2}$). But in our case, $c_e^2 = 0$, and the above assumption does not hold!

Taking the more accurate formula $u^* = \frac{1}{1+\sqrt{b^* + c_e^2}}$ with

$$b^* = \frac{c_a^2}{u_m}\,\frac{1}{1+c_e} = \frac{c_a^2}{r_a t}\,\frac{1}{1+c_e} = \frac{1}{30 \cdot 0.5}\,\frac{1}{1+0} = \frac{1}{15},$$ we have

$$u^* = \frac{1}{1+\sqrt{1/15 + 0}} = 0.795,$$ and $$k^* \simeq \left\lceil \frac{u_m}{u^*} \right\rceil = \left\lceil \frac{r_a t}{u^*} \right\rceil = \left\lceil \frac{30 \cdot 0.5}{0.795} \right\rceil = \lceil 18.87 \rceil = 19$$

the new value for u^* has been adjusted to account for the rounding in the calculation of k^*

Finally, $$CT^* = \frac{19-1}{2 \cdot 30} + \frac{1/19}{2} \cdot \frac{0.789}{1-0.789} \cdot 0.5 + 0.5 = 0.849 \text{ hrs} = 50.95 \text{ min}$$

<u>Remark:</u> The above evaluation of u^* and k^* is still an approximation, since the system of equations $\frac{c_a^2 u^*}{u_m} = b^* \Leftrightarrow \frac{c_a^2 u^*}{r_a t} = b^*$ (1) and

$$u^* = \frac{1}{1+\sqrt{b^* + c_e^2}} = \frac{1}{1+\sqrt{b^*}} \quad (2)$$ has not been solved exactly for b^* and u^*.

The accuracy of the solution could be improved by taking the current value of u^* and plugging it in (1) to get a new value for b^*, plugging this value in (2) to get a new u^*, and iterating in this way until some convergence criterion is met