IE3104: Manufacturing Systems
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Homework #4 Solutions

1. A production facility assembles inexpensive telepohnes on a production line. The assembly requires 15 tasks with precedence relationships and activity times as shown in Figure 8-16. The activity times appear next to the node numbers in the network.

a. Determine positional weights for each of the activities.

As discussed in class and in the book excerpt on the ALB problem that is posted at the library electronic reserves, the positional weight (PW) of a task is equal to the sum of the processing times of all of its successor tasks plus its own processing time. Furthermore, the successor tasks of any given task i are all those tasks j that can be reached from i through a directed path in the depicted precedence graph.

By applying the above definitions on the graph of Figure 8-16, we obtain the following PWs for the various tasks included in that graph:

Task	Positional Weight
1	100
2	94
3	46
4	43
5	37
6	47
7	23
8	20
9	29
10	16
11	16
12	20
13	12
14	8
15	5

Also, as demonstrated in class and further discussed in the provided excerpt, a systematic and efficient way to obtain the above values is by starting from the leaf nodes of the precedence graph and working backwards towards its source nodes.

b. For a cycle time of 30 units, what is the minimum number of stations that could be achieved? Find the $C = 30$ balance obtained using the ranked positional weight technique.

First notice that $\frac{100}{30} = 3.33$, which implies that at least 4 stations are needed to support this cycle time.

To obtain a design for this assembly line through the heuristic of Ranked PWs, we first need to order tasks in decreasing PWs. The corresponding list is as follows:

$$1-2-6-3-4-5-9-7-8-12-10-11-13-14-15.$$

Next, working with the above list as demonstrated in class (see also the provided excerpt), we obtain the following allocation of tasks into stations:

	1	2	3	4
Task	1, 2, 6, 3	4, 5, 7, 9	8, 12, 10, 11	13, 14, 15
Idle Time	2	1	2	15

c. Is there a solution with the same number of stations you found in part (b) but with a lower cycle time? In particular, what appears to be the minimum cycle time that gives a balance with the same number of stations you found in part (b)?

Notice that the total idle time per cycle across the entire line for any solution with 4 stations will be $4 \times 30 - 100 = 20$ (i.e., the total available processing time across the four stations minus the sum of the task processing times that defines the total amount of work that is performed during one cycle). If this idleness could be perfectly balanced across the 4 line stations, and each station got an idle time of 5 time units, then, we would be able to reduce the line cycle time down to 25 time units, and that would be the smallest possible cycle time attained by a 4-station line. But it turns out that there is no task allocation that can provide the aforementioned balancing and still respect all the precedence constraints. On the other hand, for C=26, we do find the following 4-station balance.

	1	2	3	4
Task	1, 2, 3, 5	4, 6, 7, 8	9, 11, 12	10, 13, 14, 15
Idle Time	0	0	1	3

2. Consider the assembly line balancing problem represented by the newtowrk in Figure 8-17. The performnace times are shown above the nodes.

a. Determine a balance for C=15.

Task	Positional Weight
1	68
2	53
3	43
4	33
5	30
6	31
7	25
8	21
9	12
10	14
11	8

By arranging tasks in non-increasing order, we get the following ranking:

Task	Positional Weight	Processing time
1	68	12
2	53	5
3	43	7
4	33	8
6	31	6
5	30	5
7	25	4
8	21	3
9	12	4
10	14	6
11	8	8

Considering the ratio $\frac{68}{15} = 4.53$, we can see that for $C = 15$, a lower bound to the necessary number of stations is 5. The application of the RPW heuristic to the above data provides the following design.

	1	2	3	4	5	6
Task	1	$2, 3$	$4, 6$	$5, 7, 8$	$9, 10$	8
Idle Time	3	3	1	3	5	7

b. Determine a balance for C=20.

Now, $frac6820 = 3.4$, and the RPW gives the following result:

	1	2	3	4
Task	$1, 2$	$3, 4$	$6, 5, 7, 8$	$9, 10, 11$
Idle Time	3	5	2	2

3. Consider a production line consisting of two single machine stations. The processing times at these two stations are generally distributed with a mean value of $t_0 = 11$ min and a coefficient of variation $c_0 = 0.5$. Parts are released in this line at a deterministic pace with a part inter-release time of 12 min.

Answer the following questions:

i. Argue that the above operational regime is stable.

In the considered regime, the utilization for each of the two stations is $\frac{11}{12} = 0.9167 < 1$. Hence, the line is stable.

ii. What is the line throughput in the considered operational regime?

