ISYE 3104: Manufacturing Systems
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Midterm Exam II
April 18, 2013

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. A manufacturing station fits the assumptions of a $G/G/1$ queueing station, and currently it is found to be unstable, i.e., it is not able to provide the desired target throughput. The station supervisor who had an one-day training in "lean manufacturing" contests that the problem of the station should not be considered as a "capacity" problem but as a "variability reduction" problem.

Do you agree with this position?

(A) YES [✓ (B) NO]

Explain your answer.

The stability condition for the $G/G/1$ queue is:

$$u = r_a t_p < 1$$

This condition involves only the mean of the proc. times at that station and the rate of the job arrivals, which is the inverse of the mean inter-arrival times. It has nothing to do with the second moments of the processing and inter-arrival times, that pertain to the notion of variability experienced at that station.

2. In an asynchronous transfer line, the variability in the arrival process of the downstream stations of the line (i.e., the stations that are closer to the end of the line) will be higher than the variability in the arrival process of the upstream stations (i.e. the stations that are closer to the beginning of the line).

(A) TRUE [✓ (B) FALSE]

Explain your answer.

Consider for instance a single-server station with $u \to 1$. Then, from the formula that determines C_d^2 for this station (i.e, the squared coefficient of variation of its inter-departure times), we know that C_d^2 is determined primarily by the variability of the proc. times at this station. Hence, the variability experienced by the subsequent station could be high or low depending on how variable are the proc. times in the considered station. And also this is irrespective of where this pair of stations lies in the entire line.

3. Recommend an appropriate layout among the four primary layouts discussed in class for organizing the workflow in each of the following facilities; for each case, please, provide also a brief justification for your recommendation.

i. your local gym

ii. the "front-end" of your local bank (i.e., the part of the bank operations that concerns the support of the customer service of the bank)

iii. a car wash facility

iv. a surgical operations room at a local hospital

(i) functional or process layout: The various types of equipment are organized into clusters and each such cluster is placed in one area of the entire facility. Since each patron follows his or her own program, it could have not been possible to apply any other layout in this case.

(ii) functional or process layout: For the same reason as above.

(iii) flowline or cellular layout: Typically, the steps followed at a car wash facility have a well-defined sequence, which suggests a flowline. Sometimes, however, some of these steps can be skipped, depending on the particular service that you have bought, hence the emergence of a cellular structure.

(iv) fixed product layout: Everything takes place while the patient lies on the surgical table.

4. As discussed in class, many contemporary flowlines are arranged spatially according to a U-shaped layout. What is the key element of this layout that underlies most of the advantages that it offers in the contemporary lean manufacturing paradigm?

Most of the advantages of this layout result from the proximity that it establishes among the various stations of the line.

5. We are designing a synchronous transfer line that involves a set of tasks with processing times ranging between 10 and 30 secs, and a target throughput requirement of 1000 parts over a 9 hr shift. Discuss whether the specified throughput is a feasible requirement for the considered line. Please, state clearly your answer to this question and the rationale leading to it.

The maximum cycle time, c, that would enable the line to deliver the target throughput is: $(9 \times 3600)/1000 = 32.4$ secs.

Since the maximum task proc time is 30 secs, the target throughput is feasible; we can just allocate some of these tasks by themselves at the corresponding workstations, if necessary.

Problem 1 (30 points): Consider a two-station asynchronous transfer line supporting the production of a certain item Y. The processing time of a single part of Y at the first station is distributed according to a general distribution with a mean of 1 min and st. dev of 0.25 minutes. On the other hand, the processing time of a single part of Y at the second station is distributed according to a general distribution with a mean of 1.25 min and st. dev of 0.5 minutes. Furthermore, Y is processed on this line in batches of 20 units per batch (this batch size defines, both, the processing and transfer batch size among the line stations and its ingress and egress points). Finally, batches are loaded on the line at a deterministic rate r_a. Please, answer the following questions:

i. (10 pts) What batch loading rate r_a will enable the line to produce at 85% of its production capacity?

ii. (10 pts) What is the expected cycle time for the parts moving through the line, if the line is operated at the rate that was computed in item (i) above?

iii. (10 pts) What is the average number of batches that are waiting in front of the second station?

First, let us compute $(t_{b_i}, c^2_{b_i})$, $i=1,2$ where t_{b_i} is the batch mean proc. time for station i, and $c^2_{b_i}$ is the corresponding SCV. We have:

$t_{b_1} = 20 \times 1 = 20$ min ; $t_{b_2} = 20 \times 1.25 = 25$ min

This computation was demonstrated in class when we discussed the desing of the 2-station ATL; see also at the next page.

$$^{*} \rightarrow c^2_{b_1} = \frac{c^2_{p_1}}{\beta} = \frac{0.25^2}{20} = 3.125 \times 10^{-3}$$

$$c^2_{b_2} = \frac{c^2_{p_2}}{\beta} = \frac{(0.5/1.25)^2}{20} = 8 \times 10^{-3}$$

Next, we answer the problem questions:

(i) We need $u = r_a \times t_{b_2} = 0.85 \Rightarrow r_a = 0.034\ \text{min}^{-1}$

In the above computation we have taken into consideration the fact that $t_{b_2} > t_{b_1}$.

