ISYE 4803-C: Advanced Manufacturing Systems
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Midterm Exam I
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Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. What type of workflows can be organized according to a U-shaped layout? What is the main feature of the U-shaped layout that makes it a competitive option for those workflows?

A U-shaped layout can be applied to workflows that have a linear structure; i.e., the entire workflow is organized into a sequence of stages and each stage is supported by one workstation.

A U-shaped layout for such a linear workflow
(i) enables the accommodation of the entire workflow in a more compact manner and in a way that fits better the shape of most available industrial buildings, and
(ii) enhances the proximity of the line workstations.

This second effect subsequently results in many other positive attributes of this layout, like:

(a) better supervision and control of the entire workflow
(b) easier employee sharing among the workstations
(c) a capability for centralized buffering of the WIP

etc.

2. Consider the activity that takes place at a typical emergency room. Which of the layouts discussed in class reflects the structure of the workflow that takes place in such an environment?

Explain your answer.

Arriving customers to such facilities are routed to different units for diagnosis purposes, like the X-ray and MRI units, and subsequently to some other units for treatment purposes, depending on the nature and the severity of their problem. So, the most natural layout for such an environment is the functional (also known as the process) layout.

Some (in fact, quite a few) of you suggested a fixed layout, since it provides the ability to stabilize the patient in certain ways. I think that the fixed layout characterizes the activity that might take place during particular stages of the entire process, e.g. during the application of certain treatments or when having an operation. But the overall flow of activity that takes place in an emergency room is best represented by a functional layout.

3. What is the basic definition of the *quality* of a given product or service?

The quality of a product or service is defined by the extent that it satisfies the customer needs.
For design purposes, these needs are expressed by a set of product specifications. So, a more "operational" definition of quality of a product unit is its adherence to the posed specs.

4. From the definition of the "pull" system provided in class, it can be inferred that these systems can be used only in a produce-to-order context.

(a) TRUE (b) FALSE

Explain your answer.

The "pull" system described in class is a control mechanism that regulates the release and the advancement of the various workpieces through the underlying flow line.
Whether these workpieces are produced in order to fill some standing orders or to build some anticipatory inventories is a separate issue, that has to do with the production planning policies of the company.

5. Jobs arrive at a certain station at batches of 10 units per batch, at a rate of 5 batches per hour. Each unit needs an average processing time of one minute, but 10% of these jobs need to go through some rework stage that takes an extra minute. Is this station stable?

(a) YES (b) NO

Explain your answer.

One way to answer this problem that is in line with the discussion of stability that was provided in class is as follows:

$$\text{Arrival rate } \lambda = 5 \frac{\text{batches}}{\text{hr}} \times 10 \frac{\text{units}}{\text{batch}} = 50 \frac{\text{units}}{\text{hr}} = \frac{50}{60} \frac{\text{units}}{\text{minutes}}$$

$$\text{Expected proc. time per unit } t_p = \begin{cases} 1 \text{ min} & \text{with prob. } 0.9 \\ 2 \text{ min} & \text{" " } 0.1 \end{cases}$$

Hence, $t_p = 1 \times 0.9 + 2 \times 0.1 = 1.1 \text{ min}$

So, $\lambda t_p = \frac{50}{60} \frac{\text{units}}{\text{min}} \times 1.1 \frac{\text{min}}{\text{unit}} = \frac{55}{60} < 1$

and the station is stable.

Problem 1 (20 points): We want to design a synchronous transfer line that will support an assembly process involving 5 tasks. The processing times and the precedence constraints for these tasks are as follows:

task	t_i (sec)	Imm. Pred
a	10	-
b	5	a
c	8	a
d	5	b, c
e	7	-

The required throughput is 100 parts per hour.

i. (5 pts) What is the line cycle time c that is implied by the above throughput requirement?

ii. (5 pts) What is a lower bound (or the "theoretical minimum", in the terminology of the posted excerpt on the ALB problem) to the number of workstations required for this assembly line, if it is to produce at the desired throughput?

iii. (5 pts) Explain that, in the considered case, the bound that you obtained in part (ii) is feasible (i.e., there exists a design that employs a number of workstations equal to that suggested by the bound).

iv. (5 pts) What is the maximum throughput requirement that can be supported for this assembly process? Express your answer in parts per hour.

Please, provide detailed and clear explanations for all your answers to the above questions.

(i) $c = 1/TH \Rightarrow c = \dfrac{1}{100 \frac{\text{parts}}{\text{hr}}} = \dfrac{1}{\frac{100 \text{ parts}}{3600 \text{ secs}}} = 36 \text{ sec/part}.$

(ii) $N = \left\lceil \dfrac{\sum_j t_j}{c} \right\rceil = \left\lceil \dfrac{35 \text{ sec}}{36 \text{ sec}} \right\rceil = 1$

~~(iii) Since, according to part (ii), we can fit all the tasks without violating the cycle time requirement, in one workstation, we can definitely execute all the tasks at this workstation in a way that respects the imposed precedence constraints,~~

(iii) According to the calculation in (ii), we can fit all the involved tasks in a single workstation without violating the cycle time requirement (i.e., without exceeding the specified cycle time). But then, we can definitely execute the tasks at this workstation in a way that respects the precedence constraints. So, this configuration is feasible.

