Answer the following questions (**8 points each**):

1. Provide two reasons that render important the "robustness" of the EOQ model w.r.t. deviations from the EOQ value.

1) We might need to deviate from the T^*, and therefore the Q^* value, in order to synchronize replenishments across many different products, as it happens in the case of inventory systems that operate according to the logic of the "Power-of-2-order-intervals" scheme that we discussed in class.

2) Sometimes, supplier constraints might force a deviation from T^* (and Q^*), like for instance, the cases where the supplier imposes a periodic replenishment scheme or a limit on the number of deliveries over a certain interval (e.g. per year).

3) Storage constraints might restrict the maximum order size, and therefore enforce a deviation from Q^*.

4) Also, to the extent that the different cost values as well as the annual demand rate, are just estimates of the respective quantities, the eventually computed EOQ value might not be the actual optimal value.

In all the above cases, the robustness of the EOQ model w.r.t. potential deviations from the actual EOQ value implies that the resulting increase of the total cost might not be very high w.r.t. its optimal value.

2. In a commoditized market, companies are competing primarily on the basis of . . .

cost leadership (i.e, price)

Commodities are pretty standardized products w.r.t. their functionality and quality attributes, and furthermore, they are provided in abundance.

3. Provide two ways in which location-related decisions can define competitive advantage w.r.t. the company's *responsiveness*.

1) Placing retail stores, distribution centers and sometimes even production facilities closer to the customer might enable a more immediate response to emerging demand patterns, like demand surges, etc.

2) In globalized operations, developing local presence to a certain geographical market can enable the better understanding of the local customers in terms of preferences, customs, and other factors that can have a significant impact upon the experienced demand.

3) For products with a substantial "technical" or "technological" base, proximity to the relevant centers of excellence will enable the company to stay abreast of all current developments and emerging trends.

4) For companies that utilize scarce or difficult to transport raw materials, proximity to the markets or the locations of extraction of these materials can secure an uninterrupted and responsive production flow.

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4. An inventory system is operated according to the "Order-upto-S" periodic review policy. If the demand that is experienced during a replenishment cycle is normally distributeed with mean 150 and st. deviation 10, compute the minimum safety stock that will guarantee a 90% probablity of no stock-outs during a replenishment cycle.

We want

$$P(X \leq S) = 0.9 \iff$$

$$\iff \Phi\left(\frac{S-\Theta}{\sigma}\right) = 0.9 \quad \text{(where } \Phi() \text{ is the cdf of st. normal)}$$

(from the provided tables)

$$\Rightarrow \frac{S-\Theta}{\sigma} = 1.29 \iff$$

$$\iff SS = S - \Theta = 1.29 \times \sigma = 1.29 \times 10 = 12.9 \simeq 13 \text{ units}$$

5. Any pair of items that are clustered in the same group by the Power-of-2-order-intervals heuristic may have optimal replenishment periods T^* that can differ by a factor upto

a. $\sqrt{2}$.

b. 2.

c. $2\sqrt{2}$.

d. none of the above.

Explain your answer.

According to the logic of this heuristic, for any positive integer k, the products that are assigned a replenishment period of 2^k have a T^* falling in the interval $(\sqrt{2}\,2^{k-1}, \sqrt{2}\,2^k)$. Obviously, the largest factor is defined by the extreme points of the above interval, i.e.,

$$\frac{\sqrt{2}\,2^k}{\sqrt{2}\,2^{k-1}} = 2.$$

Problem 1 (30 points): A local department store is about to place its order for one of the T-shirts that is going to sell in the next season. The purchasing price for this item is \$30 and the selling price will be \$50. Demand is expected to be uniformly distributed in the range of [400, 600] units. Finally, the company has the option of purchasing some insurance for this inventory, under which it will be able to dispose the remaining T-shirts at 50% of their purchasing price. The price of this insurance policy is \$400. Should the company purchase this insurance or not?

Basically, we should evaluate separattely each option, according to the logic that we presented in class, and consider eventually whether the expected cost reduction of the second option justifies (i.e., exceeds) the 400 \$ that we must pay for the insurance policy.

Option I (without insurance):

$C_{o_1} = 30$ \$/unit ; $C_s = 20$ \$/unit

$$G(Q_1^*) = \frac{C_s}{C_{o_1}+C_s} = \frac{20}{30+20} = \frac{20}{50} = 0.4 \Rightarrow$$

$$\Rightarrow Q_1^* = 400 + 0.4 \times 200 = 480$$

and

$$Y(Q_1^*) = 30\int_{400}^{480}(480-x)\frac{1}{200}\,dx + 20\int_{480}^{600}(x-480)\frac{1}{200}\,dx =$$

$$= \frac{30}{200}\left[480\times 80 - \frac{1}{2}(480^2-400^2)\right] +$$

$$+ \frac{20}{200}\left[\frac{1}{2}(600^2-480^2) - 480\times 120\right] =$$

$$= 480 + 720 = 1200.$$

Option II (with insurance)

$C_{o_2} = 15 \ \$/\text{unit}$ (thanks to the salvage value provided by the insurance)

C_s remains the same as in option I.