For a stable system, the average production rate at every station will be equal to the line feeding rate. Therefore, $TH = \frac{1}{12}$ min^{-1}

iii. What is the expected cycle time for a part going trough entire line?

$$
\begin{aligned}
CT_{q_1} =& \frac{c_{a_1}^2 + c_{0_1}^2}{2} \frac{u_1}{1-u_1} t_{0_1} \\
=& \frac{0 + 0.5^2}{2} * \frac{0.9167}{0.0833} * 11 \\
=& 15.13 \\
CT_1 =& CT_{q_1} + t_{0_1} = 15.13 + 11 \\
=& 26.13 \\
c_{a_2}^2 =& c_{0_1}^2 u_1^2 + c_{a_1}^2 (1 - u_1^2) \\
=& 0.5^2 * 0.9167^2 + 0 \\
=& 0.21 \\
CT_{q_2} =& \frac{c_{a_2}^2 + c_{0_2}^2}{2} \frac{u_2}{1-u_2} t_{0_2} \\
=& \frac{0.21 + 0.25}{2} * \frac{0.9167}{0.0833} * 11 \\
=& 27.84 \\
CT_2 =& CT_{q_2} + t_{0_2} = 27.84 + 11 = 38.84 \\
\text{Expected cycle time} =& CT_1 + CT_2 = 64.97 \text{ min}
\end{aligned}
$$

iv. What is the average WIP waiting to be processed at each of the two stations? Which station has the highest WIP level? Why?

From Little's law,

$$
\begin{aligned}
WIP_{q_1} = TH \cdot CT_{q_1} = \frac{1}{12} * 15.13 = 1.26 \\
WIP_{q_2} = TH \cdot CT_{q_2} = \frac{1}{12} * 27.84 = 2.32
\end{aligned}
$$

The second station has the highest WIP level. It is easy to see from the formulae that are involved in the relevant calculations that the larger value for WIP_{q_2} is due to the variability in the arrival process of station 2 that is induced by the variable processing times of station 1.

v. What is the maximum production rate that can be supported by this line in a stable mode? Please, state your answer in parts per hour.

We know that for a stable operation at each station we need that

$$
u_i = r_a \cdot t_{0_i} < 1, \quad i = 1, 2
$$

Therefore,

$$
r_a < \min_{i=1,2} \{ \frac{1}{t_{0_i}} \} = \frac{1}{11} \text{ min}^{-1}
$$

vi. Can we effect a decrease of the part cycle time by 10 percent by reducing the variability at the two line stations? Please justify your answer.

Let the new coefficient of variation for the processing times at the two line stations be $c_0' = x \cdot c_0$ for some $x \in [0, 1]$. Then, based on the computations for part (iii) of this problem, we need to solve the following problem:

$$\frac{0 + (0.5x)^2}{2} * \frac{0.9167}{0.0833} * 11 + 11 + \frac{(0.5x)^2 * 0.9167^2 + (0.5x)^2}{2} * \frac{0.9167}{0.0833} * 11 + 11 = 0.9 * 64.97$$

A solution to the above equation is $x = 0.9212$, and therefore, the answer is 'yes'.

4. Consider a single-server processing station where jobs arrive according to a Possion process with rate of 5 jobs per hour. The arrving stream of jobs is classified into two types, A and B, and it is processed on a first-come-first-serve basis. An arriving part will be type A with probability p=0.3. Processing times for part type A are normally distributed with a mean of 10 min and a st.dev. of 2 min. Processing times for part type B are normally distributed with a mean of 12 min and a st.dev.of 4 min.

i. Show that the station operation is stable.

Let X_P be a random variable denoting the processing time for any part going into service. Then,

$$E[X_P] = 0.3 * 10 + 0.7 * 12 = 11.4$$

and the utilization of the station server is

$$u = \frac{5}{60} * 11.4 = 0.95 < 1$$

Therefore, the station operation is stable.

ii. What is the expected cycle time for jobs going through the station?

To answer this question we need also to compute the SCV (squared coefficient of variation) of r.v. X_P. This computation is similar to the corresponding computation for the example with the rework cycle that was presented in class. We have:

$$\begin{aligned} Var[X_A^2] &= E[X_A^2] - E[X_A]^2 \Longrightarrow \\ 4 &= E[X_A^2] - 100 \Longrightarrow \\ E[X_A^2] &= 104 \end{aligned}$$

$$
\begin{aligned}
Var[X_B^2] =& E[X_B^2] - E[X_B]^2 \Longrightarrow \\
16 =& E[X_B^2] - 144 \Longrightarrow \\
E[X_B^2] &= 160
\end{aligned}
$$

$$
\begin{aligned}
E[X_P^2] =& p * E[X_A^2] + (1-p)[X_B^2] \\
=& 0.3 * 104 + 0.7 * 160 \\
=& 143.2
\end{aligned}
$$

$$
\begin{aligned}
Var[X_P] =& E[X_P^2] - E[X_P]^2 \\
&= 143.2 - 11.4^2 \\
&= 13.24
\end{aligned}
$$

$$
\begin{aligned}
c_p^2 =& \frac{Var[X_P]}{E[X_P]^2} \\
=& \frac{13.24}{11.4^2} \\
=& 0.1019
\end{aligned}
$$

Hence,

$$
\begin{aligned}
CT =& \frac{c_a^2 + c_p^2}{2} \frac{u}{1-u} E[X_P] + E[X_P] \\
=& \frac{1 + 0.1019}{2} * \frac{0.95}{1 - 0.95} 11.4 + 11.4 \\
=& 130.7358 \text{ min}
\end{aligned}
$$

iii. Determine the condition that must be satisfied by the type defining probability p in order to guarantee the stability of the line. Can you provide a natural interpretation for this condition?