(iv) From $TH = 1/c$, it is clear that the throughput of this assembly process will be maximized when the cycle time c is minimized. But from the task proc. times, it is evident that the minimum possible cycle time is 10 secs (for the station that will support task a. Hence,

$$\max TH = 1/10 \text{ sec} = 1/\tfrac{10}{3600} \text{ hr} = 360 \text{ hr}^{-1}.$$

Problem 2 (40 points): Consider a shoeshine shop consisting of two chairs, 1 and 2. An entering customer will first go to chair 1. When his work is completed in chair 1, he will proceed to chair 2, if that chair is empty, or he will wait in his current chair until chair 2 becomes empty. Customers arrive at this shop according to a Poisson process with rate λ but will enter the shop only if chair 1 is empty upon the time of their arrival. Processing times at each of the two chairs are exponentially distributed with respective rates μ_1 and μ_2.

Please, answer the following questions:

i. (10 pts) The exponential nature of the inter-arrival and the processing times for the considered shop enable the modeling of its operational dynamics by continuous-time Markov chain (CTMC). What is a pertinent definition of the state for this CTMC? Using your state definition, draw the state transition diagram of the CTMC. (*Hint:* Remember that a customer can be in chair 1 either getting service or being blocked there while waiting for chair 2.)

ii. (10 pts) Explain why the CTMC developed in item (i) above has a steady-state distribution for any values of λ, μ_1 and μ_2, and write down the equations that provide this distribution. (You do not need to solve these equations; just write them down.)

iii. (5 pts) What proportion of arriving customers enters the system?

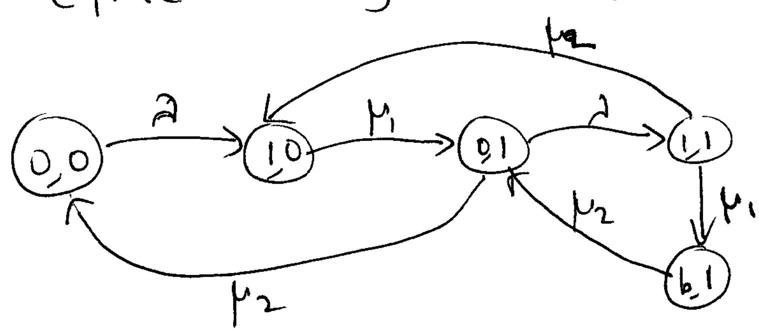
iv. (5 pts) What is the throughput of this shop?

v. (5 pts) What is the average number of customers in this shop?

vi. (5 pts) What is the average time that is spent in the shop by a customer who was able to enter it?

For items (iii) – (vi) above, express your answers in terms of the parameters λ, μ_1, μ_2 and the steady-state probabilities that are characterized in step (ii).

(i) Our state should be able to trace the status of each chair. So, it will have two components (i, j), with component i tracing the status of chair 1 and component j tracing the status of chair 2. More specifically, i can take values in the set $\{0, 1, b\}$, where 0 implies that chair 1 is empty, 1 implies that chair 1 has a customer who receives service, and b implies that chair 1 has a blocked customer, i.e., a customer who has completed service in that chair but cannot advance to chair 2 because it is occupied. On the other hand, j can take the values in the set $\{0, 1\}$, where 0 implies an empty chair 2 and 1 implies that this chair is occupied by a customer receiving service. With these definitions, it can be easily checked that the state transition diagram of the CTMC modeling this shop is as follows:



(ii) The CTMC of part (i) has a finite state space and the state transition diagram is fully connected. So, there exists a steady-state distribution that can be computed from the flow balance equations of all the process states but one, plus the normalizing equation that requests that the sum of all the state probabilities add up to one. One such system of equations is the following:

$$
\begin{cases}
P_{00}\lambda = P_{01}\mu_2 & \text{(flow balance at state (0 0))} \\
P_{10}\mu_1 = P_{00}\lambda + P_{11}\mu_2 & \text{(" " " " (1,0))} \\
P_{01}(\lambda+\mu_2) = P_{10}\mu_1 + P_{b1}\mu_2 & \text{(" " " " (0,1))} \\
P_{11}(\mu_1+\mu_2) = P_{01}\lambda & \text{(" " " " (1,1))} \\
P_{00} + P_{10} + P_{01} + P_{11} + P_{b1} = 1 &
\end{cases}
$$

(iii) $P_{00} + P_{01}$ (from PASTA)

(iv) $TH = \lambda_{eff} = \lambda(P_{00} + P_{01})$

(v) $WIP = P_{00}\cdot 0 + (P_{10} + P_{01})\cdot 1 + (P_{11} + P_{b1})\cdot 2$ (Def of expectation)

(vi) $CT = \dfrac{WIP}{TH} =$ (Little's law)

$$= \frac{P_{10} + P_{01} + 2(P_{11} + P_{b1})}{\lambda(P_{00} + P_{01})}$$