Hence, $$G(Q_2^*) = \frac{C_s}{C_{o_2}+C_s} = \frac{20}{15+20} = \frac{20}{35} \simeq 0.57 \Rightarrow$$

$$\Rightarrow Q_2^* = 400 + 0.57 \times 200 \approx 514$$

$$Y(Q_2^*) = \frac{15}{200}\int_{400}^{514}(514-x)\,dx + \frac{20}{200}\int_{514}^{600}(x-514)\,dx =$$

$$= \frac{15}{200}\left[514 \times 114 - \frac{1}{2}(514^2 - 400^2)\right] +$$

$$+ \frac{20}{200}\left[\frac{600^2-514^2}{2} - 514 \times 86\right] =$$

$$= 487.35 + 369.8 = 857.15$$

Hence, the expected cost reduction is

$$1200 - 857.15 = 342.85 < 400$$

and therefore, the insurance is too expensive for what it provides!

Problem 2 (30 points): A local distribution center receives its replenishments for one of its products every second Monday. This center estimates its holding cost for this item at \$0.05 per day, and the ordering cost at \$100 per order. Currently, it is Friday (late) afternoon, and the company needs to determine the order that it must place for the coming Monday. The demand for the next ten weeks has been estimated at the following levels:

$$50, 60, 55, 40, 45, 50, 60, 65, 70, 60$$

and the current inventory level with respect to the considered item is equal to 50 units.

If this distribution center does not perform any local deliveries during the weekend, determine the size of the order that must be placed by the company on the basis of the above information.

Since order are placed every two weeks, we can aggregate the above demand sequence so that it reflects the needs over two-week intervals, as follows:

110, 95, 95, 125, 130.

Furthermore, netting out the 50 units of the current inventory, we find that the actual needs (i.e., the "net" demand) for the considered two-week intervals are:

60, 95, 95, 125, 130.

(Remark 1: Notice that the above calculation is based on the additional info that there will be no further consumption of the current inventory of 50 units until the beginning of the next week; o/w, we should have adjusted this inventory to account for this additional consumption.)

To determine an optimal ordering plan, we need also to specify the ordering cost A and the holding cost h.

A is given to be \$100 per order.

h, in the context of this analysis, is the cost of carrying one unit of inventory from one ordering interval to the next. Since it costs 0.05 \$ for carrying one unit for one day and our ordering interval includes 14 days, we can set $h = 0.05 \times 14 = 0.7$ \$/ordering interval

(<u>Remark 2:</u> Notice that the above specification of h ignores the component of the holding cost that concerns the inventory that is carried over from the first to the second week within each ordering interval. But this cost is a "necessity"/implication of the restriction of our order placement to every second week, and it will not depend on the pattern with which we place our orders at the designated/'candidate' ordering points. Hence, it can be safely dropped from the calculations that will try to optimize this ordering pattern. On the other hand, it should be accounted in a thorough evaluation of the resulting total cost of the selected policy.)

Applying the W-W algorithm to the dynamic lot sizing problem formulated in the previous pages, we get:

$$Z_1^* = A = 100$$

$$Z_2^* = \min \begin{cases} A + hD_2 = 100 + 0.7\times 95 = 166.5 \Leftarrow \\ A + A = 100 + 100 = 200 \end{cases}$$

$$Z_3^* = \min \begin{cases} A + hD_2 + 2hD_3 = 100 + 0.7\times 95 + 2\times 0.7\times 95 = 299.5 \\ Z_1^* + A + hD_3 = 100 + 100 + 0.7\times 95 = 266.5 \\ Z_2^* + A = 166.5 + 100 = 266.5 \Leftarrow \end{cases}$$

$$Z_4^* = \min \begin{cases} Z_2^* + A + hD_4 = 166.5 + 100 + 0.7\times 125 = 354 \Leftarrow \\ Z_3^* + A = 266.5 + 100 = 366.5 \end{cases}$$

$$Z_5^* = \min \begin{cases} Z_2^* + A + hD_4 + 2hD_5 = 166.5 + 100 + 0.7\times 125 + 2\times 0.7\times 130 = \\ \qquad = 536 \\ Z_3^* + A + hD_5 = 266.5 + 100 + 0.7\times 130 = 457.5 \\ Z_4^* + A = 354 + 100 = 454 \Leftarrow \end{cases}$$

In the calculation of Z_4^* and Z_5^* above, we have used the "planning horizon" property of the W-W algorithm.

Also, from the above computation we can see that the optimal ordering plan orders for

- the demand of period 5 in period 5
- " " " " 3 & 4 " " 3
- " " " " 1 & 2 " " 1

$\Rightarrow$ Our order size for Monday should be $60 + 95 = \underline{155}$.