From the relevant stability condition, we get

$$
\frac{5}{60}[10p + 12(1-p)] < 1.0 \iff p > 0
$$

A natural interpretation of this condition is that the station will be stable as long as some of the arriving parts are of type A. Indeed, it is easy to see that if all arriving parts were type B, then the station utilization would be equal to 1. But due to the randomness in the inter-arrival and processing times of this station, a utilization of 1 is not attainable. On the other hand, the presence of some part A units in the product mix will provide the slack that is needed in order to stabilize the system (since these parts have an expected processing time less than the mean inter-arrival time).

5. Consider a workstation that produces a final product by fastening together two major sub-assemblies. Jobs arriving at this workstation consist of kits containing one unit from each of the two sub-assemblies, and if both parts are in good order, the fastening operation can be performed at an average time of t=2min. However, each of the two parts in a kit can also be defective, with corresponding probabilities $p_1 = 0.3$, and $p_2 = 0.2$. A defective part must go through some additional rework that occurs locally and requires an exponentially distributed time; the corresponding processing rates are $r_1 = 0.2\,\text{min}^{-1}$, and $r_2 = 0.1\,\text{min}^{-1}$. Part failures are independent from each other, and in the case that both parts in a kit are defective, the necessary reworks take place simultaneoulsy. Use the above information in order to determine the effective processing capacity of this station. Express your result in product units per hour.

As indicated in the following table, the kits that are processed by this station can be classified into four cases. Using the information given above, you can get the probability of each case happened, and expected processing time as follows.

Case	Probability	Expected Processing Time (min)
No defects	$0.56 = (1 - p_1)(1 - p_2)$	2
Part 1 defective	$0.24 = (p_1)(1 - p_2)$	$2 + \frac{1}{0.2} = 7$
Part 2 defective	$0.14 = (1 - p_1)(p_2)$	$2 + \frac{1}{0.1} = 12$
Both parts defective	$0.06 = p_1 \cdot p_2$	$2 + 11.67 = 13.67$

The computation of the expected processing time for each of the first three cases is obvious, when recognizing that the mean of the exponential distribution with rate r is equal to $1/r$. The expected processing time for the last case follows from the following computation:

$$\begin{aligned}
\frac{1}{r_1 + r_2} + \frac{r_1}{r_1 + r_2}\frac{1}{r_2} + \frac{r_2}{r_1 + r_2}\frac{1}{r_1} &= \frac{1}{r_1 + r_2}(1 + \frac{r_1}{r_2} + \frac{r_2}{r_1}) \\
&= \frac{1}{0.3}(1 + \frac{0.2}{0.1} + \frac{0.1}{0.2}) \\
&= 11.67
\end{aligned}$$

In the above expression, the term $\frac{1}{r_1+r_2}$ is the expected time until the first of the two parts is fixed. Indeed, this time is the expected time for the minimum of two exponentially distributed r.v's, and as I mentioned in class, this minimum is a r.v. that is exponentially distributed with rate $r_1 + r_2$.

The terms $\frac{r_1}{r_1+r_2}\frac{1}{r_2} + \frac{r_2}{r_1+r_2}\frac{1}{r_1}$ express the expected time that is necessary to complete the fixing of the second part, after the first part has been fixed. To understand this expression, notice that if part 1 was fixed first, then we need to complete the fixing of part 2. But since the fixing times are exponentially distributed, the memoryless property of this distribution implies that the remaining fixing time for part 2 is distributed in exactly the same way as its entire fixing time. Hence, the expected remaining fixing time for part 2 will be $1/r_2$. Similarly, if part 2 is fixed first, the expected remaining fixing time for part 1 will be $1/r_1$. Finally, as I mentioned in class, the probability that part 1 is fixed first is given by $\frac{r_1}{r_1+r_2}$.

Combining all the previous results, it is obvious that the expected processing time for any given kit is

$$\sum_{i=1}^{4} p_i * E[T_i] = 5.3002$$

For stability, we need $u = r * 5.3 < 1$ which implies $r < 0.1887 \text{ min}^{-1} = 11.32 \text{ hr}^{-1}$