(ii)

$$\left.\begin{aligned} CT_1 &= \frac{c_{a_1}^2 + c_{b_1}^2}{2}\,\frac{u_1}{1-u_1}\,t_{b_1} + t_{b_1} \\ u_1 &= 0.034 \times 20 = 0.68 \\ c_{a_1}^2 &= 0 \quad (\text{deterministic } \overset{\text{job}}{\text{release}} \text{ pattern}) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow CT_1 = \frac{0 + 3.125\times10^{-3}}{2}\,\frac{0.68}{1-0.68}\,20 + 20 = 20.0664 \text{ min}$$

$$\left.\begin{aligned} CT_2 &= \frac{c_{a_2}^2 + c_{b_2}^2}{2}\,\frac{u_2}{1-u_2}\,t_{b_2} + t_{b_2} \\ c_{a_2}^2 &= c_{d_1}^2 = u_{b_1}^2\,c_{b_1}^2 + (1-u_{b_1}^2)\,c_{a_1}^2 = 0.68^2 \cdot 3.125\times10^{-3} = 1.445\times10^{-3} \\ u_2 &= 0.85 \end{aligned}\right\}$$

$$\Rightarrow CT_2 = \frac{1.445\times10^{-3} + 8\times10^{-3}}{2}\,\frac{0.85}{1-0.85}\,25 + 25 = 25.669 \text{ min}$$

Finally,

$$CT = CT_1 + CT_2 = 45.7354 \text{ min}$$

(iii) $WIP_{q_2} = WIP_2 - u_2 = r_a\,CT_2 - u_2 = 0.034 \times 25.669 - 0.85 =$

$$\simeq 0.0227$$

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* Let X_b be a r.v. modeling the batch proc. time. Then:

$X_b = X_1 + X_2 + \dots + X_B$ where each X_i, $i=1,\dots,B$ is a r.v. modeling the proc. time of the i-th part in the batch. X_i's are considered iid.

Then,

$$c_b^2 = \frac{Var[X_b]}{E[X_b]^2} = \frac{Var[X_1] + Var[X_2] + \dots + Var[X_B]}{(E[X_1] + E[X_2] + \dots + E[X_B])^2} =$$

$$= \frac{B \cdot Var[X_1]}{(B\,E[X_1])^2} = \frac{1}{B}\,\frac{Var[X_1]}{E[X_1]^2} = \frac{1}{B}\,c_p^2$$

Problem 2 (30 points): We want to design a synchronous transfer line that will support an assembly process involving 5 tasks. The processing times and the precedence constraints for these tasks are as follows:

task	t_i (sec)	Imm. Pred
a	10	-
b	5	a
c	8	a
d	5	b, c
e	10	-

The required throughput is 100 parts per hour.

i. (5 pts) What is a lower bound to the minimum number of workstations required for this assembly line?

ii. (5 pts) Draw the precedence diagram that represents the precedence constraints among the tasks of this line.

iii. (10 pts) Use the heuristic of the Ranked Positional Weights to develop a design for this line.

iv. (5 pts) Compute the utilizations of the different stations in the design that you developed in step (iii).

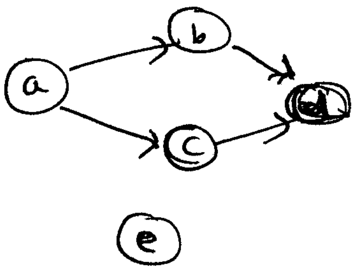
v. (5 pts) Consider a variation of the method of the Ranked Positional Weights (RPW) where instead of ordering the tasks in decreasing order of their positional weights, you order them in decreasing order of their (not necessarily immediate) successor sets. When you develop this task list, then you operate as in the case of the RPW method. Is this alternative heuristic a valid approach for the considered problem of synchronous transfer line design? In other words, will the designs obtained through this new heuristic respect the problem constraints? Please, explain your answer.

(i) We know that for the considered system, the cycle time c is the inverse of the target throughput TH.

Hence, $c = \frac{1}{TH} = \frac{1}{100 \text{ hr}^{-1}} = \frac{3600 \text{ sec}}{100} = 36 \text{ sec}$

Since the total workload, across all tasks, is 38 secs, we need at least $N = \left\lceil \frac{38}{36} \right\rceil = \lceil 1.0556 \rceil = 2$ stations.

(ii)



(iii)

Task	a	b	c	d	e
PW	28	10	13	5	10

Task ordering: a c b e d

Using the above list and a cycle time of C = 36 secs, we get the following design:

WS1	WS2
a, c, b, d	e
~~36~~ ~~26~~ ~~18~~ ~~13~~ 8	~~36~~ 26

Another, more balanced design would be:

WS1	WS2
a, c, b	e, d
~~36~~ ~~26~~ 18	~~36~~ ~~26~~ 21

(IV) The WS utilizations for these two designs are:

WS	1	2
Design 1	$\frac{36-8}{36} = 0.78$	$\frac{36-26}{36} = 0.28$
Design 2	$\frac{36-18}{36} = 0.5$	$\frac{36-21}{36} = 0.42$

(V) Consider two tasks a and b such that b is a successor of a. Then, it is easy to see that all the successors of b are also successors of a. Let Suc(a), Suc(b) denote the respective sets containing the successor tasks for tasks a and b. Then, from the above remark we have:

$$Suc(b) \subset Suc(a)$$

and the inclusion is strict since Suc(a) contains b which is not in Suc(b). Hence,

$$|Suc(a)| > |Suc(b)|$$

and a will appear before b in the suggested list. But this implies that the task precedence constraints will be observed by this list.